A Many-valued Semantics for Multi-agent System

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Abstract: We often employ epistemic logic to express the epistemic states of agents. However, it is often too complicated to build a Kripke model because we should consider all possibilities of the knowledge between agents. In this paper, we employ a many-valued logic to express the epistemic states of agents. Thus far, the representations usually show the epistemic state of a single agent, however, we apply the logic to the multi-agent system. Here, we consider that there exist three kinds of epistemic states of *known, truth-value unknown*, and *content unknown*. Furthermore, we introduce two kinds of agent communication in our semantics, i.e., teaching and asking, and show how the epistemic states of agents will change.

1 INTRODUCTION

We often employ epistemic logic to express the epistemic states of agents, e.g., $\Box_a A$ stands for agent a knows that A is true and $\neg\Box_a B$ stands for that a doesn't know that B is true. Actually, if we build the Kripke model perfectly and consider that every agent has the common sense which is shown in the model, the representation of epistemic states and the simulation of agent communication usually work very well. However, it is often too complicated to show the epistemic states of multi-agent system in the modal logic because we should consider all possibilities of knowledge between agents, e.g., agent a knows that b knows p while agent b doesn't know that agent a knows that agent b knows p.

Instead of epistemic logic, in this research, we consider employing many-valued logic to avoid those notational complications. In the 3-valued logic, we can use the third value, which can be read as "**unknown**" or "**unknown whether true or false**"(Kleene, 1952) to show such a state. (Ciucci and Dubois, 2012) provided a translation from the strong Kleene logic to epistemic logic. (Szmuc, 2019) considered the third value in the paraconsistent weak Kleene logic as an epistemic interpretation. In the 4-valued logic, the four values are usually called **true, false, neither** and **both**. Belnap considered the valued as follows(Belnap, 1977):

- The value of p is **true(T)** means that the computer is told that p is true.
- The value of p is **false(F)** means that the computer is told that p is false.
- The value of p is **neither(N)** means that the computer is not told anything about p.
- The value of p is **both(B)** means that the computer is told that p is both true and false (perhaps from different sources, or so on).

If we consider the computer as an agent, the 4 values can be seen as the epistemic states of the agent, i.e., **true(T)** and **false(f)** mean that p is known to the agent, while **neither(N)** and **both(B)** mean that p is unknown to the agent.

However, both of the representations only show the epistemic state of a single agent, while it can be the case that a certain proposition is known to some agents while unknown to others in a multi-agent system. In this paper, we give a new many-valued logic semantics to express the epistemic states in the multi-agent system. Here, we consider that each proposition is either true or false as the classical logic, while we add several additional values to show the epistemic states of agents. Therefore, all of the propositions have the same classical values for each agent, while the epistemic states is different between the agents.

This paper is organized as follows. In Section 2, we show a fundamental of many-valued semantic consequence relation and introduce some many-valued logics. In Section 3, we give two pair semantics for two readings of knowledge. In Section 4, we combine the two semantics and extend it to express the epistemic states of multi-agent system. In Section 5, we introduce two kinds of agent communication in our semantics. Finally in Section 6, we conclude.

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2 MANY-VALUED LOGIC

In the many-valued logic, we usually consider that the propositional language $\mathcal{L}$ consists of a set $\{\sim, \wedge, \vee\}$ of propositional connectives and a countable set Prop of propositional variables. We denote by Form the set of formulas defined as usual in $\mathcal{L}$, denote a formula of $\mathcal{L}$ by A, B, C, etc. and a subset of Form by Γ, Δ, Σ, etc.

Here, we show a fundamental of many-valued semantic consequence relation(Szmuc and Omori, 2018) we use in this paper.

Definition 1 (Univalent semantics)**.** A *univalent semantics* for the language $\mathcal{L}$ is a structure $M = \langle \mathcal{V}, \mathcal{D}, \delta \rangle$, where

- $\mathcal{V}$ is a non-empty set of truth values,
- $\mathcal{D}$ is a non-empty proper subset of $\mathcal{V}$,
- δ contains, for every n-ary connective $*$ in the language, a truth-function $\delta_* : \mathcal{V}^n \to \mathcal{V}$.

A *univalent interpretation* is a pair $\langle M, \mu \rangle$, where M is such a structure, and μ is an evaluation function from Prop to $\mathcal{V}$. Given an interpretation, μ is extended to a map from Form to $\mathcal{V}$ recursively, by the following clause:

- $\mu(*(A_1, \ldots, A_n)) = \delta_*(\mu(A_1), \ldots, \mu(A_n))$.

Finally, $\Gamma \models^M A$ iff for all univalent interpretation $\langle M, \mu \rangle$, if $\mu(B) \in \mathcal{D}$ for all $B \in \Gamma$, then $\mu(A) \in \mathcal{D}$.

Semantically speaking, $\mathcal{V}$ shows the possible values in a logic, $\mathcal{D}$ shows the designated values, and δ shows the truth tables of the connectives in a logic.

By this definition, we can give semantic consequence relations for many-valued logics.

- The semantic consequence relation $\models_{\mathbf{CL}}$ for classical propositional logic is obtained by setting $\mathcal{V} = \{\mathbf{t}, \mathbf{f}\}$, $\mathcal{D} = \{\mathbf{t}\}$.
- In the strong Kleene logic and logic of paradox, if we write the third value as **b** , then the truth table is written as following:

	$\sim$	$\wedge$	**t**	**b**	**f**	$\vee$	**t**	**b**	**f**
t	**f**	**t**	**t**	**b**	**f**	**t**	**t**	**t**	**t**
b	**b**	**b**	**b**	**b**	**f**	**b**	**t**	**b**	**b**
f	**t**	**f**	**f**	**f**	**f**	**f**	**t**	**b**	**f**

  - The semantic consequence relation $\models_{\mathbf{SK}}$ for strong Kleene logic is obtained by setting $\mathcal{V} = \{\mathbf{t}, \mathbf{b}, \mathbf{f}\}$, $\mathcal{D} = \{\mathbf{t}\}$.
  - The semantic consequence relation $\models_{\mathbf{LP}}$ for logic of paradox is obtained as above except that we replace $\mathcal{D} = \{\mathbf{t}\}$ by $\mathcal{D} = \{\mathbf{t}, \mathbf{b}\}$.

  Normally, **b** can be read as *unknown, undecided*, etc.

- In the weak Kleene logic, if we write the third value as **n** , then the truth table is written as following:

	$\sim$	$\wedge$	**t**	**n**	**f**	$\vee$	**t**	**n**	**f**
t	**f**	**t**	**t**	**n**	**f**	**t**	**t**	**n**	**t**
n	**n**	**n**	**n**	**n**	**n**	**n**	**n**	**n**	**n**
f	**t**	**f**	**f**	**n**	**f**	**f**	**t**	**n**	**f**

  - The semantic consequence relation $\models_{\mathbf{WK}}$ for weak Kleene logic (**WK**) is obtained by setting $\mathcal{V} = \{\mathbf{t}, \mathbf{n}, \mathbf{f}\}$, $\mathcal{D} = \{\mathbf{t}\}$.
  - The semantic consequence relation $\models_{\mathbf{PWK}}$ for paraconsistent weak Kleene logic (**PWK**) is obtained as in **WK** except that we replace $\mathcal{D} = \{\mathbf{t}\}$ by $\mathcal{D} = \{\mathbf{t}, \mathbf{n}\}$.

  Normally, **n** can be read as *meaningless, undefined, off-topic*, etc.

  From the truth table, we can see that the value of a formula will be **n** even if one atom of the formula has the value n. Therefore, the weak Kleene logic is also called infectious logic and n is called an infectious value.

- Belnap considered a 4-valued logic called first-degree entailment logic FDE. The four values are usually written as $\{$**t, f, b, n**$\}$. The truth table is as following(Omori and Wansing, 2017):

	$\sim$	$\wedge$	**t**	**b**	**n**	**f**	$\vee$	**t**	**b**	**n**	**f**
t	**f**	**t**	**t**	**b**	**n**	**f**	**t**	**t**	**t**	**t**	**t**
b	**b**	**b**	**b**	**b**	**f**	**f**	**b**	**t**	**b**	**t**	**b**
n	**n**	**n**	**n**	**f**	**n**	**f**	**n**	**t**	**t**	**n**	**n**
f	**t**	**f**	**f**	**f**	**f**	**f**	**f**	**t**	**b**	**n**	**f**

  Deutsch considered another 4-valued logic that is usually called that is called S_{fde} logic. The reading of values is the same as FDE and the truth table is as following(Ferguson, 2017):

	$\sim$	$\wedge$	**t**	**b**	**n**	**f**	$\vee$	**t**	**b**	**n**	**f**
t	**f**	**t**	**t**	**b**	**n**	**f**	**t**	**t**	**t**	**n**	**t**
b	**b**	**b**	**b**	**b**	**n**	**f**	**b**	**t**	**b**	**n**	**b**
n	**n**	**n**	**n**	**n**	**n**	**n**	**n**	**n**	**n**	**n**	**n**
f	**t**	**f**	**f**	**f**	**n**	**f**	**f**	**t**	**b**	**n**	**f**

  The semantic consequence relations of $\models_{\mathbf{FDE}}$ and $\models_{\mathbf{Sfde}}$ are both obtained by setting $\mathcal{V} = \{\mathbf{t}, \mathbf{f}, \mathbf{b}, \mathbf{n}\}$, $\mathcal{D} = \{\mathbf{t}, \mathbf{b}\}$. Also, it is easy to see from the truth table that the S_{fde} logic can also be considered as the combination of the logic of paradox and the weak Kleene logic.

3 PAIR SEMANTICS FOR MULTI-AGENT SYSTEM

We usually use the epistemic logic to show the knowledge of a multi-agent system. Normally, we can use the formula $\neg\Box_a A \wedge \neg\Box_a \neg A$ to express the fact that A is unknown to agent a. However, sometimes the Kripke model of epistemic logic is too complex, so here we give a simple way to express the states of agent knowledge by many-valued logic.

We define the classical valuation V_t : Prop $\rightarrow \{\mathbf{t}, \mathbf{f}\}$ as usual. Then we define the knowledge of agent a as the valuation V_a : Prop $\rightarrow \{1, 0\}$.

- $V_t(p) = \mathbf{t}$ means p is true.
- $V_t(p) = \mathbf{f}$ means p is false.
- $V_a(p) = 1$ means p is known to agent a.
- $V_a(p) = 0$ means p is unknown to agent a.

Before showing the semantics, we want to ask the question that **What is knowledge?**. To show this question clearly, consider the following situation,

Example 2. p, q are two propositions and p is known to agent a while q is unknown. Assume that p is true, is the formula $p \vee q$ known to a?

Agent a can acquire the knowledge that $p \vee q$ is true. Actually, it is the same in the epistemic logic that $\Box p \rightarrow \Box(p \vee q)$ no matter what is q. Can we say that $p \vee q$ is known to a?

Here, we show two semantics for two readings of knowledge.

- Semantics (I) stands for the case that we consider that $p \vee q$ is unknown to a. In other words, we consider that a formula A is known to a agent if and only if all of the atoms of A are known to the agent. Therefore, the state **unknown** is infectious and the semantics should be similar with that of weak Kleene logic which we introduce in Section 2.
- Semantics (II) stands for the case that we consider that $p \vee q$ is known to a. In other words, we consider that a formula A is known to a agent if and only if the agent knows whether A is true or not. Therefore, the semantics should be similar with that of epistemic logic.

3.1 Semantics (I)

If we consider the state of **unknown** as the first case, we should give a semantics for the infectious logic. Actually, we have already shown a pair semantics to express the infectious logic in(Song et al., 2021).

Definition 3. A *two-valued interpretation* for the language $\mathcal{L}$ is a pair $\langle V_t^w, V_a^w \rangle$, where V_t^w : Prop $\rightarrow \{\mathbf{t}, \mathbf{f}\}$ and V_a^w : Prop $\rightarrow \{0, 1\}$. Valuations V_t^w, V_a^w are then extended to interpretations I_t, I_m by the following conditions.

- $I_t^w(p)$=$\mathbf{t}$ iff $V_t^w(p)$=$\mathbf{t}$
- $I_a^w(p)$=1 iff $V_a(p)$=1
- $I_t^w(\sim A)$=$\mathbf{t}$ iff $I_t^w(A)$=$\mathbf{f}$
- $I_a^w(\sim A)$=1 iff $I_a^w(A)$=1
- $I_t^w(A \wedge B)$=$\mathbf{t}$ iff $I_t^w(A)$=$\mathbf{t}$ and $I_t^w(B)$=$\mathbf{t}$
- $I_a^w(A \wedge B)$=1 iff $I_a^w(A)$=1 and $I_a^w(B)$=1

In this paper, we consider that $A \vee B$ as the same as $\sim(\sim A \wedge \sim B)$. If we read the additional value V_a^w as **unknown**, it can be considered as the semantics that shows the epistemic states of agents.

Definition 4. A *four-valued interpretation* of $\mathcal{L}$ is a function I_4^w : Prop $\rightarrow \{\mathbf{t}1, \mathbf{t}0, \mathbf{f}0, \mathbf{f}1\}$. Given a four-valued interpretation I_4^s, this is extended to a function that assigns every formula a truth value by the following truth functions:

A	$\sim A$	$A \wedge B$	$\mathbf{t}1$	$\mathbf{t}0$	$\mathbf{f}0$	$\mathbf{f}1$
$\mathbf{t}1$	$\mathbf{f}1$	$\mathbf{t}1$	$\mathbf{t}1$	$\mathbf{t}0$	$\mathbf{f}0$	$\mathbf{f}1$
$\mathbf{t}0$	$\mathbf{f}0$	$\mathbf{t}0$	$\mathbf{t}0$	$\mathbf{t}0$	$\mathbf{f}0$	$\mathbf{f}0$
$\mathbf{f}0$	$\mathbf{t}0$	$\mathbf{f}0$	$\mathbf{f}0$	$\mathbf{f}0$	$\mathbf{f}0$	$\mathbf{f}0$
$\mathbf{f}1$	$\mathbf{t}1$	$\mathbf{f}1$	$\mathbf{f}1$	$\mathbf{f}0$	$\mathbf{f}0$	$\mathbf{f}1$

We introduce three different sets of designated values as follows:

- $\mathcal{D}_1^w := \{\mathbf{t}1\}$;
- $\mathcal{D}_2^w := \{\mathbf{t}1, \mathbf{t}0\}$;
- $\mathcal{D}_3^w := \{\mathbf{t}1, \mathbf{t}0, \mathbf{f}0\}$.

Based on these sets of designated values, we define three consequence relations as follows.

Definition 5. For all $\Gamma \cup \{A\} \subseteq$ Form, $\Gamma \models_i^w A$ iff for all four-valued interpretations I_4^w, $I_4^w(A) \in \mathcal{D}_i^w$ if $I_4^w(B) \in \mathcal{D}_i^w$ for all $B \in \Gamma$, where $i \in \{1, 2, 3\}$.

We have already shown the facts in (Song et al., 2021) that:

- $\models_1$ is the weak Kleene logic;
- $\models_2$ is the classical logic;
- $\models_3$ is the paraconsistent weak Kleene logic.

3.2 Semantics (II)

If we consider the state of **unknown** as the second case, we give the semantics as following:

Definition 6. A *two-valued interpretation* for the language $\mathcal{L}$ is a pair $\langle V_t^s, V_a^s \rangle$, where V_t^s : Prop $\rightarrow \{\mathbf{t}, \mathbf{f}\}$ and V_a^s : Prop $\rightarrow \{0, 1\}$. Valuations V_t^s, V_a^s are then extended to interpretations I_t^s, I_a^s by the following conditions.

- $I_t^s(p)$=$\mathbf{t}$ iff $V_t^s(p)$=$\mathbf{t}$
- $I_a^s(p)$=1 iff $V_a^s(p)$=1
- $I_t^s(\sim A)$=$\mathbf{t}$ iff $I_t^s(A)$=$\mathbf{f}$
- $I_a^s(\sim A)$=1 iff $I_a^s(A)$=1

- $I_t^s(A \wedge B)$=**t** iff $I_t^s(A)$=**t** and $I_t^s(B)$=**t**
- $I_a^s(A \wedge B)$=1 iff ($I_t^s(A \wedge B)$=**t** and $I_a^s(A)$=1 and $I_a^s(B)$=1) or ($I_t^s(A)$=**f** and $I_a^s(A)$=1) or ($I_t^s(B)$=**f** and $I_a^s(B)$=1))

The definition of $I_a^s(A \wedge B)$ may seem strange. Actually, we just consider it as the **S5** system of the epistemic logic. For example, we consider the $I_a^s(A \wedge B) = 1$ as $\Box_a(A \wedge B) \vee \Box_a \neg(A \wedge B)$. Therefore there exists three possible cases in all **S5** models that

- $A \wedge B$ and $\Box_a A \vee \Box_a \neg A$ and $\Box_a B \vee \Box_a \neg B$;
- $\neg A$ and $\Box_a A \vee \Box_a \neg A$;
- $\neg B$ and $\Box_a B \vee \Box_a \neg B$.

which is the same as the definition we showed above. Also, we can see the semantics more clearly by the truth table.

Definition 7. A *four-valued interpretation* of $\mathcal{L}$ is a function I_4^s : Prop $\rightarrow \{\mathbf{t}1, \mathbf{t}0, \mathbf{f}0, \mathbf{f}1\}$. Given a four-valued interpretation I_4^s, this is extended to a function that assigns every formula a truth value by the following truth functions:

A	$\sim A$	$A \wedge B$	**t**1	**t**0	**f**0	**f**1
t1	**f**1	**t**1	**t**1	**t**0	**f**0	**f**1
t0	**f**0	**t**0	**t**0	**t**0	**f**0	**f**1
f0	**t**0	**f**0	**f**0	**f**0	**f**0	**f**1
f1	**t**1	**f**1	**f**1	**f**1	**f**1	**f**1

We introduce three different sets of designated values as follows:

- $\mathcal{D}_1^s := \{\mathbf{t}1\}$;
- $\mathcal{D}_2^s := \{\mathbf{t}1, \mathbf{t}0\}$;
- $\mathcal{D}_3^s := \{\mathbf{t}1, \mathbf{t}0, \mathbf{f}0\}$.

Based on these sets of designated values, we define three consequence relations as follows.

Definition 8. For all $\Gamma \cup \{A\} \subseteq$ Form, $\Gamma \models_i^s A$ iff for all four-valued interpretations I_4^s, $I_4^s(A) \in \mathcal{D}_i$ if $I_4^s(B) \in \mathcal{D}_i^s$ for all $B \in \Gamma$, where $i \in \{1, 2, 3\}$.

Then, we can show the facts that:

- $\models_1^s$ is the strong Kleene logic;
- $\models_2^s$ is the classical logic;
- $\models_3^s$ is the logic of paradox.

We first deal with the case in which **t**1 is the only designated value. To show the first fact, we prepare a lemma.

Lemma 1. *For all strong Kleene three-valued valuation v_3^s for $\mathcal{L}$, there is a four-valued valuation v_4^s such that for all $A \in$ Form, (i) $I_4^s(A) = \mathbf{t}1$ iff $I_3^s(A) = \mathbf{t}$, and (ii) $I_4^s(A) = \mathbf{f}1$ iff $I_3^s(A) = \mathbf{f}$.*

Proof. Given a three-valued valuation v_3^s, we define v_4^s : Prop $\rightarrow \{\mathbf{t}1, \mathbf{t}0, \mathbf{f}0, \mathbf{f}1\}$ as follows:

$$v_4^s(p) = \begin{cases} \mathbf{t}1 & v_3^s(p) = \mathbf{t} \\ \mathbf{f}0 & v_3^s(p) = \mathbf{b} \\ \mathbf{f}1 & v_3^s(p) = \mathbf{f} \end{cases}$$

Then we prove the desired result by induction on the complexity of the formula. For the base case, the desired result holds by the definition of v_4^s. For the induction step, we split the cases depending on the form of the formula A.

If A is of the form $\sim B$, then for (i), we have $I_4^s(A) = \mathbf{t}1$ iff $I_4^s(\sim B)$=**t**1 iff $I_4^s(B)$=**f**1 (by def. of I_4^s) iff $I_3^s(B)$=**f** (by IH) iff $I_3^s(\sim B)$=**t** (by def. of I_3^s) iff $I_3^s(A)$=**t**. For (ii), $I_4^s(A) = \mathbf{f}1$ iff $I_4^s(\sim B)$=**f**1 iff $I_4^s(B)$=**t**1 (by def. of I_4^s) iff $I_3^s(B)$=**t** (by IH) iff $I_3^s(\sim B)$=**f** (by def. of I_3^s) iff $I_3^s(A)$=**f**.

If A is of the form $B \wedge C$, then for (i), $I_4^s(A)$=**t**1 iff $I_4^s(B \wedge C)$=**t**1 iff $I_4^s(B)$=**t**1 and $I_4^s(C)$=**t**1 (by def. of I_4^s) iff $I_3^s(B)$=**t** and $I_3^s(C)$=**t** (by IH) iff $I_3^s(B \wedge C)$=**t** (by def. of I_3^s) iff $I_3^s(A)$=**t**. For (ii), $I_4^s(A)$=**f**1 iff $I_4^s(B \wedge C)$=**f**1 iff $I_4^s(B)$=**f**1 or $I_4^s(C)$=**f**1(by def. of I_4^s) iff $I_3^s(B)$=**f** or $I_3^s(C)$=**f**(by IH) iff $I_3^s(B \wedge C)$=**f** (by def. of I_3^s) iff $I_3^s(A)$=**f**.

The case for disjunction is similar. □

We are now ready to prove one of the directions.

Proposition 1. *For $\Gamma \cup \{A\} \subseteq$ Form, if $\Gamma \models_1^s A$ then $\Gamma \models_{\mathbf{SK}} A$.*

Proof. Suppose $\Gamma \not\models_{\mathbf{SK}} A$. Then, there is a three-valued valuation v_3^s : Prop $\rightarrow \{\mathbf{t}, \mathbf{b}, \mathbf{f}\}$ such that $I_3^s(B) = \mathbf{t}$ for all $B \in \Gamma$ and $I_3^s(A) \neq \mathbf{t}$. Now, in view of (i) of Lemma 1, there is a four-valued valuation v_4^s such that $I_4^s(B)$=**t**1 for all $B \in \Gamma$ and $I_4^s(A) \neq \mathbf{t}1$, namely $\Gamma \not\models_1^s A$, as desired. □

For the other direction, we prepare another lemma.

Lemma 2. *For all four-valued valuation v_4^s for $\mathcal{L}$, there is a strong Kleene three-valued valuation v_3^s such that for all $A \in$ Form, (i) $I_3^s(A) = \mathbf{t}$ iff $I_4^s(A) = \mathbf{t}1$, and (ii) $I_3^s(A) = \mathbf{f}$ iff $I_4^s(A) = \mathbf{f}1$.*

Proof. Given a four-valued valuation v_4^s, we define v_3^s : Prop $\rightarrow \{\mathbf{t}, \mathbf{b}, \mathbf{f}\}$ as follows:

$$v_3^s(p) = \begin{cases} \mathbf{t} & v_4^s(p) = \mathbf{t}1 \\ \mathbf{b} & v_4^s(p) = \mathbf{t}0 \text{ or } v_4^s(p) = \mathbf{f}0 \\ \mathbf{f} & v_4^s(p) = \mathbf{f}1 \end{cases}$$

Then we prove the desired result by induction. □

Then, the proof is similar to the above case.

Proposition 2. *For $\Gamma \cup \{A\} \subseteq$ Form, if $\Gamma \models_{\mathbf{SK}} A$ then $\Gamma \models_1^s A$.*

Proof. Suppose $\Gamma \not\models_1^s A$. Then, there is a four-valued valuation $v_4^s : \mathsf{Prop} \to \{\mathbf{t}1, \mathbf{t}0, \mathbf{f}0, \mathbf{f}1\}$ such that $I_4^s(B) = \mathbf{t}1$ for all $B \in \Gamma$ and $I_4^s(A) \neq \mathbf{t}1$. Now, in view of (i) of Lemma 2, there is a three-valued valuation v_3^s such that $I_3^s(B) = \mathbf{t}$ for all $B \in \Gamma$ and $I_3^s(A) \neq \mathbf{t}$, namely $\Gamma \not\models_{\mathbf{SK}} A$, as desired. □

In view of the above propositions, we obtain the following.

Theorem 1. *For all* $\Gamma \cup \{A\} \subseteq \mathsf{Form}$, $\Gamma \models_{\mathbf{SK}} A$ *iff* $\Gamma \models_1^s A$.

In other words, this semantics is equivalent to the strong Kleene logic.

Then, consider the case for the logic of paradox, in which $\mathbf{t}1$, $\mathbf{t}0$ and $\mathbf{f}0$ are taken as designated values. In fact, the proofs are basically the same with the cases for the strong Kleene logic.

Theorem 2. *For all* $\Gamma \cup \{A\} \subseteq \mathsf{Form}$, $\Gamma \models_{\mathbf{LP}} A$ *iff* $\Gamma \models_3^s A$.

Finally, we consider the case in which $\mathbf{t}1$ and $\mathbf{t}0$ are designated. Actually, the proof is just the same as $\models_2^w$ of Semantics (I) which we showed in (Song et al., 2021). Therefore, we can obtain the following theorem.

Theorem 3. *For all* $\Gamma \cup \{A\} \subseteq \mathsf{Form}$, $\Gamma \models_{\mathbf{CL}} A$ *iff* $\Gamma \models_2^s A$.

4 A MANY-VALUED SEMANTICS FOR MULTI-AGENT SYSTEM

In the previous section, we give two different semantics for the different readings of **unknown**. If we consider the two reading of **unknown** as two different epistemic states, we can give a more general semantics.

Actually, we can consider the epistemic states as following:

- The agent knows that A is true or agent knows that A is false. ($I_a^s(A) = 1$ in Semantics (I) or $I_a^w(A) = 1$ in Semantic (II))
- The agent doesn't know the content of A. ($I_a^w(A) = 0$ in Semantics (I))
- The agent considers that A is possibly true and A is possibly false, i.e., the agent knows the contents of A but doesn't know whether A is true or not. ($I_a^s(A) = 0$ in Semantic (II))

Therefore, we combine the two semantics above to extend the valuation V_a to three-valued $\{1, 0.5, 0\}$ and read the values as following:

- $V_a(p) = 1$: Agent a knows the content of p and whether p is true or not.
- $V_a(p) = 0$: Agent a doesn't know the content of p.
- $V_a(p) = 0.5$: Agent a knows the content of p but doesn't know whether p is true or not.

According to the consideration above, first, we give the semantics for a single agent.

4.1 Many-valued Semantics for Single Agent

Definition 9. A many-valued semantics for the language $\mathcal{L}$ is a pair $\langle V_t, V_a \rangle$, where $V_t : \mathsf{Prop} \to \{\mathbf{t}, \mathbf{f}\}$ and $V_a : \mathsf{Prop} \to \{0, 0.5, 1\}$. Valuations V_t, V_a are then extended to interpretations I_t, I_a by the following truth table. Here, to show the truth table clearly, we write the valuation $I_6 : \mathsf{Prop} \to \{\mathbf{t}1, \mathbf{t}0.5, \mathbf{t}0, \mathbf{f}0, \mathbf{f}0.5, \mathbf{f}1\}$ as a 6-valued logic instead of the pair (I_t, I_a).

A	$\sim A$
t1	**f**1
t0.5	**f**0.5
t0	**f**0
f0	**t**0
f0.5	**t**0.5
f1	**t**

$A \wedge B$	**t**1	**t**0.5	**t**0	**f**0	**f**0.5	**f**1
t1	**t**1	**t**0.5	**t**0	**f**0	**f**0.5	**f**1
t0.5	**t**0.5	**t**0.5	**t**0	**f**0	**f**0.5	**f**1
t0	**t**0	**t**0	**t**0	**f**0	**f**0	**f**0
f0	**f**0	**f**0	**f**0	**f**0	**f**0	**f**0
f0.5	**f**0.5	**f**0.5	**f**0	**f**0	**f**0.5	**f**1
f1	**f**1	**f**1	**f**0	**f**0	**f**1	**f**1

We introduce three different sets of designated values as follows:

- $\mathcal{D}_1 := \{\mathbf{t}1\}$, if we ask the agent that what true statement it knows;
- $\mathcal{D}_2 := \{\mathbf{t}1, \mathbf{t}0.5, \mathbf{t}0\}$, if we ask that what true statements are(may be answered by an omniscient agent);
- $\mathcal{D}_3 := \{\mathbf{t}1, \mathbf{t}0.5, \mathbf{f}0.5\}$, if we ask the agent that what statement the agent considers that may be true.

Based on these sets of designated values, we define the consequence relations as follows.

Definition 10. For all $\Gamma \cup \{A\} \subseteq \mathsf{Form}$, $\Gamma \models_i A$ iff for all four-valued interpretations I_6, $I_6(A) \in \mathcal{D}_i$ if $I_6(B) \in \mathcal{D}_i$ for all $B \in \Gamma$, where $i \in \{1, 2, 3\}$.

Then, we can show the facts that:

- $\models_2$ is the classical logic;

- $\models_3$ is the S_{fde} logic.

The proof of $\models_2$ is just the same as above because it is just the classical logic if we only take care of V_t and I_t. To deal with the case of $\models_3$, we prepare two lemmas.

Lemma 3. *For all S_{fde} valuation v_{sfde} for $\mathcal{L}$, there is a six-valued valuation v_6 such that for all $A \in$ Form,*

- *(i) $I_6(A) = \mathbf{t}1$ iff $I_{sfde}(A) = \mathbf{t}$;*
- *(ii) $I_6(A) \in \{\mathbf{t}0.5, \mathbf{f}0.5\}$ iff $I_{sfde}(A) = \mathbf{b}$;*
- *(iii) $I_6(A) \in \{\mathbf{t}0, \mathbf{f}0\}$ iff $I_{sfde}(A) = \mathbf{n}$;*
- *(iv) $I_6(A) = \mathbf{f}1$ iff $I_{sfde}(A) = \mathbf{f}$;*

Proof. Given a S_{fde} valuation v_{sfde}, we define v_6 : Prop $\rightarrow \{\mathbf{t}1, \mathbf{t}0.5, \mathbf{t}0, \mathbf{f}0, \mathbf{f}0.5, \mathbf{f}1\}$ as follows:

$$v_6(p) = \begin{cases} \mathbf{t}1 & v_{sfde}(p) = \mathbf{t} \\ \mathbf{f}0.5 & v_{sfde}(p) = \mathbf{b} \\ \mathbf{f}0 & v_{sfde}(p) = \mathbf{n} \\ \mathbf{f}1 & v_{sfde}(p) = \mathbf{f} \end{cases}$$

Then we prove the desired result by induction. □

Lemma 4. *For all six-valued valuation v_6 for $\mathcal{L}$, there is a strong Kleene three-valued valuation v_{sfde} such that for all $A \in$ Form,*

- *(i) $I_{sfde}(A) = \mathbf{t}$ iff $I_6(A) = \mathbf{t}1$;*
- *(ii) $I_{sfde}(A) = \mathbf{b}$ iff $I_6(A) \in \{\mathbf{t}0.5, \mathbf{f}0.5\}$;*
- *(iii) $I_{sfde}(A) = \mathbf{n}$ iff $I_6(A \in \{\mathbf{t}0, \mathbf{f}0\}$;*
- *(iv) $I_{sfde}(A) = \mathbf{f}$ iff $I_6(A) = \mathbf{f}1$;*

Proof. Given a six-valued valuation v_6, we define v_{sfde} : Prop $\rightarrow \{\mathbf{t}, \mathbf{b}, \mathbf{n}, \mathbf{f}\}$ as follows:

$$v_{fde}(p) = \begin{cases} \mathbf{t} & v_6(p) = \mathbf{t} \\ \mathbf{b} & v_6(p) = \mathbf{t}0.5 \text{ or } v_6(p) = \mathbf{f}0.5 \\ \mathbf{n} & v_6(p) = \mathbf{t}0 \text{ or } v_6(p) = \mathbf{f}0 \\ \mathbf{f} & v_6(p) = \mathbf{f}1 \end{cases}$$

Then we prove the desired result by induction. □

Also, it is easy to see that $\models_1$ is the logic if we replace the designate values $\mathcal{D}$ of S_{fde} by $\mathcal{D} = \{\mathbf{t}\}$.

4.2 Many-valued Semantics for Multi-agent System

Consider that there are several agents, therefore there should be several valuations of V_a. Then we give a many-valued semantics for multi-agent system.

Definition 11. A many-valued semantics for the language $\mathcal{L}$ is a pair $\langle V_t, \{V_a\}_{a \in Ag} \rangle$, where Ag is a non-empty set of agents, V_t : Prop $\rightarrow \{\mathbf{t}, \mathbf{f}\}$ and V_a : Prop $\rightarrow \{0, 0.5, 1\}$. Valuations V_t, V_a are then extended to interpretations I_t, I_a by the following conditions.

- I_t is the same as the classical logic.
- $I_a(p) = V_a(p)$
- $I_a(\sim A) = 1 - I_a(A)$
- $I_a(A \wedge B) = 1$ iff ($I_t(A \wedge B) = \mathbf{t}$ and $I_a(A) = 1$ and $I_a(B) = 1$) or ($I_t(A) = \mathbf{f}$ and $I_a(A) = 1$ and $I_a(B) \neq 0$) or ($I_t(B) = \mathbf{f}$ and $I_a(B) = 1$ and $I_a(A) \neq 0$))
- $I_a(A \wedge B) = 0.5$ iff ($I_t(A \wedge B) = \mathbf{f}$ and $I_a(A) = 0.5$ and $I_a(B) = 0.5$) or ($I_t(A) = \mathbf{t}$ and $I_a(A) = 0.5$ and $I_a(B) \neq 0$) or ($I_t(B) = \mathbf{t}$ and $I_a(B) = 0.5$ and $I_a(A) \neq 0$))
- $I_a(A \wedge B) = 0$ iff $I_a(A) = 0$ or $I_a(B) = 0$

Also, we introduce some different sets of designated values as follows. To see it clearly, we write the value as $I_t, I_{a_1}, ... I_{a_n}$.

- $\mathcal{D}_1^k := \{\mathbf{t}, I_{a_1}, ..., I_{a_n} : I_{a_k} = 1\}$, if we ask the agent k that what true statements k knows;
- $\mathcal{D}_2^k := \{\mathbf{t}, I_{a_1}, ..., I_{a_n} : I_{a_j} \in \{1, 0.5, 0\} (j \in (1, n))\}$, if we ask that what true statements are(may be answered by an omniscient agent);
- $\mathcal{D}_3^k := \{\mathbf{t}, I_{a_1}, ..., I_{a_n} : I_{a_k} \in \{1, 0.5\}\} \cup \{\mathbf{f}, I_{a_1}, ..., I_{a_n} : I_{a_k} = 0.5\}$, if we ask the agent k what agent k considers that may be true.

Based on these sets of designated values, we define three consequence relations as follows.

Definition 12. For all $\Gamma \cup \{A\} \subseteq$ Form, $\Gamma \models_i^k A$ iff for all interpretations I, $I(A) \in \mathcal{D}_i^k$ if $I(B) \in \mathcal{D}_i^k$ for all $B \in \Gamma$, where $i \in \{1, 2, 3\}$.

We can show the facts that $\models_2^k$ is the classical logic and $\models_3^k$ is the S_{fde} logic. The proof is the same as case of a single agent.

5 AGENT COMMUNICATION

We can see that, in the many-valued semantics for multi-agent system, the valuation shows both the classical values and the epistemic states of each agent. In other words, the valuation can be considered as a model like the Kripke model we use in dynamic epistemic logic. Therefore, we can consider several kinds of valuation change like the dynamic operators to express the agent communication. First, we give a definition of consequence relation which is like the epistemic logic.

Definition 13. Let Ag be a non-empty set of agents, Prop be the set of propositions and $V = \{V_t, \{V_a\}_{a \in Ag}\}$ be the valuation where V_t : Prop $\rightarrow \{t, f\}$ and V_a : Prop $\rightarrow \{1, 0.5, 0\}$. Then we can define the satisfied functions $\models_a^t$ and $\models_a^{mt}$ as following:

$$\begin{array}{lll} V \models_a^t A & \text{iff} & I_t(A) = \mathbf{t} \text{ and } I_a(A) = 1 \\ V \models_a^{mt} A & \text{iff} & (I_t(A) = \mathbf{t} \text{ and } I_a(A) = 1) \\ & & \text{or } I_a(A) = 0.5 \end{array}$$

Semantically speaking, $V \models^t_a A$ means that agent a knows that A is true, and $V \models^{mt}_a A$ means that agent a considers that A may be true. Actually, $\models^t_a$ stands for the $\models^k_1$ and $\models^{mt}_a$ stands for the $\models^k_3$ we showed in Section 4.

In dynamic epistemic logic, there are several operators to show the communication between agents. (Van Ditmarsch, 2014) showed the semantics of agent announcement that one agent tells others some statement instead of the public announcement. (Hatano et al., 2015) considered the channel communication as the semi-private announcement. Unlike the studies of dynamic epistemic logic above, we do not use a Kripke model so that we cannot show all of the belief changes. However, it is very simple to show a certain kind of changes of knowledge by our semantics. Moreover, we can give some new ideas by considering two kinds of agent communication: *teaching* and *asking*.

5.1 Teaching

Here, we consider that *teaching* is the communication from agent a to a set of agent G that a will tell every proposition he/ she knows to the member of G. We write such agent communication by the operator $\downarrow^a_G$. Semantically speaking, *teaching* is considered as the act that the teacher teaches the knowledge to the students. If G has only one member, then the act can be considered as the semi-private announcement. If G is the set of all agents, then the act can be considered as the agent announcement.

Definition 14. Let the original model be V. After the act that agent a teaches the group G, the new valuation $V^{\downarrow^a_G}$ is defined as following:

For all $p \in \mathsf{Prop}$,

- If $i \notin G$, then $V_i^{\downarrow^a_G}(p) = V_i(p)$, and
- If $i \in G$, then

$$V_i^{\downarrow^a_G}(p) := \begin{cases} 1 & \text{if } V_a(p) = 1 \\ V_i(p) & \text{otherwise.} \end{cases}$$

Semantically speaking, the students of a can acquire the knowledge that is known to a, and the epistemic states of other agents will not change.

5.2 Asking

Consider the situation that a student asks a teacher questions. The student possesses several propositions that he/ she doesn't know whether true or false, and if the teacher knows that, the student can obtain the answer to know that proposition. It seems that *asking* is just the opposite of *teaching* so that the results that a teaches b and b asks a are the same. However, actually sometimes the two cases are different for the epistemic states of teacher may also be changed by asked questions. For example, a is the teacher and b is the student, let $V_a(p) = 0$ and $V_b(p) = 0.5$, b doesn't know whether p is true or not, so he/ she will ask a about p, while the teacher doesn't know even the content of p at the moment. Therefore, after *asking*, student b cannot obtain an answer, while teacher a becomes to know the content of p. Then, we define the model $V^{\uparrow^a_b}$ that show the model after b asks a questions as following:

Definition 15. For all $p \in \mathsf{Prop}$,

- If $i \notin \{a, b\}$, then $V_i^{\uparrow^a_b}(p) = V_i(p)$;
- If $i = a$, then

$$V_i^{\uparrow^a_b}(p) := \begin{cases} 1 & \text{if } V_a(p) = 0.5 \text{ and } V_b(p) = 1 \\ V_i(p) & \text{otherwise.} \end{cases}$$

- If $i = b$, then

$$V_b^{\uparrow^a_b}(p) := \begin{cases} 0.5 & \text{if } V_a(p) = 0.5 \text{ and } V_b(p) = 0 \\ V_b(p) & \text{otherwise.} \end{cases}$$

6 CONCLUSION AND FUTURE WORK

In this paper, we showed a many-valued semantics to express the epistemic states in multi-agent system, instead of epistemic logic. Firstly, we introduced two pair semantics to express the different considerations of knowledge, and showed that they could be considered as the two three-valued logic: weak Kleene logic and strong Kleene logic. Secondly, we gave a new semantics by combining the two semantics and the two states of unknown, and extended it to express the epistemic states of multi agents. Moreover, we gave two kinds of agent communication,*teaching* and *asking*, and showed the results of them could be different.

There remain several future works.

- In this paper, we ignored the situation of misunderstanding, i.e., agent a believes p is true while in fact p is false. Here, we considered the multi-agent system as a knowledge system, which we often used **S5** system in the epistemic logic. To consider a belief system, we may add a new epistemic value to express the state of misunderstanding.
- Using our semantics, there can be more kinds of agent communication besides what we introduced. For example, *discussing* makes a group of agents exchange their knowledge, *forgetting* lets some agents lose some information they knew before, etc..

- In this paper, we showed the relations between our semantics and several many-valued logics. Actually, we can also find some connection with other logics. For example, Fan considered a epistemic operator $\triangle$ to express *knowing whether* in (Fan et al., 2015), which has the similar meaning of the value 0.5 in our semantics. In the awareness logic(Fagin and Halpern, 1987), the epistemic states were considered as known with *awareness, unknown with awareness* and *unaware*, which had the similar readings of our semantics. Thus, we can make use of our semantics in the epistemic logic and the awareness logic.

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