A Multi-stage Integer Linear Programming Problem for Personnel and Patient Scheduling for a Therapy Centre

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Abstract: In this paper, we propose a multi-stage integer linear programming problem to solve the scheduling of speech-language pathologists involved in conventional treatments as well as in augmentative and alternative communication therapies. In order to reduce the complexity of this problem, we suggest a hierarchical approach that breaks the problem into three sub-problems: patient selection for augmentative and alternative communication therapies, therapists' shift assignment, and routing optimization of home-based rehabilitation services. The resulting models were tested on data collected in a physiotherapy centre in *Acireale* (*Catania, Italy*), using AMPL optimization package and Genetic Algorithm implemented in Matlab. From the results of the case study, the model ensures to maximize the number of patients eligible for augmentative and alternative communication therapies, to assign sustainable therapist schedule, and to optimize the home therapy routing.

1 INTRODUCTION

The nurse scheduling problem is one of the main issues in healthcare system. It aims to assign a number of nurses to a number of shifts in order to satisfy hospital demand (Van den Bergh et al., 2013). Scheduling in healthcare is often planned manually and it is time-consuming. Therefore, the automatic assignment of shifts can lead to improvements in efficiency, personnel and patient satisfaction, and staff workload.

This research aims at presenting the multi-stage integer linear programming problem for determining the proper scheduling of speech-language pathologists. The model is tested on a case study conducted in a speech therapy centre in *Acireale* (*Catania, Italy*), where qualified therapists are involved in conventional treatments as well as in Augmentative Alternative Communication (for simplicity, AAC) therapies. In addition, all the therapists, apart from the therapy sessions at the centre, have to provide rehabilitation services in patients' homes. In this paper, we deal with the following problems encountered by the personnel and patients of the speech therapy centre:

1. selection of patients for AAC therapy according to their priority levels;
2. assignment of therapists' shifts (for conventional and AAC therapies) to optimize their workload;
3. planning of the routes/reducing time for the delivery of home-based therapy.

Therefore, we propose a hierarchical approach that breaks the problem into three sub-problems: the selection of the maximum number of patients for AAC therapies, the achievement of an equitable distribution of therapists' workload, and decrease in the transfer time of therapists, who have to change location during the working day, respectively (Ogulata et al., 2008). The first sub-problem is to determine the maximum number of patients benefiting from AAC therapies, with respect to predetermined staff capacity. Because of the extremely high demand of this service, selection of patients must be done before the scheduling. In this step, the selection of patients for AAC treatment among the total number of patients is made according to their priority decided by doctors. In the second sub-problem, we minimize the penalty of each soft constraint and, in particular, we find the optimal assignment of therapists' shifts on weekdays from Monday to Saturday. Thus, the important goal of this study is to provide a balanced schedule for every speech therapist. The AAC therapy is scheduled throughout a week, in order to replicate what really happens in the centre we analyzed. Finally, in the third sub-problem, we determine the minimum cost

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on the journeys made by therapists so as to minimize their travel time. The remainder of this paper is organized as follows. Section 2 is devoted to the related literature. Section 3 presents the proposed methodology which encompasses parameters, model formulation, and solution method. Section 4 describes the case study and the numerical experiment results. Finally, Section 5 draws the conclusions and illustrates further research issues.

2 RELATED WORK

The problem addressed in this paper relates to a model described in (Ogulata et al., 2008), where a hierarchical mathematical programming problem is proposed to generate weekly staff scheduling. The model is decomposed into three hierarchical stages: the selection of patients, the assignment of patients to the staff, and the scheduling of patients throughout a day.

In the past years, several approaches were proposed, such as tabu search (Burke et al., 2006), genetic algorithms (Aickelin and Dowsland, 2004), learning methodologies (Aickelin et al., 2007; Euchi et al., 2020), scatter search (Burke et al., 2010), and mathematical programming (Ogulata and Erol, 2003; Wolfe and Young, 1965; Warner, 1976). The approach used is to penalize the violation of the constraints in the objective function. In real applications, it is often difficult to find feasible solutions. In (Legrain et al., 2015), the authors study the scheduling process for two types of nursing teams, regular teams from care units and the float team that covers for shortages in the hospital. The corresponding multi-objective model and heuristics are presented. In (El Adoly et al., 2018), the authors study a nurse scheduling problem to minimize the overall hospital cost, and maximize nurses' preferences, while taking into consideration the governmental rules and hospital standards. The mathematical model presented is based on multi-commodity network flow model. In (Berrada et al., 1996; Blöchliger, 2004), a multi-objective approach is introduced that differentiates between hard and soft constraints. In (Valouxis and Housos, 2000), a non-optimal solution is generated by solving the mathematical model, and a post-optimization phase using tabu search is performed. In (Wong et al., 2014), the authors solve the nurse scheduling problem in a Hong Kong emergency department with a two-phase heuristic implemented in Excel. In (Shao et al., 2014), the authors present an algorithm for supporting weekly planning of therapists. In particular, it allows one to match patient demand with therapist skills while minimizing treatment, travel, administrative and mileage reimbursement costs. Solutions are found with a parallel Greedy Randomized Adaptive Search Procedure (GRASP) that exploits a novel decomposition scheme and employs a number of benefit measures that explicitly address the trade-off between feasibility and solution quality.

This paper is builds on the work of (Ogulata et al., 2008), but with the following extensions: 1) in our model all the patients receive basic treatments at the centre and some of them are eligible for the AAC therapy program; 2) only some therapists in the centre are qualified to deliver AAC therapies; 3) AAC qualified therapists may also deliver conventional treatments; 4) some patients (AAC and not) receive home-based rehabilitation services.

3 PROPOSED METHODOLOGY

This section presents the assumption of the model and the formulation. The assumptions of the model are defined as below:

- the number of patients eligible to start the AAC program is known and fixed;
- the number of therapists in the speech centre is known and constant;
- the velocity of the vehicles used for delivering home-based therapy is constant, and the traffic conditions are not taken into consideration.

The overall problem was broken down into three hierarchical sub-problems, since it was rather difficult to solve the entire problem within an acceptable time for even small size problem instances (Ogulata and Erol, 2003). The first sub-problem, called "AAC patient selection" aims to get the list of patients whose AAC therapy will be scheduled for the following weeks. These patients receive special therapies only from qualified AAC therapists, while continuing with conventional treatments delivered by the other therapists. The second stage called "Shift assignment" aims to get the weekly shifts for both AAC and basic therapists. Lastly, the third stage called "Travelling therapist problem" aims to get the best route of therapists for delivering home-based sessions during a working day. Mathematical programming models corresponding to each stage are explained in detail in the following subsections.

3.1 Problem I: Augmentative Alternative Communication Patient Selection

The purpose of this stage is to select patients that will be scheduled for the following weeks from the candidate list, considering therapists' capacity and priority of patients. The first step of the process is then to determine the maximum number of patients benefiting from the AAC therapy, with respect to the predetermined staff capacity. Moreover, patients may have different priority levels. This difference must be included in an efficient scheduling plan. Priority of patients are categorized into three levels as high, normal, and low according to specialized doctors. In addition, AAC therapy sessions are longer than conventional ones; hence, it is important to balance the distribution of patients among therapists.

Indices and Parameters

- P: number of patients;
- p: patients index;
- w_p: priority level of patients;
- t_p^b: basic treatment time of pth patient;
- t_p^{AAC}: AAC treatment time of pth patient;
- H: total weekly hours;
- T^{AAC}: total weekly hours of AAC sessions.

Decision Variables. Decision variables at this stage are defined as followed;

$$x_p = \begin{cases} 1 & \text{if } p\text{th patient is selected,} \\ 0 & \text{otherwise.} \end{cases}$$

Objective Function Problem I. In the objective function (1), total number of selected patients is maximized considering priority factor of patients.

$$\max \sum_{p=1}^{P} w_p x_p \tag{1}$$

Subject to:

$$\sum_{p=1}^{P} x_p (t_p^b + t_p^{AAC}) \leq H; \tag{2}$$

$$t_p^b x_p \leq 1.5, \quad \forall p \in P; \tag{3}$$

$$\sum_{p=1}^{P} t_p^{AAC} x_p \leq T^{AAC}; \tag{4}$$

Constraint (2) ensures that the total sum of the therapy times for all patients must not exceed the total time available in a working week. Inequality (3) expresses that each patient does an hour and a half weekly each basic treatment. Finally, inequality (4) establishes that the sum over all the time for all AAC patients is less or equal than the total hours devote to AAC sessions per week. We remark that when patients complete the AAC program, it is necessary to update the list of eligible ones, and a new optimal selection is performed.

3.2 Problem II: Shift Assignment

In this section, we present the second sub-problem in which we focus on the planning of the shifts. The assignment of shifts is based on the schedules and the availability of the therapy centre to which we are referring. In particular, we differentiate the shifts in the following way: the morning shift, the afternoon shift and the shift for AAC therapy, during the working week from Monday to Saturday, excluding Saturday afternoon. The shift assignment is going to be the same and is repeated for each week. Some patients change as the weeks change but the number of patients in the first sub-problem can be catered for every week. The aim of this sub-problem is to minimize the sum of all the deviations of the soft constraints, multiplied each by an appropriate weight.

Indices

- $C = \{1, \ldots, c\}$ set of therapists working for the AAC program.
- $T = \{c+1, \ldots, t\}$ set of shift workers.
- $I = C \cup T = \{1, \ldots i, \ldots, t\}$ set of the total number of therapists in the centre.
- $L = \{1, 2, 3\}$ set of shifts, with the typical element of the set denoted by l, where
  - $l = 1 = M$: morning shift;
  - $l = 2 = A$: afternoon shift;
  - $l = 3 = AAC$: AAC shift.
- $J = \{1, \ldots, j, \ldots, 6\}$ set of working days. The days will be identified as follows:
  - $j = 1$: Monday;
  - $j = 2$: Tuesday;
  - $j = 3$: Wednesday;
  - $j = 4$: Thursday;
  - $j = 5$: Friday;
  - $j = 6$: Saturday.
- S: set of soft constraints;
- W_s: weight parameter $\forall s \in S$ assigned to each violation of soft constraints.

Decision Variables

$$X_{ijl} = \begin{cases} 1 & \text{if the therapist } i \text{ is assigned to the shift } l \\ & \text{on the day } j, \\ 0 & \text{otherwise.} \end{cases}$$

Hard Constraints

$$\sum_{i \in C} X_{ij3} = 2, \quad j = 2,4; \tag{5}$$

$$\sum_{i \in I} X_{i22} \geq 3; \tag{6}$$

$$\sum_{i \in I} X_{i51} \geq 1; \tag{7}$$

$$\sum_{l \in L} X_{ijl} = 1, \quad \forall i \in C,\, j \in J; \tag{8}$$

$$X_{ij1} + X_{ij2} = 1, \quad \forall i \in T,\, j \in J; \tag{9}$$

$$X_{i62} = 0, \quad \forall i \in I; \tag{10}$$

$$X_{ij2} = 0, \quad \forall i \in C,\, j = 2,4; \tag{11}$$

$$X_{ij3} = 0, \quad \forall i \in T, \forall j \in J \tag{12}$$

$$X_{ij3} = 0, \quad \forall i \in C,\, j = 1,3,5; \tag{13}$$

$$6 \sum_{j \in J} X_{ij3} \geq T^{AAC}, \quad \forall i \in C; \tag{14}$$

Constraint (5),(6) and (7) ensure that two therapists are required for AAC shifts, three therapists are required for Tuesday afternoons and one for Friday mornings, respectively. Constraints (8) and (9) state that each therapist only has to do one shift a day. Constraint (10) specifies that the centre is closed on Saturday afternoon. Constraints (11) and (13) state that each AAC therapist has not to do afternoon shift on Tuesday and Thursday, and has not to do AAC shift on Monday, Wednesday and Friday. Constraint (12) ensures that shift workers have not to do AAC shift. Finally, inequality (14) establishes that the therapists have to do at least T^{AAC} hours per week. We emphasize that we set the constraints according to the specific centre under consideration. They can be modified as needed and adapted to other situations.

Soft Constraint. Now, we present the soft constraints and introduce the variables that take into account the deviations of the constraints from their predetermined goals. These variables will then be minimized in the objective function in order to obtain the best possible solution, trying to reduce the deviations from these constraints. We denoted by $d^+_{si} \geq 0$, $d^+_{sij} \geq 0$ and $d^+_{sij} \geq 0$, $d^-_{si} \leq 0$ the positive and the negative deviations, respectively, associated to the soft constraint $s \in S$, $i \in I$ and $j \in J$.

The soft constraints are the following:

$$6 \sum_{j \in J} (X_{ij2} + X_{ij2}) + 6 \sum_{j \in J} X_{ij3} + d^+_{1i} \geq 36, \quad \forall i \in C; \tag{15}$$

$$6 \sum_{j \in J} X_{ij3} - d^-_{2i} \leq 18, \quad \forall i \in C; \tag{16}$$

$$6 \sum_{j \in J} (X_{ij2} + X_{ij1}) + d^+_{3i} \geq 36, \quad \forall i \in T; \tag{17}$$

$$X_{ij3} + X_{i(j+1)1} - (d^+_{4ij} + d^-_{4ij}) = 1, \quad \forall i \in C,\, j \in J \tag{18}$$

$$X_{ij2} + X_{i(j+1)2} - (d^+_{5ij} + d^-_{5ij}) = 1, \quad \forall i \in T,\, j \in J \tag{19}$$

$$X_{ij1} + X_{i(j+1)1} - (d^+_{6ij} + d^-_{6ij}) = 1, \quad \forall i \in T,\, j \in J \tag{20}$$

Constraint (15) and (16) establish that it is preferable that the therapists do at least thirty six hours a week. Constraint (18) states that is preferable that the therapists do no more than eighteen hours per week of AAC sessions. Finally, equalities (17), (19) and (20) ensure that is preferable that they do not have two consecutive mornings or afternoons.

Objective Function Problem II. The overall objective function to be minimized is given by the sum of all the deviations of the soft constraints described above, each multiplied by an appropriate weight, chosen on the basis of the importance of the violated constraint.

$$\begin{aligned} \min \Big(& W_1 \sum_{i \in C} d^+_{1i} + W_2 \sum_{i \in C} -d^-_{2i} + W_3 \sum_{i \in T} d^+_{3i} \\ & + W_4 \sum_{i \in C} \sum_{j \in J} (d^+_{4ij} - d^-_{4ij}) + W_5 \sum_{i \in T} \sum_{j \in J} (d^+_{5ij} - d^-_{5ij}) \\ & + W_6 \sum_{i \in T} \sum_{j \in J} (d^+_{6ij} - d^-_{6ij}) \Big); \end{aligned} \tag{21}$$

The minimization of the objective function, subject to the constraints already described, guarantees a solution that satisfies all the hard constraints and violates the soft constraints as little as possible.

3.3 Problem III: Travelling Therapist Problem

In this section, we optimize the routing from one location to another one during the working day. The problem can be defined as an asymmetric multiple Traveling Salesman Problem with Time Windows (mTSPTW) (Bektas, 2006), and additional constraints, such as an upper bounded variable of the number of therapists, and the maximum traveling time or distance of each therapist. We also include time window at each location. Usually, the mTSP is specified as an integer programming formulation.

Sets and Parameters

- $G = (V, E)$;
- $V = \{v_1, \dots v_h, \dots, v_k, \dots, v_n\}$ set of vertices;
- $E = \{(v_h, v_k)\}$ set of edges, which satisfy the symmetric property;
- $I = \{1, \dots i, \dots, t\}$ set of the total number of therapists in the centre, where i is the general one;
- TW indicates the time window. It is important to remark the role of this parameter as each therapist takes at least 2 hours for home-based therapies (considering transfer time and therapy session), before leaving for a new destination. Morevoer, the daily working hours are limited.
- $\hat{c}_{hk}$, where $\hat{c}_{hk} = c_{hk} + c_k^{TW}$ is the total cost considered;
- c_{hk} ordinary cost (distance or duration) associated with E. The costs could be symmetric if $c_{hk} = c_{kh}$, $\forall (v_h, v_k) \in E$ and asymmetric otherwise;
- c_k^{TW} cost of the time window TW, where every therapist has to do the therapy in each location, which takes about 2 hours;
- $\bar{I}$ upper bound of the therapist i, namely, the actual number of therapist used, i.e. the number of available therapists;
- c_i cost of the involvement of a therapist $i \in I$, i.e a fixed cost aiming to minimize their number;
- D maximum length of any tour in the solution.

Decision Variables

$$y_{hki} = \begin{cases} 1 & \text{if therapist } i \text{ chooses the edge}(v_h, v_k), \\ 0 & \text{otherwise.} \end{cases}$$

Objective Function Stage III

$$\min \sum_{h=0}^{n} \sum_{k=0}^{n} \hat{c}_{hk} \sum_{i=1}^{t} y_{hki} + t \cdot c_i \tag{22}$$

Subject to

$$\sum_{h=0}^{n} \sum_{i=1}^{t} y_{hki} = 1, \quad \forall k = 1, \dots, n, \tag{23}$$

$$\sum_{k=0}^{n} \sum_{i=1}^{t} y_{hki} = 1, \quad \forall h = 1, \dots, n, \tag{24}$$

$$\sum_{h=1}^{n} \sum_{i=1}^{t} y_{1ki} = t, \quad \forall k = 1, \dots, n, \tag{25}$$

$$\sum_{k=1}^{n} \sum_{i=1}^{t} y_{h1i} = t, \quad \forall h = 1, \dots, n, \tag{26}$$

$$\sum_{h=1}^{n} \sum_{k=1}^{n} c_{hk} \cdot y_{hki} \leq D, \quad \forall i \in I \tag{27}$$

$$+ \text{sub tour elimination constraints} \tag{28}$$

The objective function (22) represents the minimization of the cost of the journey, where $\hat{c}_{hk}$ is expressed as a weight on each edge, based on the distance or the cost of the journey, and c_i is the cost of involvement of the therapist i. Constraints (23) and (24) state that in each node v_h only one edge enters and exits $\forall h, k = 1, \dots n$. Constraints (25) and (26) are the usual assignment constraints for the starting and the ending point, using the binary variable. Constraints (27) ensures that the tour length of each therapist is under the specified bound D.

4 CASE STUDY

In order to apply our models, a data set of the speech therapy centre of *Acireale*, *Sicily* (*Italy*) is used. In this centre, there are two therapists assigned to work on the AAC project and six conventional therapists, who are working 6 days a week for 6 hours a day. To solve the mathematical models, AMPL and CPLEX solver for the first and the second problem and GA Matlab code for the third problem were used.

4.1 Problem I

In this stage, priority of patients, which was categorized into three levels as high, medium, and low, according to specialized doctors' view, is reflected in the model. Determination of these weights (w_p) for each priority level depends on the decision maker's preferences. The difference between the weights for different levels of priorities should be selected large enough to maintain a certain hierarchy between priorities. Our selection of weights is just a case for an illustration of the model. We fixed $w_p = 0.8$, $w_p = 0.5$ and $w_p = 0.2$ for high, medium and low level, respectively. We considered the number of patients equal to 50, ($P = 50$), who ask to participate in the AAC program in addition to basic therapy. This particular therapy can only be carried out by some therapists because the staff must be qualified for this additional therapy. In fact, Table 1 shows that only 21 patients were selected among those who asked to participate in the special AAC program, as a consequence the following Table 1 represents the patients that are selected for the AAC treatment, considering only the two therapists who are involved in the AAC shift. Unselected patients are not ignored, but will continue to be followed through basic therapy, because they do not have severe language difficulties and do not urgently need additional therapy.

Table 1: Selected Patients Number.

Weight	Patient i	Selected
$w_i = 0.2$	1	0
	⋮	⋮
	15	0
$w_i = 0.5$	16	1
	⋮	⋮
	26	1
	27	0
	⋮	⋮
	40	0
$w_i = 0.8$	41	1
	⋮	⋮
	50	1

In this case study, the treatment time was classified into two categories; t_p^b and t_p^{AAC} were assigned to symbolize basic treatment (45 min) and AAC treatment (≥ 60 min), respectively. Finally, we fixed 24 weekly hours dedicated to basic therapy by each AAC therapist and 12 hours carried out simultaneously by both AAC therapists. Therefore the total hours available to AAC therapists is $H = 60$.

4.2 Problem II

In this stage, we considered two therapists $i = 1, 2$ who perform both basic shifts and AAC treatments and six shift workers, $i = 3, \ldots 8$. In the centre under study, the weekly work is structured from Monday to Friday, morning (M) and afternoon (A), and on Saturday only in the morning. In particular, the AAC treatment is carried out only on Tuesday, Thursday and Saturday mornings, indicated with the index $l = 3$. We remind that the centre is closed on Saturday afternoons. In the objective function (21), we fixed $W_1 = 0.84, W_2 = 0.3, W_3 = 0.58, W_4 = 0.88, W_5 = 0.67, W_6 = 0.68$, which represent the weight associated with the soft constraint. The greater the weight, the greater the importance of the soft constraint. We fixed $t_p = 0.75$ and $t_p^{AAC} = 2$ to define the following constraint:

$$0.75 x_p \leq 1.5, \quad \forall p \in P; \tag{29}$$

$$2 \sum_{p=1}^{P} x_p \leq 12. \tag{30}$$

In Table 2, we provide the shifts for the eight therapists that we have considered.

Table 2: Therapists' shifts.

	Morning	Afternoon	AAC
Mon	4, 7, 8	1, 2, 3, 5, 6	
Tue	3, 5, 6	4, 7, 8	1, 2
Wed	7, 8	1, 2, 3, 4, 5, 6	
Thu	3, 4, 5, 6	7, 8	1, 2
Fri	7, 8	1, 2, 3, 4, 5, 6	
Sat	3, 4, 5, 6, 7, 8	*closed*	1, 2

4.3 Problem III

In this subsection we investigate the problem of moving from one location to another one considering fixed time windows. In fact, during the working day, therapists have to move from one therapy centre to another one, from one centre to another location to deliver home-based rehabilitation services, or from one house to another one. We considered $\bar{I} = 8$ therapists, who have the starting point at the centre of *Acireale* and we supposed that patients' houses are in the other locations considered, to simulate that some therapies are carried out directly at home. The corresponding multiple traveling salesmen problem was implemented using the genetic algorithm with multi-chromosome representation as in (Király and Abonyi, 2015). The algorithm considers that each therapist starts at the first location, and ends at the first location, but travels to a unique set of cities in between. We assume that the first location is the central location placed in *Acireale, Italy*, then each therapist has her own patients in different places. As a consequence, except for the starting point, each location is visited by exactly one therapist. The algorithm uses a special, so-called multi-chromosome genetic representation to code solutions into individuals. Special genetic operators (even complex ones) are used. The number of therapists that every day have to travel from one location to another is minimized during the algorithm. The algorithm also considers additional constraints, such as the minimum number of locations that the therapists visit and the maximum distance travelled by each therapist. We fixed $D = 80$ kilometers as the maximum tour length for each therapist, since a working day lasts only six hours. We considered the objective function (22), where the weights, considered as distances and costs, associated with the edges, are defined as

$$c_{hk} = \sqrt{(x_h - x_k)^2 + (y_h - y_k)^2}, \quad \forall v_h, v_k \in V. \quad (31)$$

We solved this problem using a genetic algorithm (GA), implemented in Matlab (Király and Abonyi, 2015), tested on MacBook Air (2021), processor Apple M1 8 Core, 3.2 GHz, RAM 8 GB.
For instance, we obtained the total distance traveled by all the therapists equal to 379 kilometers, obtained by 474 number of iterations and 19 time in milliseconds until the solution was given. The following plots explain better the solution reached.

Figure 1: City Locations.

Figure 1 represents the different 40 locations chosen in the example. It is necessary that therapists move from one location to another, because some therapies, especially on younger people, are carried out in places where they spend a lot of their life, for example at home or in parks or even at school.

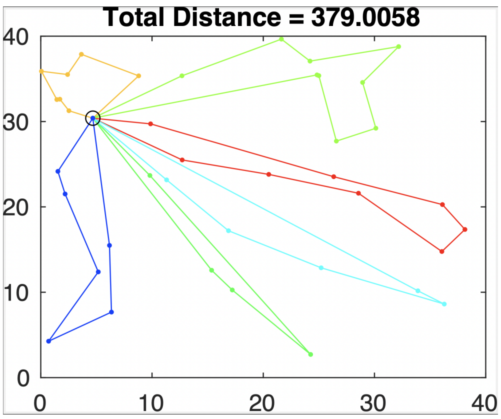

Figure 2: Total Distance.

Figure 2 shows the routes solution of each therapist. As a conclusion, we underline that the minimum number of therapists needed to reach all the 40 locations in one day is six. As a consequence of the six hours a day of each therapist, the time spent for travel influence the number of patients that could be treated. The Figure 2 underlines that some therapists are forced to face even long distances on a daily basis to satisfy the request of their patients.

5 CONCLUSIONS

This paper presented a multi-stage integer linear programming problem to solve the scheduling of speech-language pathologists involved in conventional treatments as well as Augmentative Alternative Communication therapies. In order to reduce the complexity of this problem, we developed a mathematical model based on a hierarchical approach. Thus, the problem was broken down into three sub-problems. Our aims were: the selection of the maximum number of patients, who can use the Augmentative Alternative Communication therapy program in addition to basic therapy; the achievement of an equitable distribution of therapists' workload to optimize work shifts and distribute them optimally during the week; the decrease of the time-wasting of therapists during transfers, who have to move for home-based therapies and have to change location during the working day. The model was tested on a therapy centre and the solution time was acceptable for the hierarchical implementation, with AMPL optimization package and Genetic Algorithm implementation in Matlab to find the solution in a faster way and to avoid the limitations of AMPL software. The model presented has some limitations that encourage us to further investigate the problem and improve our achievements. In fact, we did not take into consideration the preferences of therapists about their shifts, and the staggered entry times due to COVID-19 pandemic. As a future research, we can also explore the model with a higher number of therapists and patients.

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