Bi-objective Risk-averse Facility Location using a Subset-based Representation of the Conditional Value-at-Risk

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Abstract: For many real-world decision-making problems subject to uncertainty, it may be essential to deal with multiple and often conflicting objectives while taking the decision-makers' risk preferences into account. Conditional value-at-risk (CVaR) is a widely applied risk measure to address risk-averseness of the decision-makers. In this paper, we use the subset-based polyhedral representation of the CVaR to reformulate the bi-objective two-stage stochastic facility location problem presented in (Nazemi et al., 2021). We propose an approximate cutting-plane method to deal with this more computationally challenging subset-based formulation. Then, the cutting plane method is embedded into the ε-constraint method, the balanced-box method, and a recently developed matheuristic method to address the bi-objective nature of the problem. Our computational results show the effectiveness of the proposed method. Finally, we discuss how incorporating an approximation of the subset-based polyhedral formulation affects the obtained solutions.

1 INTRODUCTION

On the one hand, many real-world problems have multiple conflicting objectives, e.g., cost versus customer satisfaction or profitability versus environmental concerns. On the other hand, it is important to incorporate uncertainty about the considered parameters into the decision-making process. Optimization under uncertainty and multi-objective optimization have evolved mostly separately over the past decades. However, it is desirable to develop optimization techniques to deal with these two domains simultaneously (Gutjahr and Pichler, 2016).

Due to its sound theoretical background and its proven algorithmic performance, stochastic programming is a widely applied approach in computational optimization for handling data uncertainty (see, e.g., (Birge and Louveaux, 2011) and (Shapiro et al., 2014), and references therein). The standard two-stage stochastic programming problem formulation considers the expected value as a performance measure. It assumes a risk-neutral attitude of the decision-maker and performs well in the context of repetitive decision-making problems ((Birge and Louveaux, 2011)). However, decision-makers may have other risk preferences, especially in non-repetitive applications, e.g., in large-scale financial investments or in the management of high-impact disasters. In the case where more robust solutions are of interest, one of the recently most widely recommended risk measures is the Conditional Value at Risk (CVaR). It can be adjusted to the actual degree of risk-averseness of the decision-maker by specifying a confidence level α ($\alpha \in [0,1)$) (CVaR(α)). The closer the value of α to 0, the more risk-averse is the decision-maker. CVaR has been applied in different streams of the optimization literature, such as, e.g., financial engineering (e.g., (Krokhmal et al., 2002), (Roman et al., 2007), (Huang et al., 2021), (Zheng and Zheng, 2021)) disaster management (e.g., (Noyan, 2012), (Elçi and Noyan, 2018), (Nazemi et al., 2021)), water management (e.g., (Zhang et al., 2016), (Simic, 2019), (Chen et al., 2021)), and sustainable supply chain (e.g., (Rahimi and Ghezavati, 2018), (Rahimi et al., 2019)). We refer to (Filippi et al., 2020) for a recent survey on application of CVaR to several optimization domains different from financial engineering. (Huang et al., 2021) propose a two-stage distributionally robust model including CVaR as a risk measure for two different practical applications, a multi-

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product assembly problem and a portfolio selection problem. They also extend the model to a multi-stage one. Two decomposition algorithms based on the cutting-plane method are developed to solve the models. Finally, they conclude that the proposed models show a more robust performance compared to the risk-neutral model. (Noyan, 2012) incorporates the classical CVaR representation into the objective function of a two-stage mean-risk stochastic problem in disaster management where the demand and the damage level of the transportation network are uncertain parameters. She develops decomposition algorithms to deal with the computationally challenging nature of the problem. In line with this study, (Elçi and Noyan, 2018) consider a chance-constrained two-stage mean-risk stochastic relief network design model. They also develop a Benders decomposition-based algorithm to solve formulations with alternative representations of the CVaR. (Zhang et al., 2016) present a risk-averse multi-stage stochastic water allocation problem. A mean-CVaR objective is considered in each stage of the problem. Finally, they propose a nested L-shaped method to solve the problem. In addition to the aforementioned works that address single-objective problems, there are also some studies in the literature that consider multi-objective optimization under uncertainty incorporating CVaR as the risk measure into two-stage stochastic programming. (Zheng and Zheng, 2021) consider a bi-objective portfolio optimization model with a focus on minimizing CVaR as a risk measure and maximizing the expected return rate. They also incorporate transaction costs into the objective functions. A NSGA-II metaheuristic method (Deb et al., 2002) based on sparsity strategy (Zitzler et al., 2001) (SMP-NSGAII) is developed to deal with the bi-objective model. (Rahimi et al., 2019) propose a risk-averse sustainable multi-objective mixed-integer non-linear model to design a supply chain network under uncertainty by incorporating CVaR into their two-stage stochastic model. The objectives aim at minimizing the design cost and maximizing the profit. (Nazemi et al., 2021) incorporate a risk-neutral, a risk-averse, and the CVaR measure into a bi-objective two-stage facility location model in a disaster management context to analyze a wide range of risk preferences, including risk-neutral and worst-case approaches. They integrate different uncertain two-stage models into two well-known exact multi-objective frameworks, namely the ε-constraint and the balanced-box methods. Additionally, they also develop a matheuristic approach and they analyze and evaluate the combination of different uncertainty representations and multi-objective frameworks. In most of the aforementioned studies, the classical representation of CVaR has been employed to model risk-averseness. However, as mentioned earlier, alternative variants of CVaR are also proposed in the literature. In this paper, we focus on the existing two-stage bi-objective model presented in (Nazemi et al., 2021) and extend this work by reformulating their two-stage risk-averse model using a subset-based polyhedral representation of the CVaR. To address this alternative formulation, we propose a scenario cutting-plane algorithm. We conduct a computational analysis on the test instances used by (Nazemi et al., 2021) to illustrate how the subset-based variant performs in comparison to the classical formulation on their model.

The remainder of this paper is organized as follows. In Section 2, we describe the existing bi-objective facility location problem under uncertainty presented by (Nazemi et al., 2021) along with its corresponding subset-based polyhedral mathematical reformulation. In Section 3, we summarize the developed solution approach to solve the problem. In Section 4 we discuss the computational results. Finally, we present the conclusion and address potential future work in Section 5.

2 TWO-STAGE RISK-AVERSE STOCHASTIC OPTIMIZATION

The problem addressed in (Nazemi et al., 2021) is a bi-objective two-stage location-allocation model under demand uncertainty motivated by last-mile network design in a slow-onset disaster context. A finite discrete set of demand scenarios ($s \in S$, $S = \{1, ..., N\}$) with equal probabilities ($\frac{1}{N}$) is used to incorporate stochastic information. The problem aims at finding the best location to position temporary local distribution centers (LDC) in the response phase of a disaster, such that the affected people can walk to these centers to receive their relief aids. The model tries to find the trade-off between two conflicting objectives, i.e., minimization of operational cost on opening LDCs and maximization of the coverage or in other words, minimization of the amount of uncovered demand. We use the same notation as introduced in (Nazemi et al., 2021) to describe the model. The problem is formulated on a network, where the sets of nodes representing the demand and potential LDC locations are denoted by I and J, respectively, assuming that $J \subseteq I$. The assumption is that the affected people in each demand node ($i \in I$) will walk only to an LDC if their distance (d_{ij}) from an opened LDC is less than a certain distance threshold (d_{max}). Uncertainty is assumed on the amount of demand at each node ($q_i^{(s)}$). The first-stage decision (here-and-now) concerns the selection of sites to open LDCs. Binary variables $y_j \in \{0,1\}$ indicate whether or not an LDC is opened at node $j \in J$. Each LDC is assumed to have a limited capacity c_j and a given fixed cost γ_j. Decision variables $x_{ij}^{(s)} \in \{0,1\}$ and $u_j^{(s)}$ represent the second-stage decisions (wait-and-see) that determine the allocation of demand nodes to the opened LDCs and the amount of relief items delivered to each of them, respectively. The second-stage decisions are determined based on first-stage values and realized uncertain demand information. We next elaborate on our choice of the alternative subset-based polyhedral representation of CVaR incorporated into the second stage of the stochastic model.

2.1 Two-stage Subset-based CVaR (SSCVaR)

(Fábián, 2008) obtains a subset-based polyhedral representation of the classical CVaR for the special case of scenarios with equal probabilities where $\alpha = 1 - \frac{k}{N}$; k is the cardinality of any subset of scenarios. The cardinality of all scenarios is equal to N (i.e., $\bar{S} \subseteq S, |\bar{S}| = k$ and $|S| = N$):

$$\text{CVaR}_{1-\frac{k}{N}}(X) = \frac{1}{k} \max_{\bar{S} \subseteq S : |\bar{S}| = k} \sum_{s \in \bar{S}} X^{(s)} \quad (1)$$

where $X^{(s)}$ is the value of the random variable X in scenario s. Equation (1) leads to the following subset-based polyhedral representation of CVaR (Fábián, 2008):

$$\text{CVaR}_{1-\frac{k}{N}}(X) = \{\min \rho : \rho \geq \frac{1}{k} \sum_{s \in \bar{S}} X^{(s)}, \forall \bar{S} \subseteq S : |\bar{S}| = k\} \quad (2)$$

We utilize the subset-based representation (2) to obtain the MIP formulation of the two-stage model (**MB**):

$$\min \sum_{j \in V} \gamma_j y_j \quad (3)$$

$$\min \ \text{CVaR}_\alpha\Big(\sum_{i \in I} q_i^{(s)} - \sum_{j \in J} u_j^{(s)}\Big) = \rho \quad (4)$$

$$\text{s.t. } \rho \geq \frac{1}{k} \sum_{s \in \bar{S}} \Big(\sum_{i \in I} q_i^{(s)} - \sum_{j \in J} u_j^{(s)}\Big) \ \ \forall \bar{S} \subseteq S : |\bar{S}| = k \quad (5)$$

$$\sum_{j \in J} \psi(d_{ij}) x_{ij}^{(s)} \leq 1 \qquad \forall i \in I, s \in S \quad (6)$$

$$u_j^{(s)} \leq c_j y_j \qquad \forall j \in J, s \in S \quad (7)$$

$$u_j^{(s)} \leq \sum_{i \in I} q_i^{(s)} \psi(d_{ij}) x_{ij}^{(s)} \quad \forall j \in J, s \in S \quad (8)$$

$$x_{ij}^{(s)} \leq y_j \qquad \forall i \in I, \forall j \in J, s \in S \quad (9)$$

$$y_j \in \{0,1\} \qquad \forall j \in J \quad (10)$$

$$x_{ij}^{(s)} \in \{0,1\} \qquad \forall i \in I, \forall j \in J, s \in S \quad (11)$$

$$u_j^{(s)} \in \mathbb{Z}^+ \qquad \forall j \in J, s \in S \quad (12)$$

$$\rho \geq 0 \quad (13)$$

The first objective (3) minimizes total opening costs of LDCs. The second objective (4) minimizes the CVaR of the uncovered demand of the affected people. The auxiliary variable ρ represents the solution value of the r.h.s. of (2), which coincides with the CVaR as given by (1). Constraints (5) make sure that ρ is set to the correct value. Constraints (6) ensure that the demand at node i can only be covered at most once by an opened LDC if it is located within the coverage threshold of LDC j ($\psi(d_{ij})$=1, if $d_{ij} \leq d_{max}$ and 0 otherwise). Constraints (7) are the capacity constraints, one for each LDC j. Constraints (8) guarantee that the amount of demand considered to be covered by LDC j is at most the actual demand assigned to this node within its coverage threshold. Constraints (9) guarantee that a demand node can only be assigned to an opened LDC. Constraints (10)-(13) give the domains of the decision variables.

3 SOLUTION APPROACH

In this section, we describe the designed cutting-plane (approximation) method to deal with the potentially large number of scenario subsets in the model. To address the bi-objective nature of the problem, we embed the cutting-plane method into the multi-objective frameworks applied in (Nazemi et al., 2021), namely the ε-constraint (e) (Laumanns et al., 2006), the balanced-box (Boland et al., 2015) (BB) and a specific matheuristic (Mat) developed in (Nazemi et al., 2021). The matheuristic method embeds the bi-objective generalization of two heuristic approaches, namely local branching (Fischetti and Lodi, 2003) and relaxation induced neighborhood search (RINS) (Danna et al., 2005), into the ε-constraint method. The RINS is a variable fixing scheme employed when moving along the Pareto frontier from one solution to another in the ε-constraint framework. Additionally, a local branching approach is applied for the solutions, for which only fixing the variables does not solve the problem to optimality within a certain time limit. In a bi-objective setting, it is usually not possible to find a solution which simultaneously optimizes all of the considered objectives, but a set of optimal or efficient trade-off solutions exists, which are incomparable among each other and together dominate all other feasible solutions. The image of an efficient solution in objective space is called non-dominated point (NDP) and all NDPs together form the *Pareto frontier*. For further background on multi-objective optimization, we refer to (Ehrgott, 2005).

3.1 Scenario Subset-based Approximation

Formulation (MB) contains constraints corresponding to subsets of S. The size of this set of constraints could grow fast, depending on the cardinality of the subsets (k) and the set of scenarios S.

As mentioned in Section 1, different decomposition-based techniques have been proposed to solve two-stage stochastic programming models with a single objective (e.g., (Shapiro et al., 2014)). (Elçi and Noyan, 2018) propose a Benders decomposition-based framework for the subset-based variant of CVaR with a mean-risk objective function. In the single objective two-stage stochastic model, the model is decomposed by scenario once the first-stage variables are fixed.

In our scenario subset cutting-plane approach, we iteratively solve the so-called single objective problem which relaxes the subset-based constraints (5), and dynamically generates them using a delayed cut generation algorithm. To be more precise, the single objective problem contains the objective function (4) as the main objective, where the value of the other objective (3) is fixed.

Given an optimal solution $(\hat{y}, \hat{\rho}, \hat{u})$ of the relaxed model in each iteration, we check whether there is any violated constraint of the type (5), and identify a subset $\hat{S}$ if there is a violation. To solve this so-called *separation* problem, a few simple calculations (steps 3-8 of Algorithm 1) are required. At each iteration, we have the exact optimal second stage objective values associated with the current $(\hat{y}, \hat{\rho}, \hat{u})$ vector; in particular, these objective values can be expressed as $\rho^s = \frac{1}{k}(\sum_{i \in I} q_i^{(s)} - \sum_{j \in J} \hat{u}_j^{(s)})$ for each $s \in S$. Let $S^* = \{s \in S : \sum_{s \in S^*} \rho^s > \hat{\rho}\}$. If $S^* = \{s \in S : \sum_{s \in S^*} \rho^s \leq \hat{\rho}\}$ then the candidate solution is labeled as the new incumbent. Otherwise, the identified violated constraint of the form (5) associated with S^* is introduced as a cutting-plane. The pseudo-code of the delayed cut generation algorithm is presented in Algorithm 1.

Initial Cut. We propose a heuristic algorithm in order to initialize the relaxed model. This procedure is based on the following steps:

1. Rank the scenarios in each demand point based on their corresponding demand value (q_j^s) in descending order.
2. Sum the ranks for each scenario over I, the set of demand points.
3. Sort the scenarios based on their cumulative total rank in descending order.
4. Select the first k scenarios of the sorted set.

Algorithm 1: Delayed cut generation.

1: initialize the relaxed model with the initial cut, $t \leftarrow 1$
2: **while** $\sum_{s \in \hat{S}} \rho^s > \hat{\rho}$ **do**
3:     **for** each $s \in S$ **do**
4:         $\rho^s \leftarrow \frac{1}{k}(\sum_{i \in V} q_i^{(s)} - \sum_{j \in V} \hat{u}_j^{(s)})$
5:         (sort ρ^s in descending order)
6:     **end for**
7:     $S^* \leftarrow$ the first k values (largest) of the sorted ρ^s values
8:     $\hat{S} \leftarrow S^*$
9:     $t \leftarrow t + 1$
10: **end while**

In our bi-objective setting, due to the presence of integer variables not only in the first stage, but also in the second stage, using cutting-plane technique for every single point along the Pareto frontier causes computationally no benefit. However, preliminary tests showed that, employing the cutting-plane algorithm in order to calculate the first extreme point of the Pareto frontier and, consequently, fixing the generated subset cuts to the model, leads to a high-quality approximation of the entire Pareto frontier. We refer to this approach towards solving model MB by $\overline{\text{MB}}$ in the following.

4 COMPUTATIONAL STUDY

In this section, we present computational results for the instances used by (Nazemi et al., 2021) to show the performance of the proposed method with respect to the alternative subset-based polyhedral reformulation of the model in comparison to the classical representation of CVaR.

All experiments are executed on a single thread cluster where each node consists of two Intel Xeon X5570 CPUs at 2.93 GHz and 8 cores, with 48GB RAM. The model is implemented in C++ using CPLEX 12.9. As in (Nazemi et al., 2021) a time limit (TL) of 7200 s is considered. We note that we use the generic callback feature of CPLEX in order to generate the cuts in the MB model. Unlike lazy constraint callbacks, which prevent CPLEX from utilizing "dynamic search", generic callbacks allow CPLEX to choose among the options of dynamic search and classical branch-and-cut as it suits.

4.1 Test Instances

Here, we briefly explain the key points of the test instances and refer the reader to (Nazemi et al., 2021) for further details. They are derived from a real-world data set from a drought case presented in (Tricoire et al., 2012) for the region of Thies in western Senegal. The region is divided into 32 rural areas where each has between 9 and 31 villages. In total, the region contains 500 nodes. Opening costs for LDCs are assumed to be identical for all locations (5000 cost units). In addition, it is assumed that the affected people in a demand node walk to the closest LDC if the distance is less than 6km. For each instance, first, 10 sample scenarios ($|S| = 10$) are generated to cope with demand uncertainty. Then, in order to compare the algorithms for a larger number of scenarios, samples with sizes of $|S| = 100$, 500, and 1000 are generated.

4.2 Solution Quality Indicators

In order to assess the performance of heuristic methods for bi- and multi-objective optimization, several different quality indicators have been proposed in the literature (see, (Zitzler et al., 2003)). In this study, instead of the hypervolume indicator (I_H) used by (Nazemi et al., 2021), we employ the hypervolume gap (gH) and the multiplicative ε-indicators.

For a bi-objective framework with minimization objective functions, assume X is the set of feasible solutions in decision space and $Z = f(X)$ is the image of this set in objective space. We assume $A \subseteq Z$ is an approximation set of a Pareto frontier, and $R \subseteq Z$ is a reference set.

The hypervolume gap (gH) definition is as follows:

$$\text{gH\%} = \frac{I_H(R) - I_H(A)}{I_H(R)} \cdot 100 \tag{14}$$

The closer the value of gH% to 0, the higher is the quality of the approximated Pareto frontier.

The multiplicative epsilon indicator (I_ε) ((Zitzler et al., 2003)) gives the minimum factor ε that the approximation set (A) in the objective space has to be multiplied with, such that it dominates the reference set (R). The closer the value of I_ε to 1, the better is the approximated Pareto frontier. The ε value calculated here can be interpreted as the gap between A and R in the objective space.

4.3 Results

We compare all combinations of multi-objective criterion space approaches used in (Nazemi et al., 2021) (e, BB, and Mat) with two linear counterparts of the classical (MA) and the approximation of subset-based polyhedral ($\overline{\text{MB}}$) representations of the two-stage CVaR model for different levels of risk. {e-MA, BB-MA, Mat-MA, e-$\overline{\text{MB}}$, BB-$\overline{\text{MB}}$, Mat-$\overline{\text{MB}}$} denotes the set of all methods evaluated in this study.

The run time for the two different representations of the CVaR approach, MA, and $\overline{\text{MB}}$, is reported in Table 1. MA is solved to optimality, whereas $\overline{\text{MB}}$ is the MB which is solved using the explained cutting-plane technique to approximate its Pareto frontier. It is worth mentioning that the cutting-plane method solves the first extreme point of the Pareto frontier to optimality and all other non-dominated solutions are computed without generating additional cuts. Results for different levels of α are reported: the two special cases identical with the expected value (risk-neutral) and the worst-case (risk-averse) point of view, and one "middle" level and their corresponding k-values ($\alpha = 1 - \frac{k}{N}$). The results in this table show that the combination of the BB with $\overline{\text{MB}}$ solves the instances slightly faster than its combination with MA. In the following, we compare the quality of the obtained solutions using MA and $\overline{\text{MB}}$.

After analyzing the two exact multi-objective techniques (e and BB), we apply the Mat method in combination with the MA and $\overline{\text{MB}}$ models to solve the test instances. As shown in Table 2, the $\overline{\text{MB}}$-Mat method is slightly faster than MA-Mat. Additionally, we solve a few instances with a larger number of scenarios. We report the run time and the number of found non-dominated points (NDP) in Table 3. The results show that the approximately solved subset-based representation of the CVaR method ($\overline{\text{MB}}$) pays off when the number of scenarios increases.

Performance comparison of the Mat method and the ε-constraint method for small instances where the latter could find the optimal Pareto frontier is shown in Tables 4 and 5. To measure the quality, we report the hypervolume gap (gH%) and the value of the I_ε indicator. Analyzing the results in Table 4 confirms

Table 1: Run time comparison of two representations of CVaR, MA, and $\overline{\text{MB}}$, for different α-level/k-value for instances of size 21-500 with sample size 10.

	α/k value											
	0/10				**0.7/3**				**0.9/1**			
#Node	**e-MA**	**BB-MA**	**e-$\overline{\text{MB}}$**	**BB-$\overline{\text{MB}}$**	**e-MA**	**BB-MA**	**e-$\overline{\text{MB}}$**	**BB-$\overline{\text{MB}}$**	**e-MA**	**BB-MA**	**e-$\overline{\text{MB}}$**	**BB-$\overline{\text{MB}}$**
21	1	2	1	2	2	1	1	2	2	2	1	1
44	16	22	15	19	17	23	13	14	20	26	10	17
56	24	31	19	23	18	22	15	23	19	33	14	16
72	36	51	29	35	45	48	30	41	64	103	36	40
90	62	110	59	80	73	78	59	75	99	179	64	73
106	123	229	118	133	151	161	111	132	210	380	141	175
120	143	241	135	163	144	158	121	160	138	262	100	115
163	445	818	392	552	417	506	369	503	636	1365	TL	596
182	577	1550	481	738	932	1053	701	776	1326	2548	1010	1192
203	956	2078	879	1208	616	699	552	716	515	TL	TL	558
254	6103	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL
264	3903	6800	TL	2449	6453	TL	TL	TL	TL	TL	TL	4007
275	7133	TL	6585	TL	TL	TL	TL	TL	TL	TL	TL	TL
295	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL
326	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL
355	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL
388	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL
410	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL
436	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL
449	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL
472	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL
482	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL
500	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL	TL

Table 2: Run time comparison of a combination of ε-constraint method (e), and the proposed matheuristic (Mat) method with MA and $\overline{\text{MB}}$ for different α/k-values for test instances of size 21-500 with sample size of 10 scenarios.

	α/k value											
	0/10				**0.7/3**				**0.9/1**			
#Node	**e-MA**	**Mat-MA**	**e-$\overline{\text{MB}}$**	**Mat-$\overline{\text{MB}}$**	**e-MA**	**Mat-MA**	**e-$\overline{\text{MB}}$**	**Mat-$\overline{\text{MB}}$**	**e-MA**	**Mat-MA**	**e-$\overline{\text{MB}}$**	**Mat-$\overline{\text{MB}}$**
21	1	0.51	1	0.42	2	0.43	1	0.43	2	0.58	1	0.46
44	16	3	15	3	17	3	13	2	20	3	10	3
56	24	4	19	4	18	4	15	3	19	5	14	3
72	36	11	29	8	45	9	30	6	64	11	36	7
90	62	18	59	16	73	23	59	19	99	21	64	14
106	123	32	118	37	151	36	111	28	210	44	141	29
120	143	37	135	40	144	39	121	36	138	43	100	38
163	445	183	392	102	417	132	369	98	636	167	392	104
182	577	212	481	140	932	178	701	128	1326	172	1010	136
203	956	206	879	198	616	217	552	172	515	210	TL	194
254	6103	414	TL	462	TL	589	TL	356	TL	568	TL	514
264	3903	479	TL	652	6453	470	TL	405	TL	478	TL	445
275	7133	686	6585	1082	TL	781	TL	711	TL	819	TL	636
295	TL	726	TL	1068	TL	729	TL	733	TL	836	TL	775
326	TL	893	TL	1027	TL	2407	TL	856	TL	1182	TL	914
355	TL	1447	TL	1378	TL	1499	TL	1339	TL	1619	TL	1333
388	TL	1611	TL	1711	TL	1835	TL	1726	TL	1931	TL	2115
410	TL	2133	TL	2251	TL	4062	TL	2167	TL	2847	TL	1901
436	TL	2439	TL	2413	TL	2531	TL	2340	TL	3608	TL	2220
449	TL	2633	TL	2857	TL	2953	TL	2546	TL	2903	TL	2534
472	TL	2955	TL	3185	TL	3167	TL	2971	TL	3194	TL	3155
482	TL	5054	TL	4044	TL	3715	TL	3216	TL	3885	TL	3162
500	TL	6238	TL	4442	TL	5653	TL	2955	TL	4278	TL	2358

that the combination of Mat and MA gives an upper bound for the Pareto frontier (as Mat is a heuristic), whereas the combination of the ε-constraint method with $\overline{\text{MB}}$ finds a lower bound for the Pareto frontier. A lower bound is obtained because the ε-constraint method computes exact solutions of the approximated problem, but approximates the CVaR objective from below, as the $\overline{\text{MB}}$ only works with a reduced number of subsets that are not necessarily worst for each non-dominated point. Therefore, we obtain *negative* gH% values and I_ε values ≤ 1. Nonetheless, both methods provide high-quality approximations of Pareto fron-

Table 3: Performance comparison of a combination of ε-constraint method (e), and the proposed matheuristic (Mat) method with MA and $\overline{\text{MB}}$ for α-level 0.7 and its corresponding *k*-value for larger number of scenarios. T[s] indicates the run time in seconds.

	α/*k* value: 0.7/3								
		e-MA		**Mat-MA**		**e-$\overline{\text{MB}}$**		**Mat-$\overline{\text{MB}}$**	
#Node	*#Scenario*	T[s]	#NDP	T[s]	#NDP	T[s]	#NDP	T[s]	#NDP
21	100	161	17	27	17	56	17	5	17
	500	6476	16	348	16	322	16	52	16
	1000	TL	6	1668	17	TL	13	282	17
44	100	1076	30	178	30	316	30	34	30
	500	TL	5	1374	32	TL	19	365	32
	1000	TL	3	TL	3	TL	7	1239	34
56	100	2267	39	234	39	583	39	51	39
	500	TL	3	2675	40	TL	22	776	38
	1000	TL	3	TL	2	TL	3	1747	41
72	100	4186	50	180	50	826	50	107	50
	500	TL	3	TL	2	TL	15	983	51
	1000	TL	2	TL	2	TL	2	TL	2

tiers as both gH%, and I_ε represent a gap between the approximated sets and the reference set.

To measure the performance of the combination of the Mat method and $\overline{\text{MB}}$, we re-evaluate the second stage of $\overline{\text{MB}}$ by solving the MA model, where the first stage decisions are fixed to the values obtained from the $\overline{\text{MB}}$. In other words, in order to assess its true quality, the solution provided by $\overline{\text{MB}}$ is re-evaluated by using the exact second-stage objective value as obtained through MA. As can be seen in Table 5, the e-$\overline{\text{MB}}$ and Mat-$\overline{\text{MB}}$ frameworks generate high-quality Pareto frontiers.

After that, we also assess the performance of Mat-$\overline{\text{MB}}$ (with re-evaluated second stage) on large-size instances where the optimal Pareto frontier is not known. For this purpose, we consider a union of the Pareto frontiers (U) obtained by e-MA, BB-MA, and Mat-MA as the reference set. Table 6 shows the hypervolume gap (gH%) values. Negative values indicate that Mat-$\overline{\text{MB}}$ with re-evaluated second stage values obtains better approximations than the union of the other methods.

5 CONCLUSIONS

In this paper, we investigate the subset-based polyhedral representation of CVaR to capture risk in a two-stage bi-objective location-allocation model introduced in (Nazemi et al., 2021), the purpose of which is the optimal design of a relief network. The model aims at finding the trade-off between a deterministic objective (minimization of location cost) and an uncertain one (minimization of uncovered demand) where uncertainty is assumed in demand values. We introduce an approximate cutting-plane method to deal with the computationally challenging subset-based formulation of the two-stage stochastic optimization problem containing the CVaR.

We evaluate the performance of the proposed method by conducting a computational study on the instances used by (Nazemi et al., 2021). Our experiments show that the proposed approximate cutting-plane approach pays off for a large number of scenarios and they illustrate how incorporating an approximation of the subset-based formulation affects the quality of the produced solutions. In future research, we would like to investigate whether the performance of the cutting-plane method can be further improved by employing additional enhancements.

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Table 4: Quality comparison using I_ε and hypervolume gap (gH%) for approximated Pareto frontiers where the optimal Pareto frontier is known from solutions of solved instances using ε-constraint method (e), with the sample size of 10 scenarios.

α/k value: 0.7/3

	e-MA (Reference set)		Mat-MA		e-$\overline{\text{MB}}$	
#Node	**gH%**	I_ε	**gH%**	I_ε	**gH%**	I_ε
21	0	1	0.00	1.00	-0.01	0.99
44	0	1	0.00	1.00	0.00	1.00
56	0	1	0.00	1.00	0.00	0.91
72	0	1	0.42	1.22	-0.02	0.95
90	0	1	0.05	1.32	0.00	0.87
106	0	1	0.06	1.33	-0.01	0.94
120	0	1	0.02	1.35	0.00	1.00
163	0	1	0.00	1.00	0.00	0.96
182	0	1	0.00	1.00	0.00	0.94
203	0	1	0.02	1.02	0.00	0.97

Table 5: Quality comparison using I_ε and hypervolume gap (gH%) for re-evaluated approximated Pareto frontiers using e-$\overline{\text{MB}}$ and Mat-$\overline{\text{MB}}$, where the optimal Pareto frontier is known from solutions of solved instances using ε-constraint method (e), with the sample size of 10 scenarios.

α/k value: 0.7/3

	e-MA (Reference set)		e-$\overline{\text{MB}}$		Mat-$\overline{\text{MB}}$	
#Node	**gH%**	I_ε	**gH%**	I_ε	**gH%**	I_ε
21	0	1	0.00	1.00	0.00	1.00
44	0	1	0.00	1.00	0.00	1.00
56	0	1	0.00	1.00	0.00	1.00
72	0	1	0.01	1.03	0.01	1.03
90	0	1	0.00	1.04	0.00	1.04
106	0	1	0.01	1.00	0.01	1.00
120	0	1	0.00	1.00	0.00	1.00
163	0	1	0.00	1.00	0.00	1.00
182	0	1	0.00	1.04	0.00	1.04
203	0	1	0.00	1.00	0.00	1.00

Table 6: Performance of Mat-$\overline{\text{MB}}$ on large-size instances, where the optimal Pareto-frontier is not known. The union of the Pareto frontiers obtained by e-MA, BB-MA, and Mat-$\overline{\text{MB}}$ is considered as a reference set. The sample size of scenarios is 10.

α/k value: 0.7/3

	U (Reference set)	Mat-$\overline{\text{MB}}$
#Node	**gH%**	**gH%**
254	0.00	0.00
275	0.00	0.00
295	0.00	-0.02
326	0.00	-0.06
355	0.00	0.01
388	0.00	-0.10
410	0.00	-0.05
436	0.00	-0.02
449	0.00	-0.03
472	0.00	-0.08
482	0.00	-0.02
500	0.00	0.00

REFERENCES

Birge, J. R. and Louveaux, F. (2011). *Introduction to stochastic programming*. Springer Science & Business Media.

Boland, N., Charkhgard, H., and Savelsbergh, M. (2015). A criterion space search algorithm for biobjective integer programming: The balanced box method. *INFORMS Journal on Computing*, 27(4):735–754.

Chen, Y., Li, J., Lu, H., and Yan, P. (2021). Coupling system dynamics analysis and risk aversion programming for optimizing the mixed noise-driven shale gas-water supply chains. *Journal of Cleaner Production*, 278:123209.

Danna, E., Rothberg, E., and Le Pape, C. (2005). Exploring relaxation induced neighborhoods to improve mip solutions. *Mathematical Programming*, 102(1):71–90.

Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: Nsga-ii. *IEEE transactions on evolutionary computation*, 6(2):182–197.

Ehrgott, M. (2005). *Multicriteria optimization*, volume 491. Springer Science & Business Media.

Elçi, Ö. and Noyan, N. (2018). A chance-constrained two-stage stochastic programming model for humanitarian relief network design. *Transportation research part B: methodological*, 108:55–83.

Fábián, C. I. (2008). Handling cvar objectives and constraints in two-stage stochastic models. *European Journal of Operational Research*, 191(3):888–911.

Filippi, C., Guastaroba, G., and Speranza, M. G. (2020). Conditional value-at-risk beyond finance: a survey. *International Transactions in Operational Research*, 27(3):1277–1319.

Fischetti, M. and Lodi, A. (2003). Local branching. *Mathematical programming*, 98(1-3):23–47.

Gutjahr, W. J. and Pichler, A. (2016). Stochastic multi-objective optimization: a survey on non-scalarizing methods. *Annals of Operations Research*, 236(2):475–499.

Huang, R., Qu, S., Yang, X., and Liu, Z. (2021). Multi-stage distributionally robust optimization with risk aversion. *Journal of Industrial & Management Optimization*, 17(1):233.

Krokhmal, P., Palmquist, J., and Uryasev, S. (2002). Portfolio optimization with conditional value-at-risk objective and constraints. *Journal of risk*, 4:43–68.

Laumanns, M., Thiele, L., and Zitzler, E. (2006). An efficient, adaptive parameter variation scheme for metaheuristics based on the epsilon-constraint method. *European Journal of Operational Research*, 169(3):932–942.

Nazemi, N., Parragh, S. N., and Gutjahr, W. J. (2021). Bi-objective facility location under uncertainty with an application in last-mile disaster relief. *Annals of Operations Research*, https://doi.org/10.1007/s10479-021-04422-4.

Noyan, N. (2012). Risk-averse two-stage stochastic programming with an application to disaster management. *Computers & Operations Research*, 39(3):541–559.

Rahimi, M. and Ghezavati, V. (2018). Sustainable multi-period reverse logistics network design and planning under uncertainty utilizing conditional value at risk (cvar) for recycling construction and demolition waste. *Journal of Cleaner Production*, 172:1567–1581.

Rahimi, M., Ghezavati, V., and Asadi, F. (2019). A stochastic risk-averse sustainable supply chain network design problem with quantity discount considering multiple sources of uncertainty. *Computers & Industrial Engineering*, 130:430–449.

Roman, D., Darby-Dowman, K., and Mitra, G. (2007). Mean-risk models using two risk measures: a multi-objective approach. *Quantitative Finance*, 7(4):443–458.

Shapiro, A., Dentcheva, D., and Ruszczyński, A. (2014). *Lectures on stochastic programming: modeling and theory*. SIAM.

Simic, V. (2019). Interval-parameter conditional value-at-risk two-stage stochastic programming model for management of end-of-life vehicles. *Environmental Modeling & Assessment*, 24(5):547–567.

Tricoire, F., Graf, A., and Gutjahr, W. J. (2012). The bi-objective stochastic covering tour problem. *Computers & operations research*, 39(7):1582–1592.

Zhang, W., Rahimian, H., and Bayraksan, G. (2016). Decomposition algorithms for risk-averse multistage stochastic programs with application to water allocation under uncertainty. *INFORMS Journal on Computing*, 28(3):385–404.

Zheng, Y. and Zheng, J. (2021). A novel portfolio optimization model via combining multi-objective optimization and multi-attribute decision making. *Applied Intelligence*, pages 1–12.

Zitzler, E., Laumanns, M., and Thiele, L. (2001). Spea2: Improving the strength pareto evolutionary algorithm. *TIK-report*, 103.

Zitzler, E., Thiele, L., Laumanns, M., Fonseca, C. M., and Da Fonseca, V. G. (2003). Performance assessment of multiobjective optimizers: An analysis and review. *IEEE Transactions on evolutionary computation*, 7(2):117–132.