Stochastic Programming Model for Elective Surgery Planning: An Effect of Emergency Surgery

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Abstract: This paper introduces a stochastic programming model for a hospital with two surgery types: elective and emergency surgeries. We propose a model that decides the number of the elective surgeries per day according to a scheme that makes best use of the operating rooms. Specifically, we model when the demand capacity for emergency surgery in the operating room of one day is uncertain. We created multiple surgery times, performed random sampling, and conducted numerical experiments. In the results, emergency surgery changed the allocation of elective surgery. In this paper, we report on the proposed model and numerical results, and discuss these and the future research prospects.

1 INTRODUCTION

Operating room management is one of the critical factors in the management of hospitals. It is important both for achieving high patient treatment quality and suppressing hospital costs. From the viewpoint of the treatment quality, a long waiting time for patients caused by a delay relative to the scheduled end time of a previous surgery is a serious problem. From an economic point of view, about 60% of hospital income comes from surgery (Jacson et al., 2002). In contrast, about 40% of expenditure comes from surgery (Marcro et al., 1995). Therefore, hospitals are creating operating room schedules to improve the usage rate of operating rooms and reduce the cost of surgery.

Surgery can be broadly divided into two types: elective surgery and emergency surgery. Elective surgery refers to surgery that is scheduled in advance. Emergency surgery is not scheduled through discussions between the patient and the doctor. Rather, emergency surgery becomes necessary suddenly due to a life-threatening condition in the patient. Emergency surgery treatment must be done on the same day. Many hospitals use the same operating room for both elective surgery and emergency surgery. At Erasmus University Medical Center Rotterdam, the operating room dedicated to emergency surgery was closed, so both types of surgery are being performed in the same operating room (van Veen-Berkx et al., 2016). The closure was due to a published study that reported that performing the two different types of surgery in the same operating room is more efficient. However, in the case of performing elective surgery and emergency surgery in the same operating room, operating room management needs to take into account the uncertainty of emergency surgery. This is because not considering the uncertainty of emergency surgery in operating room management would result in poor service, such as delays in elective surgery. Therefore, additional costs may be incurred under this arrangement, including overtime operating room usage, labor costs, and hospitalization costs if carried forward.

The flow until the schedule is set starts with the surgeon and patient deciding the date of the elective surgery by mutual agreement. This surgery date will also depend on the available reserved capacity for elective surgery on each day. Since when emergency surgery will occur is uncertain, in order to manage elective surgery efficiently, the manager must aim at the creation of a robust surgery planning schedule that considers the timing uncertainty associated with emergency surgery.

In elective surgery planning, costs associated with overuse of the operating room and costs associated with performing elective surgery should be minimized. However, few studies have considered the uncertainty of emergency surgery (Cardoen et al., 2010, Zhu et al., 2019). If uncertainty is not taken into account, service degradation such as delayed procedures can occur, which incurs associated additional costs.

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Therefore, surgery planning carries forward overtime use of the operating room and elective surgery. It is considered necessary to minimize the costs incurred by this. Gerchak et al. (1996) proposed a dynamic programming model for pre-scheduling patients with single-period elective surgery. Lamiri et al. (2006, 2009) suggested considering the uncertainty of emergency surgery demand and proposed optimization models and algorithms for surgery planning. Their optimization models use the time of emergency surgery as a random variable generated for each surgery according to an exponential distribution. We propose elective surgery plans with multiple planning periods. However, it is unclear whether the best approach is to follow Lamiri et al. (2006, 2009) and generate a probabilistic emergency surgery time and create a deterministic schedule. Akiyama et al. (2021) proposed a probabilistic planning model for scheduling surgery. Comparing the propose model and the Lamiri model, it can be seen that the proposed model is superior to the Lamiri model because the amounts of overtime are shorter.

In this study, we propose a stochastic programming model for elective surgery planning. The effectiveness of the proposed model was verified by considering the uncertainty of duration of emergency surgery, as well as the allocation of elective surgery. An emergency surgery scenario created using Monte Carlo sampling was used to analyze how elective surgery was assigned. Our results were that the surgery was carried over if the surgical capacity for a later period was sufficient. The question of whether the commonly used one-week planning period is correct was also considered.

The structure of the remainder of the manuscript is as follows. Section 2 shows the proposed formulation of elective surgery planning. Section 3 describes the problem setting used for the numerical analysis. Section 4 presents and discusses the results. Section 5 summarizes the paper and describes future research.

2 THE MODEL

In this section, we introduce proposed formulation of elective surgery planning under uncertainty in emergency surgery.

2.1 Elective Surgery Planning

The elective surgery planning determines the number of elective surgeries per day using an uncertain demand model for emergency surgery. In operating room scheduling, it is necessary to schedule all surgeries performed during a given planning period. In this study, the capacity of all operating rooms subject to operating room scheduling is available for surgery. Thus, the total daily capacity of operating rooms is the same as the time per day that they are open. The cost carried forward and total capacity are considered here. We consider overtime, defined as the amount of time beyond the intended closing time, of the operating room, but not the penalties for low operating room usage.

Notation

Index Sets

T: The set of days for scheduling.

I: The set of elective surgery index values.

S: The set of emergency surgery scenarios.

Parameters

p_i: time needed for performing elective surgery i, which is assumed to be a given constant.

B_i: earliest period for performing elective surgery i.

CE_{it}: cost ratio of performing elective case i within day t.

T_t: total available regular capacity of all ORs on day t.

CO_t: cost ratio per unit of overtime on day t.

W_t^s: capacity needed for emergency surgery on day t.

Variables

O_t: expected operating room overtime on day t.

Y_{it}: probability of allocated elective surgery i on day t.

X_{it}^s: 1 if elective surgery i is to be scheduled in scenario s on day t; 0 otherwise.

Formulation

$$\text{Minimize} \quad \sum_{i \in I} \sum_{t=B_i}^{|T|+1} CE_{it} Y_{it} + \sum_{t \in T} CO_t O_t \tag{1}$$

subject to

$$O_t = \mathbf{E_s}[(W_t^s + \sum_{i \in I} p_i X_{it} - T_t)], \forall t \in T, \tag{2}$$

$$\sum_{t=1}^{|T|+1} X_{it} = 1, \forall i \in I, \tag{3}$$

$$X_{it}^s = \{0,1\}, \forall i \in I, \forall t \in |T|+1, \forall s \in S, \tag{4}$$

$$Y_{it} = \mathbf{E_s}[X_{it}^s], \forall i \in I, \forall t \in T, \forall s \in S, \tag{5}$$

$$Y_{it} \geq 0, \forall i \in I, \forall t \in T. \tag{6}$$

In the above formulation, the objective function (1) minimizes carry-forward costs and overtime acceptance costs. Constraint (2) defines overtime from the closing time of operating room on day t. Constraint (3) guarantees that each elective surgery is allocated only once. Constraint (4) is a binary constraint . Constraint (5) is an equation to find the allocated distribution of elective surgery. Constraint (6) represents a non-negativity constraint on Y_{it}.

3 ANALYTICAL METHOD

In this study, we conducted numerical experiments on how elective surgery was assigned for 100 emergency surgery scenarios. The procedure was as follows. Based on the surgery data of an actual hospital (Akiyama et al., 2021), we assume five types of elective surgery and one type of emergency surgery. We selected the top five types with the largest sample size from about 9,000 actual surgery data. The time of each of the five elective surgeries follows a lognormal distribution, and ten random numbers are generated for each type of elective surgery to create a total of 50 data. The operation time of emergency surgery is also lognormal. It is assumed that a normal distribution is followed, and 10 units are created. In the verification of this study, two operating rooms are assumed. The open time per room is 480 minutes. In addition, elective surgery is performed every day. It is assumed that 4 elective surgeries and one emergency surgery occur every day. The scheduling period is 6 to 8 days. In the case of 6 days, a total of 24 elective surgeries will occur and 6 emergency surgeries will occur. For these 24 elective surgeries, further random sampling is performed from the created 50 data.

The above data are applied to the proposed model to determine the optimal number of elective surgery assignments for each day, considering the uncertainty of emergency surgery. We generate emergency surgery scenarios by Monte Carlo sampling. The probability of the occurrence of a scenario follows a uniform distribution. We assume that a schedule for emergency surgery will always be accepted. The total available capacity of the operating rooms here is about 960 minutes, which is the capacity of the two operating rooms.

Table 1 shows the expected value and standard deviation of the surgery duration, and the end time, which is the time when the surgery needs to be completed.

Table 1: Duration for elective and emergency surgery [min].

Surgery ID	A	B	C	D	E	Emergency
Expected value	154	177	293	215	235	91
Standard deviation	95	77	78	87	64	79

4 RESULTS AND DISCUSSION

In this study, we performed and analyzed multiple numerical experiments. Here, we mention the analysis of one of these in detail as an example.

Table 2 shows the elective surgery requests. As described in Section 3, we assumed in the created surgical data that randomly occurring elective surgery was performed. The values in the table are the duration of each elective surgery in minutes and the letters in parentheses refer to the surgery ID.

Figures 1–3 show which surgery was carried over and how frequently among the 100 emergency surgery scenarios for each day. In the figures 1–3, the vertical axis gives the execution rate, which is the ratio of which day the elective surgery was assigned among 100 emergency surgery scenarios.The horizontal axis shows the “elective surgery number”. The depth axis shows the day. Regarding the surgery numbers on the horizontal axis, theses represent the dates, and A to D indicate the four elective surgeries scheduled for that day. The longer the period, the more often elective surgery will be carried over. The results show that this carrying over occurred when there was unused capacity of the operating room in a later period. This implies that as a result of extending the planning period, the number of carry-overs has increased because the number of days with unused surgical capacity has increased. On the other hand, however, it would be represented that even if the planning period is extended, overtime will increase if the num-

Table 2: Elective surgery requests: surgery duration (surgery ID) [min].

Surgery	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
1(Elective)	332(C)	126(B)	383(C)	353(D)	212(B)	33(A)
2(Elective)	309(E)	209(B)	263(B)	212(A)	256(B)	286(B)
3(Elective)	214(C)	269(C)	269(E)	231(B)	277(D)	59(B)
4(Elective)	71(A)	253(E)	253(A)	335(C)	189(A)	404(C)

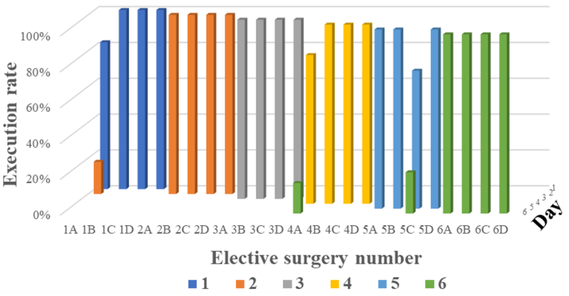

Figure 1: Execution rate for 6 days.

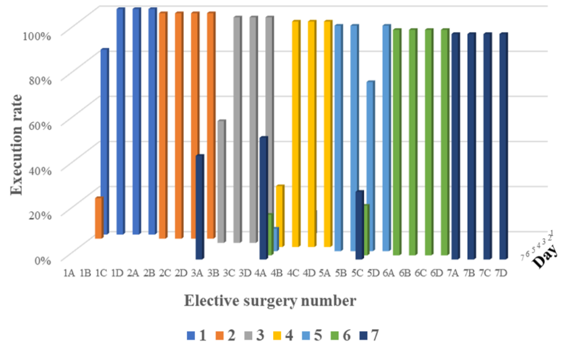

Figure 2: Execution rate for 7 days.

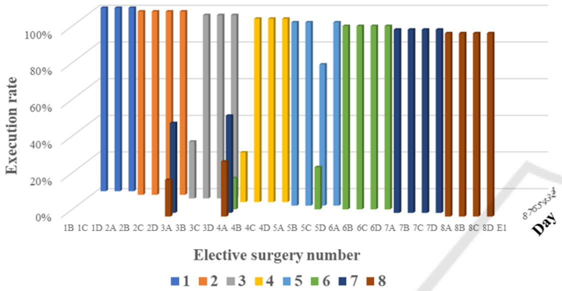

Figure 3: Execution rate for 8 days.

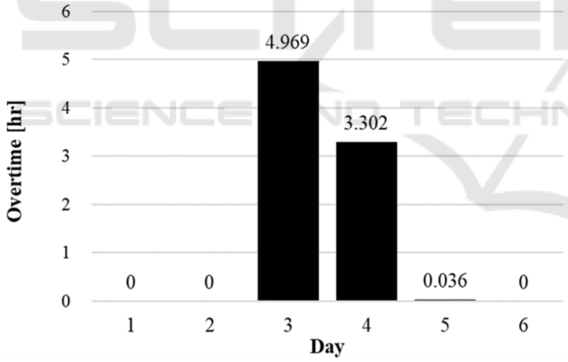

Figure 4: Overtime for 6 days.

ber of days exceeding the surgical capacity increases. Overtime will also increase if surgical capacity is exceeded on the last day of the planning period.

Figures 4–6 show overtime as a function of day. In these figures, the vertical axis shows the time and the horizontal axis shows the day. In this result, we found that the length of overtime decreases in proportion to the length of the period. Operating room capacity for later days must be available in order to accept elective surgery.

Most hospitals create operating room schedules for a period of one week, but the results here indicate that a longer planning period reduces overtime. This raises the question of whether the planning period currently in use is appropriate. However, although there is a possibility that overtime can be shortened by extending the planning period, there is also a overtime that it will increase. Three cases were carried forward. We found that the surgery with the longest operation time was selected among these three cases during the scheduling period, and that these surgeries could be significantly delayed if performed on time.

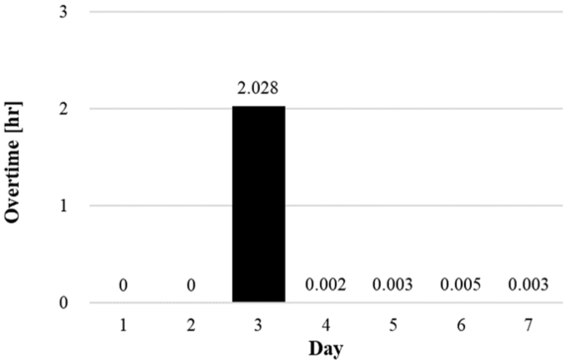

Figure 5: Overtime for 7 days.

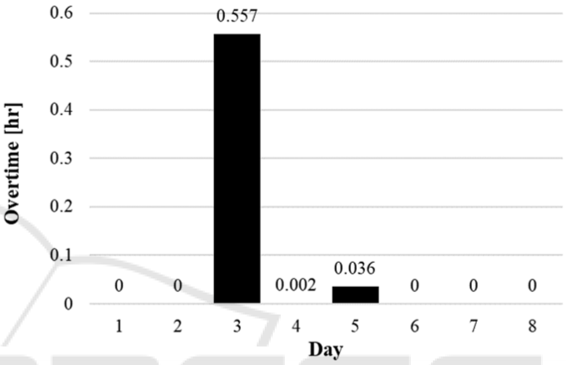

Figure 6: Overtime for 8 days.

5 CONCLUDING REMARKS

In this study, we developed an elective surgery planning model for establishing a surgical plan for multi-period elective surgery considering the uncertainty of the occurrence of emergency surgery. The proposed model considers the uncertainty of emergency surgery by scenario. Considering the uncertainty duration of emergency surgery for the elective surgery scheduling, the number of elective surgery allocations scheduled during the period was determined. In the results, the elective surgery was assigned to another day to avoid overtime in the operating room due to elective surgery during the scheduling period. Currently, most hospitals create operating room schedules for one week. However the results here indicate that a longer planning period reduces overtime. Therefore, there is room to consider whether one week is appropriate for the length of the planning period for scheduling.

As potential future research, the uncertainty in the number of emergency surgeries should be taken into consideration. In addition, the operating room capacity of each operating room should be separately considered, rather than just the total operating room capacity. Therefore, we would like to extend the proposed model to a model of multiple operating rooms. We would also like to further examine whether the appropriate planning period is one week.

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REFERENCES

Akiyama, R., Ito, M., and Takashima, R., Hoshino, K., Hashimoto, M., Yamamoto, H., Fujii, H. (2021). Stochastic programming model for elective surgery planning considering emergency surgery, *Informs Healthcare 2021*.

Cardoen, B., Demeulemeester, E., Beliën, J., (2010). Operating room planning and scheduling: A literature review. *European Journal of Operational Research*, 201(3), 921–932.

Gerchak, Y., Gupta, D., and Henig, M. (1996). Reservation planning for elective surgery under uncertain demand for emergency surgery. *Management Science*, 42 (3), 321–334.

Jackson, R. (2002). The bushiness of surgery. *Health Management Technology,* 23(7), 20–22.

Lamiri, M., Xie, X., Dolgui, A., and Grimaud, F. (2006). A stochastic model for operating room planning with elective and emergency demand for surgery. *European Journal of Operational Research*, 185, 1026–1037.

Lamiri, M., Grimaud F., and Xie, X. (2009). Optimization methods for a stochastic surgery planning problem. *Int. J. Production Economics*, 120, 400–410.

Macario, A., Vitez, T. S., Dunn, B., and McDonald, T. (1995). Where are the costs in perioperative care?: Analysis of hospital costs and charges for inpatient surgical care. *Anesthesiology*, 83(6), 1138–1144.

van Veen-Berkx, E., Elkhuizen, S.G., Kuijper, B., and Kazemier, G. (2016). Dedicated operating room for emergency surgery generates more utilization, less overtime, and less cancellations. *The American Journal of Surgery*, 211(1), 122–128.

Zhu, S., Fan, W., Yang, S., Pei, J., and Pardalos, P. M. (2019). Operating room planning and surgical case scheduling: a review of literature. *Journal of Combinatorial Optimization*, 37, 757–805.