On Configuring Directional Transmission for Path Exposure Reliability in Energy Harvesting Wireless Sensor Networks

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Abstract: In this paper, we consider a path exposure problem in energy harvesting wireless sensor networks (EH-WSNs) where nodes are equipped with directional communication devices. Nodes harvest energy from ambient environment (e.g., solar power), and manage fluctuations in their stored energy by adjusting some of their directional transmission parameters. Using a probabilistic graph model we formalize a problem, denoted DirEXPO-RU, that quantifies the ability of a network to detect and report traversal along a given path as probability representing the reliability of the network in performing the path monitoring task. A problem that arises in managing the network resources to maximize this reliability measure is to adjust the transmission beam width of each node, given that nodes beam centers are given as input. We develop a heuristic algorithm to deal with the problem, and use the algorithm in a framework for computing lower bounds on the reliability of the overall network. The obtained numerical results show improvement in the achieved network reliability.

1 INTRODUCTION

Research work on wireless sensor networks (WSNs) where nodes are capable of directing their transmission (and/or reception) has been receiving attention for a number of years now (see, e.g., the surveys of (Amac Guvensan and Gokhan Yavuz, 2011; Tao and Wu, 2015) for applications in area and barrier coverage). In general, directional communication offers improvement over omnidirectional transmission in terms of achieving longer range (in a certain direction) for a given level of energy consumption.

Our interest in this paper is on investigating the benefits of using this mode of transmission in the class of energy harvesting wireless sensor networks (EH-WSNs) that is currently receiving significant attention. In particular, we are interested in quantifying the benefits of directional transmission to achieve good network performance while taking a complex network reliability measure as our objective function. We pursue this goal in the context of investigating a path exposure reliability measure.

The path exposure problem is a well-known WSN problem where we want to compute the probability of detecting a moving target along a given path (over a period of time) through a WSN field. Early results on the problem appears in (Megerian et al., 2002; Clouqueur et al., 2003). In (Megerian et al., 2002), e.g., the authors present a model that utilizes energy sensed from a moving target over a period of time. The model is then used to find a path of minimal exposure using a shortest path algorithm. In (Clouqueur et al., 2003), the authors analyze target detection probability using different data fusion models, and present a method to minimize network deployment cost while achieving desired exposure levels. Subsequently, in (Liu et al., 2009), the authors formalize a minimal path exposure problem for WSNs employing directional communication. They develop a directional sensing model used to define two weighted graphs that are used to reduce the minimal path exposure problem into two discrete geometry problems. Their results include developing two approximation algorithms to solve the problem.

Our work here on a path exposure problem builds on the results obtained in (Elmorsy et al., 2013; Basabaa and Elmallah, 2019; Basabaa and Elmallah, 2020). In (Elmorsy et al., 2013), the authors formalize a path exposure problem, called EXPO, on conventional WSNs for surveillance applications where nodes are subject to communication and/or sensing failure. In (Basabaa and Elmallah, 2019; Basabaa and Elmallah, 2020), we consider a restricted version of the DirEXPO-RU problem, called EXPO-RU. In (Basabaa and Elmallah, 2019), the EXPO-RU problem is formalized and an efficient algorithm is pre-

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Table 1: Overview of some related results.

Reference	Problem	Obtained results			
		Non-iterative improvable algorithms		Iterative improvable algorithms	
		LB	UB	LB	UB
Current paper	DirEXPO-RU	Configuring node's beam width			
(Basabaa and Elmallah, 2021)	EXPO-RU		✓		✓
(Basabaa and Elmallah, 2019)	EXPO-RU	✓		✓	
(Basabaa and Elmallah, 2020)	EXPO-RU	✓	✓		
(Elmorsy et al., 2013)	EXPO			✓	✓

sented to compute lower bounds on the exact solution of the EXPO-RU problem. In (Basabaa and Elmallah, 2020), a dynamic programming approach is presented to compute bounds on exact solutions of the EXPO-RU problem from certain network subgraphs.

The main focus of this paper is on developing approaches to configure the beam width for each node in the network, so as to obtain good path exposure reliability for the entire network, as described in the next sections.

The rest of the paper is organized as follows: Section 2 gives an overview of related work. The used system model is presented in Section 3. Section 4 discusses problem formulation and configuration approaches. Based on the proposed configuration approaches, Section 5 presents an efficient heuristic algorithm to configure the transmission beams of nodes. Section 6 explains a framework to compute bounds for the DirEXPO-RU problem, and lastly the performance of the proposed approaches is evaluated in Section 7.

2 LITERATURE REVIEW

Our work here lies in the intersection of three main research areas: the general area of EH-WSNs, research specific to the use of directional communication in WSNs, and research on developing reliability assessment methods using probabilistic graph models. In the following, we touch on each of the three areas.

2.1 Work on EH-WSNs

We refer the reader to the surveys in (Adu-Manu et al., 2018; Sudevalayam and Kulkarni, 2011). Specific algorithmic contributions in the area include the work of (Jakobsen et al., 2010; Peng and Low, 2013; Martínez et al., 2014; Peng et al., 2015; Li et al., 2020). In (Jakobsen et al., 2010), the authors present a distributed routing algorithm for EH-WSNs that aims at dynamically finding sustainable routes to the sink. In (Peng and Low, 2013), the authors present an energy neutral WSN routing algorithm based on the Directed Diffusion paradigm by adding an admission control step before a node can reinforce a flow that serves a particular interest. In (Martínez et al., 2014), the authors consider wastage of harvested energy when a node's battery reaches its limit and can not be charged. The authors formalize a route selection problem that aims at reducing network wide overcharging-wastage in each time slot of the network's operation. In (Peng et al., 2015), the authors present an energy neutral clustering algorithm for EH-WSNs that divides system's time into time slots. In each time slot, the network is partitioned into a user-specified number of clusters and the data is collected in each cluster. In (Li et al., 2020), the authors propose a priority task scheduling algorithm for EH-WSNs, where the transmission method and order of collected data are determined according to task priority and node's remaining energy.

2.2 Work on Directional Communication in WSNs

Examples of work done in this area include the work of (Duan et al., 2019) where the authors consider data collection in a rechargeable sensor network that is powered by directional wireless power transfer (DWPT). The DWPT provides directional energy beams using higher antenna gain for increasing energy efficiency. Also, the work of (Zhang et al., 2009; Zhu et al., 2019), where the authors in (Zhang et al., 2009) study a strong barrier coverage problem in directional sensor networks. They present an integer linear programming model, and present two efficient algorithms to solve the problem. The authors in (Zhu et al., 2019) discuss the problem of deploying environmental energy harvesting directional sensor networks with the minimum cost, where targets have different coverage requirements and the average energy harvesting rate of each node should not be less than its consumption rate. They formulate a mixed integer linear programming problem, and propose three heuristics algorithms to solve the problem.

2.3 Work on WSNs Reliability for a Path Exposure Problem

Here, we position the work done in this paper among a few results that we have recently obtained on the EXPO-RU problem. Table 1 gives an overview of the results. In Table 1, the EXPO problem is the path exposure reliability problem on conventional WSNs where nodes are subject to failure (either in communication or sensing). Methods of assessing the network reliability are classified as being either *iteratively improvable* or *non-iteratively improvable*. The earlier type can achieve exact results if allowed to work to completion. Else, they provide either a lower bound (LB), or and upper bound (UB). In contrast, the latter type produces a single bound (LB or UB) on each input. Our work in this paper on configuring beam width in directional transmission utilizes an iteratively improvable LB algorithm to obtain numerical results.

3 OVERVIEW OF SYSTEM MODEL

In this section, we introduce the needed assumptions and notations about the node model, the network model, and the directional transmission model.

3.1 Node Operation Model

We assume that each node x in a given EH-WSN is equipped with a directional communication device, and an omnidirectional sensing device. Energy harvested at each node x fluctuates over time. For the purpose of simulating the network (e.g., to obtain the probability distributions mentioned below), we assume that time is divided into equal length slots.

Since wireless transmission is the most energy consuming activity in many WSNs, we assume that such fluctuations affect the transmission range of a node (but does not affect much its sensing range). An energy management unit (EMU) in each node controls a node's state during any time slot. Our work employs a multi-state node model for each node. In its basic form, the model associates 3 states with each non-sink node x, defined as follows.

- If x's stored energy level during a time slot lies in some high range (say, e.g., [70%-100%]) of its total energy storage capacity then the EMU puts x in a *full* energy state where the node's maximum transmission range, denoted R_{full}, is high.
- Else, if x's stored energy level during a time slot is in some lower range (say, e.g., [40%-70%]) of its total energy storage capacity then the EMU puts x in a *reduced* energy state where the node's maximum transmission range, denoted R_{red}, is lower.
- Else, if x's stored energy level is lower than the above, we consider the node to be in a *failed* state.

3.2 Network Reliability Model

Similar to our previous work in (Basabaa and Elmallah, 2019), we adopt a network reliability model that abstracts the above node dynamics over a long operation time by associating with each node x a probability distribution where for each possible state $s \in \{full, reduced, fail\}$ we know the probability $p_s(x)$ (also denoted $p(x,s)$) that node x is in state s (here, $p_{fail}(x) = 1 - p_{full}(x) - p_{red}(x)$). In addition, we assume that different nodes are assigned different states independently of each other. Such an assumption is common in the literature to simplify the analysis, and also to take care of applications where different nodes perform different tasks independent of each other.

In overall, an EH-WSN is modelled over a long period of time by a *probabilistic graph* (G, p) where $G = (V \cup \{sink\}, E)$ is a directed graph on a set V of EH wireless nodes, a non-EH and fully operational sink node, and a set E of **directed** communication links (also referred to as directed edges or arcs). The length of each arc $(x,y) \in E$ emanating from a node x depends on the energy state of x and also the directional transmission parameters associated with x, as explained below. We note that G is a directed multigraph with parallel arcs. Parallel arcs from a node a to a node b arise since if (a,b) exists when a is in the full energy state then (a,b) may also exist when a is in the reduced energy state (yet, they are considered two different arcs).

3.3 Node Directional Communication Model

We assume that all nodes in G are deployed in a 2-dimensional space, where each node has an (x,y)-coordinate. The transmission beam of each node x (when x is in either the full, or the reduced state) can be obtained from the node's *directionality* parameters, denoted $DIR_{comm}(x)$. Our model uses the following parameters: $DIR_{comm}(x) = \{\Theta_{mid}, \alpha_{full}, \alpha_{red}, R_{full}, R_{red}\}$ where

- Θ_{mid} is a counterclockwise (CCW) angle between two rays emanating from x. The first ray is a horizontal ray (i.e., parallel to the x-axis) that extends to the right. The second ray defines the middle of x's transmission beam, see, e.g., Fig. 1.
- For $\alpha = \alpha_{full}$, the angle $\Theta_{mid} - \alpha$ (or, $\Theta_{mid} + \alpha$) is a CCW angle between two rays emanating from x. The first ray is a horizontal ray as above. The second ray defines the start (respectively, the end) of x's full range transmission beam (thus, x's beam is of width 2α). When $\alpha = 180°$, the beam is of width $360°$, and node transmission becomes omnidirectional. A similar definition applies when $\alpha = \alpha_{red}$ to describe the reduced transmission beam.
- R_{full} (or, R_{red}) is the maximum x's transmission range when x is in the full (respectively, reduced) state, and the angle α is sufficiently narrow (e.g., $\alpha = 1°$).
- For a given half-width beam angle $\alpha = \alpha_{full}$, the actual full transmission range of a node x depends on R_{full} and the angle α. We use $R_{full}(\alpha)$ to refer to such an actual transmission range. In general, such a transmission range decreases as α increases. Similarly, we use $R_{red}(\alpha)$ to refer to a node's reduced transmission angle when $\alpha = \alpha_{red}$.

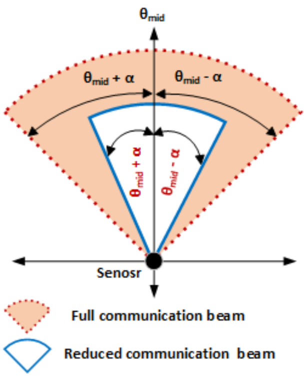


Figure 1: Node directional communication model.

In Section 7, we experiment with grid networks having diagonal links. Each grid network has a sink node located at (x, y)-coordinates $(0, 0)$, and the grid has horizontal (and vertical) links parallel to the x-axis (respectively, the y-axis), in addition to the diagonal links. The horizontal (and vertical) distance between two horizontally (respectively, vertically) adjacent nodes is set to 100 units. For full energy transmission, we set $R_{full}(\alpha = 1°) = 360$ units, and use a function that reduces the transmission range linearly so that $R_{full}(\alpha = 180°) = 180$ units (for omnidirectional transmission). Likewise, for reduced energy transmission, we set $R_{red}(\alpha = 1°) = 180$ units, and $R_{red}(\alpha = 180°) = 100$ units.

4 PROBLEM FORMULATION AND CONFIGURATION APPROACHES

In this section, we start by reviewing (in Section 4.1) the definition and key remarks on a WSN path exposure reliability assessment problem, called EXPO-RU (for path exposure with range uncertainty). The problem is defined on EH-WSNs where fluctuations in a node's stored energy affect primarily its transmission range.

In the context of designing EH-WSNs with directional communication nodes, we aim at developing methods to configure node transmission dynamically so as to increase the overall network path exposure reliability. Achieving this goal is non-trivial since the EXPO-RU assessment problem has been shown in (Basabaa and Elmallah, 2021) to be intractable (#P-hard), and there is no simple optimization criterion that can be used to adjust the node directionality parameters so as maximize the overall network reliability. As a starting point, and working toward our goal, we present in Section 4.2 two approaches for adjusting the directionality parameters, so as to obtain good network reliability performance.

4.1 Review of the EXPO-RU Problem

The EXPO-RU problem is defined in, e.g., (Basabaa and Elmallah, 2019). Several results on the problem appears in (Basabaa and Elmallah, 2020).

In the problem, we are given a WSN, and a path **P** across the network that we need to monitor against unauthorized traversal. Nodes that are in close proximity of **P** can sense the path. Such nodes are called *sensing nodes* (specified as part of the input). The input also specifies an integer $k_{req} \geq 1$ of sensing nodes that need to sense and report an intrusion event for the network to succeed in its monitoring task. As mentioned above, the EH-WSN reacts to fluctuations in a node's stored energy by adjusting the node's transmission range. The EXPO-RU problem uses a probabilistic graph (G, p) to model the network. In addition, the problem formulation assumes that different nodes behave independently of each other.

During a short random time interval, the network G is in some **network state** S where each node x is in some state $s_x \in \{full, reduced, fail\}$. We use $S = \{(x, s_x) : x \in V, s_x \in \{full, reduced, fail\}\}$ to refer to any such network state. Since nodes assume states independent of each other, the probability that a given network state S arises is $\Pr(S) = \prod_{(x,s_x) \in S} p(x, s_x)$. Each network state S is either *operating* or *failed*. To be operating, nodes in S should assign states such that at least k_{req} sensing nodes in S can reach the sink node. Else, S is in a *failed* state.

The EXPO-RU problem is called to compute the probability that the network G is in an operating state S. We denote such probability by $\mathrm{Expo}(G, p, \mathbf{P}, k_{req})$, or $\mathrm{Expo}(G, p)$ for short. Complete enumeration of all network states can be used to compute $\mathrm{Expo}(G, p)$ exactly. However, such an algorithm has a running time that grows exponentially with the number of nodes in G. More generally, it has been shown in (Basabaa and Elmallah, 2021) that the EXPO-RU problem is #P-hard.

In addition to the concept of network state, we need the following concepts to discuss our method for bounding the network reliability measure.
A **network configuration** C is a partial network state where some nodes (but not necessarily all) are assigned states (an empty configuration is valid) and the remaining unassigned nodes are free in C. A **Pathset** is an operating network configuration that has at least k_{req} sensing nodes that can reach the sink and monitor the given intrusion path $\mathbf{P}$. A *minpath* is a minimal pathset.

Example. Fig. 2 shows an instance of a 3×3 grid network with one unit of horizontal (or, vertical) spacing. The dashed lines (coloured red) represent communications in full power whereas the solid lines (coloured blue) represent communications in reduced power. In Fig. 2, if $k_{req} = 2$, then the configuration $C = \{(3, reduced), (4, full)\}$ is a pathset and, indeed, it is a minpath.

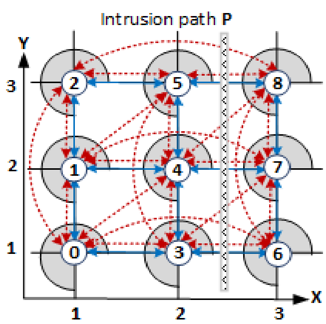


Figure 2: An instance of a 3×3 grid network.

In this paper, we use the name DirEXPO-RU to refer to the EXPO-RU problem when nodes employ directional transmission.

4.2 Approaches for Adjusting Directional Transmission

Given a node x in a state $s \in \{full, reduced\}$, we present in this section two approaches for configuring the directionality parameters of the node-state pair (x, s). We assume that the given problem instance specifies (as part of the input) the angle Θ_{mid} that determines the direction of the middle line of x's transmission beam (e.g., one may use the direction of the line segment between x and the sink node). To adjust the half-width angle $\alpha(x, s)$ for node x in state s, we experiment with the following two approaches that aim at maximizing the performance measure $Expo(G, p)$.

4.2.1 Approach 1

In this approach, we adjust the angle $\alpha(x, s)$ so as to maximize the out-degree, denoted $deg^+(x, s)$, of node x. Equivalently, we seek to maximize the number of out-neighbours reachable from node x in state s. We recall that the actual transmission range $R_{full}(\alpha)$ (or, $R_{red}(\alpha)$) decreases as α increases. The rationale of this approach is that the more directed links that exist in any network state S of the resulting probabilistic graph (G, p), the more likely that $Expo(G, p)$ increases.

We next remark that maximizing $deg^+(x, s)$ for the node-state pair (x, s) is a local operation to node x that does not depend on the directionality setting of other nodes. Our implementation of this approach (present in next section) performs sequential search for finding an optimized angle $\alpha(x, s)$ in the interval $[1°, 180°]$ in increments of some sufficiently small angle δ (e.g., $\delta = 1°$).

4.2.2 Approach 2

The above approach seeks to maximize the number of out-neighbours of node x in state s. In this approach, we consider the quality of such out-neighbours. In particular, this approach makes an effort to adjust the angle $\alpha(x, s)$ so as to give preference to include out-neighbour y of x depending on the quality of the best found directed path, denoted $P^*_{y,sink}$, from y to the sink node. Based on the quality of such a best found path, we associate with node y a preference weight, denoted $w(y)$, that takes on a value that is proportional to the goodness of the corresponding best found directed path $P^*_{y,sink}$.

We next remark that unlike the first approach, finding such a best path $P^*_{y,sink}$ depends on the directionality setting of nodes other than node x (including node y). To simplify the search for such a best path, we adopt a heuristic solution that assumes that each node z, $z \neq x$, operates in an omnidirectional (full or reduced) mode.

As done in approach 1, we perform a sequential search for a good setting of the angle $\alpha(x,s)$ in the range $[1°, 180°]$ in increments of some sufficiently small angle δ (e.g., $\delta = 1°$). At each search step, the angle $\alpha(x,s)$ is assigned a certain value, and each node $z \in V$, $z \neq x$, is assumed to be omnidirectional. With the angle $\alpha(x,s)$ assigned a specific value in each search step, node x can reach a subset of its possible out-neighbours, denoted $S_{\alpha(x,s)}$. With each possible out-neighbour y of x assigned a preference weight $w(y)$, we define the weight of the set $S_{\alpha(x,s)}$ to be $w(S_{\alpha(x,s)}) = \sum_{y \in S_{\alpha(x,s)}} w(y)$. Then, the search algorithm selects an angle $\alpha(x,s)$ that gives the highest $w(S_{\alpha(x,s)})$.

The details of computing the preference weight $w(y)$ of a possible out-neighbour y of x follows the following steps.

1. Let (a,b) be an arc in the probabilistic graph (G,p) that exists when node a is in state $s \in \{full, reduced\}$. We recall that typically the graph G has many parallel arcs since an arc (a,b) that exists when node a is in the full state may also exist when a is in the reduced state (and the two arcs are considered to be different). We associate with each such an arc (a,b) a cost, denoted $cost(a,b)$, that takes on a small value when the node state probability $p(a,s)$ takes on a high value (e.g., $cost(a,b) = -\log p(a,s)$).
2. The cost of a directed path $P_{(y,sink)}$ from a node y to the sink, denoted $cost(P_{(y,sink)})$, is the sum of the costs of its arcs.
3. We set the directed graph G_x to be G with node x deleted. Taking arc costs as distances in G_x and the sink node as a destination, we solve a single-destination shortest paths problem to find a shortest path from each potential out-neighbour y of x to the sink. We denote such a shortest path by $P^*_{y,sink}$.
4. Finally, we assign a preference value $w(y)$ to node y so that the value is inversely proportional to the $cost(P^*_{y,sink})$ (e.g., $w(y) = 1/cost(P^*_{y,sink})$).

We conclude this section by noting that the two above approaches also apply when each node has multiple states (i.e., not only the basic 3-state model used to explain the approaches).

5 MORE ALGORITHMIC DETAILS

In this section, we present a heuristic algorithm, called HWAS (for half-width angle selection), that utilizes the two proposed approaches (explained in Section 4.2) to configure the transmission beams of each individual node x in the network given its working direction Θ_{mid}. The devised HWAS algorithm produces two different types of results based on the selected approach.

5.1 Algorithm Idea

Consider an instance (G,p) of the DirEXPO-RU problem with directional parameters $\mathrm{DIR_{comm}}(x) = \{\Theta_{mid}, \alpha_{full}, \alpha_{red}, R_{full}, R_{red}\}$ for each node $x \in G$, where Θ_{mid}, R_{full}, and R_{red} are given. Our devised algorithm configures the directionality parameters for each individual node $x \in G$ by finding the half-width angle $\alpha(x,s)$ for node x in state s using *Approach 1* and *Approach 2*.

The algorithm utilizes an associative array, denoted $Best_\alpha$, to store such $\alpha(x,s)$ values. In other words, $Best_\alpha$ provides a key-value mapping, where a key is a tuple (x,s) of node x in state s (e.g., $s \in \{full, reduced\}$), and the corresponding value is $\alpha(x,s)$.

In particular, the algorithm configures the directionality parameters of the node-state pair (x,s) by performing the following steps:

1. Perform a sequential search for a good setting of the angle $\alpha(x,s)$ in the range $[1°, 180°]$ in increments of some sufficiently small angle δ (e.g., $\delta = 1°$), which implies adjusting the transmission beam of node x in state s to reach a subset of its possible out-neighbours, denoted $S_{\alpha(x,s)}$. After processing all $\alpha(x,s)$, we obtain a set $\{S_{\alpha_1(x,s)}, ..., S_{\alpha_{180}(x,s)}\}$ of possible out-neighbours of x that correspond to values of $\alpha(x,s)$.
2. We evaluate the obtained set, and select a member $S_{\alpha(x,s)}$ using either **Method-1** or **Method-2** (explained below).
3. Then, we set $Best_\alpha(x,s) = \alpha(x,s)$ that corresponds to the best subset $S_{\alpha(x,s)}$ of out-neighbours of x in state s.

5.1.1 Method-1

This method is based on *Approach 1* (explained in Section 4.2). It is a baseline method that selects $\alpha(x,s)$ for each node $x \in G$ based on the number of out-neighbours of x. The approach aims at increasing the reachablity of node x to maximize the performance measure $\text{Expo}(G, p)$. In particular, Method-1 works by selecting a member $S_{\alpha(x,s)}$ of $\{S_{\alpha_1(x,s)}, \ldots, S_{\alpha_{180}(x,s)}\}$ that has the highest number of out-neighbours of x. We set $Best_\alpha(x,s) = \alpha(x,s)$ corresponding to the selected member $S_{\alpha(x,s)}$.

5.1.2 Method-2

This method is based on *Approach 2*. It is a cost-based method that selects a subset $S_{\alpha(x,s)}$ for each node $x \in G$ based on *good out-neighbours* of x in state s, and hence can improve the performance measure $\text{Expo}(G, p)$. In particular, Method-2 works by selecting a member $S_{\alpha(x,s)}$ of the set $\{S_{\alpha_1(x,s)}, \ldots, S_{\alpha_{180}(x,s)}\}$ that has the highest weight, $w(S_{\alpha(x,s)})$, as explained in Section 4.2. Then, we set $Best_\alpha(x,s) = \alpha(x,s)$ corresponding to the selected member $S_{\alpha(x,s)}$ of out-neighbours of x in state s.

After computing α_{full} and α_{red} for each node $x \in G$, the actual $full$ (respectively, *reduced*) transmission range of a node x is calculated as explained in Section 4.2. Therefore, $full$ (respectively, *reduced*) arcs for each node $x \in G$ are added based on node's directionality parameters $\text{DIR}_{\text{comm}}(x)$.

Function $\text{HWAS}(G, p)$
Input: An instance (G, p) of the DirEXPO-RU problem, and $\text{DIR}_{\text{comm}}(x)$ for each node $x \in G$
Output: Compute $\alpha(x,s)$ where $x \in G$ and $s \in \{full, reduced\}$

1. set $Best_\alpha = \{\}$
2. **foreach** (node $x \in G$)
   {
3. **foreach** (node state $s \in \{full, reduced\}$)
   {
4. **for** ($\alpha = 1^\circ$ *to* 180° **step** δ)
   {
5. find set $\{S_{\alpha_1(x,s)}, \ldots, S_{\alpha_{180}(x,s)}\}$ of subsets of out-neighbours of x as explained in the text
   }
6. evaluate the obtained set and select a subset $S_{\alpha(x,s)}$ for x in state s using **Method-1** or **Method-2** as explained in the text
7. set $Best_\alpha[(x,s)] = \alpha(x,s)$ that corresponds to $S_{\alpha(x,s)}$
   }
   }
8. **return** $Best_\alpha$

Figure 3: Function HWAS for the DirEXPO-RU problem.

5.2 Algorithm Details

Fig. 3 shows a pseudo code for function HWAS. Step 1 initializes array $Best_\alpha$ used to store $\alpha(x,s)$ for all nodes in G. The nested loop in Steps 2 and 3 iterates over each node-state pair (x,s) to find its half-width angle $\alpha(x,s)$. Steps 4 and 5 perform a sequential search for a good setting of an angle $\alpha(x,s)$ in the range $[1^\circ, 180^\circ]$ (in increment of $\delta = 1^\circ$). This process generates a set $\{S_{\alpha_1(x,s)}, \ldots, S_{\alpha_{180}(x,s)}\}$ of possible subsets for node x's neighbours. Step 6 evaluates each generated set, and selects a subset $S_{\alpha(x,s)}$ using either **Method-1** or **Method-2**. Step 7 sets $Best_\alpha(x,s) = \alpha(x,s)$ corresponding to the best subset $S_{\alpha(x,s)}$ of the out-neighbours of x in state s. Finally Step 9 returns $Best_\alpha$.

5.3 Running Time

Consider an input problem instance on a 3-state network with n nodes. Denote by d_{max} the maximum possible out-degree of any node ($d_{max} \le n-1$). Thus, the maximum possible number of arcs in the network is $m \le d_{max}(n-1)$.

Method-1 iterates a fixed number of times (depending on the increment angle δ) over every node-state pair (x,s). Each iteration examines at most d_{max} possible out-neighbours of x. Thus, the algorithm requires $O(n \cdot d_{max})$ time.

Similarly, Method-2 iterates over every node-state pair (x,s). Each iteration solves a single-destination shortest paths in time $\Theta(n+m)$ time (see, e.g., (Cormen et al., 2009)). Thus, the algorithm requires $O(n \cdot (n+m))$ time.

6 COMPUTING BOUNDS

One main contributions in this work is obtaining lower bounds (LBs) on $\text{Expo}(G, p, k_{req})$ for the DirEXPO-RU problem where each input graph G is constructed by our devised HWAS configuration methods. To compute the Expo measure for any given problem instance, we use an iteratively improvable method, called the factoring method. Reference (Ball et al., 1995) is among the early references to this general method. Subsequently the method has been used to obtain lower and upper bounds on many reliability problems, including the class of path exposure problems (see, e.g., (Elmorsy et al., 2013; Basabaa and Elmallah, 2019)). In (Elmorsy et al., 2013), the authors adapt the factoring method to compute lower bounds (LBs) and upper bounds (UBs) of the EXPO problem by generating s-disjoint pathsets and cutsets, respectively.

In more details, the factoring algorithm systematically generates a set of pairwise statistical disjoint (**s-disjoint**, for short) configurations that can be used to obtain bounds from the sum of disjoint products. We call two configurations C_1 and C_2 s-disjoint if at least one node, say x, that appears in both C_1 and C_2 is assigned two different states in the two configurations. So, $\Pr(C_i) + \Pr(C_j)$ is the probability of obtaining at least C_i and/or C_j. For the DirEXPO-RU problem, e.g., the configurations $C_1 = \{(1, reduced), (2, fail)\}$, and $C_2 = \{(1, full), (2, fail)\}$ are s-disjoint since node 1 is assigned two different states in C_1 and C_2.

For the EXPO-RU problem (with omnidirectional transmission), the work in (Basabaa and Elmallah, 2019) develops an efficient function, called E2P (for extension to a pathset), that extends (if possible) a given configuration C to a pathset. The authors use the function within the factoring method to obtain LBs on the solutions. In this context, the method generates a set $\{P_1, P_2, \cdots, P_r\}$ of pathsets such that

$$\text{Expo}(G, p, k_{req}) \geq \sum_{i=1}^{r} Pr(P_i) \qquad (1)$$

In the next section, we present results based on using this latter method to compute LBs on problem instances generated by our HWAS configuration methods.

7 NUMERICAL RESULTS

In this section, we present selected numerical results to evaluate and compare the performance of the devised directional transmission configuration approaches.

Test networks. The selected results are for a class of networks that can be viewed as extended 2-dimensional square grid networks (denoted x-grids). Any such $W \times W$ network G has W rows (and columns) indexed as $0, 1, 2, \cdots, W-1$ from bottom to top (respectively, left to right). Each node has (x, y)-coordinates. The sink node is placed at the origin at coordinates $(0, 0)$. Rows (respectively, columns) run horizontally (respectively, vertically) parallel to the x-axis (respectively, the y-axis). The horizontal (or vertical) distance between two consecutive nodes in the grid is set to 100 units.

As explained in Section 3.3, when a node is in the full energy state its actual transmission range is assumed (for simplicity) to decrease linearly from $R_{full}(\alpha = 1^\circ) = 360$ units to $R_{full}(\alpha = 180^\circ) = 180$ units (the omnidirectional case) as the half-width beam angle α increases. Thus, e.g., with omnidirectional transmission of an internal node x, the node can reach 8 other nodes (corresponding to 4 horizontal and vertical neighbours, and 4 diagonal nodes). Likewise, when a node is in the reduced energy state its actual transmission range decreases linearly from $R_{red}(\alpha = 1^\circ) = 180$ units to $R_{red}(\alpha = 180^\circ) = 100$ units as the half-width beam angle α increases. Thus, with omnidirectional transmission of an internal node x, the node can reach 4 other nodes. In each x-grid, the intrusion path **P** runs vertically between the rightmost two columns. Only nodes that lie on the immediate left and right of **P** sense the path.

Node-state Probabilities. We obtain results with 95% confidence and each point in each obtained curve is the average of 20 runs, where each run assigns to each node x random $p_{full}(x)$ and $p_{red}(x)$ such that $p_{full}(x) + p_{red}(x) + p_{fail}(x) = 1$ and each run completes within 71 seconds.

Reliability Lower Bounding Method. In each run, a LB on $\text{Expo}(G, p)$ is obtained by the factoring algorithm after performing 1000 iterations.

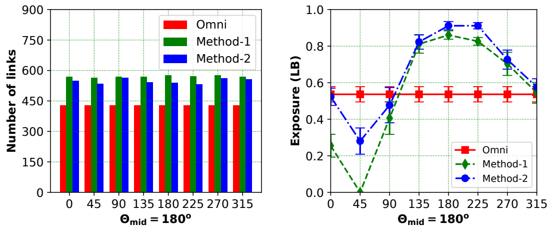
Graph Legend. Labels *Method-1* and *Method-2* refer to results obtained using our devised HWAS algorithm based on *Approach 1* and *Approach 2* , respectively. The *Omni* label refer to results obtained using omnidirectional transmission.

7.1 Effect of Varying Θ_{mid}

In this set of experiments, we explore the ability of our two devised approaches to configure the beamwidths of each node so as to achieve good LBs on the exposure reliability measure. We utilize a 6×6 x-grid, and vary Θ_{mid} in the range $[0^\circ, 360^\circ]$ (all nodes use the same Θ_{mid} value). We recall that for each node x and state $s \in \{full, reduced\}$, the proposed approaches are used to configure the angles $\alpha(x, s)$, and consequently the out-neighbours of node x.

Fig. 4a presents a histogram of the average number of links in the network that results from using random node-state probabilities (explained above). The average number of links achieved by the omnidirectional configuration is constant (independent of Θ_{mid}). Method-1 has a particular strength in maximizing the out-degree of each node, Thus, it achieves the highest average number of total links in the network. Method-2, however, produces a comparable number of links in the graph.

Fig. 4b presents curves of the average LB on the computed $\mathrm{Expo}(G, p)$ measure. Method-2 has a particular strength in selecting for each node x reachability to out-neighbours that are deemed to be good in reaching the sink node. Thus, one expects the resulting LBs to outperform LBs obtained from the two other methods. This expected behaviour is confirmed in the figure.



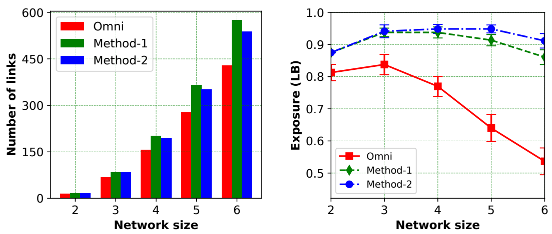
(a) Number of links (b) Exposure

Figure 4: Links and exposure versus $\Theta_{mid} \in [0°, 360°]$.

7.2 Effect of Varying Network Size

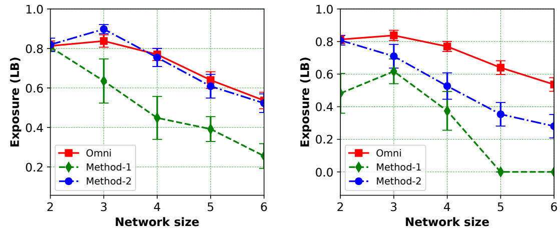
Here, we show the effects of using different networks of size $W \times W$, where $W \in [2,6]$ and $\Theta_{mid} = 180°$, on the obtained number of links and exposure. The proposed approaches are used to configure the transmission beams of each node for both the *full* and *reduced* power states. Fig. 5a and Fig. 5b show the obtained number of links and exposure, respectively, for all methods: Method-1, Method-2, and the *Omni* method.

Fig. 5a shows that the number of links increases as network's size increases for all used methods as a large network will have a higher number of nodes, and hence more links. However, Method-1 obtains the highest number of links compared to Method-2 and the *Omni* method. Fig. 5b shows that the obtained results by Method-1 and Method-2 outperform the *Omni* method for different network sizes. The results illustrate the advantages of using directional sensors over omnidirectional as they provide a higher level of tunability needed in optimizing their performance when network's size increases.

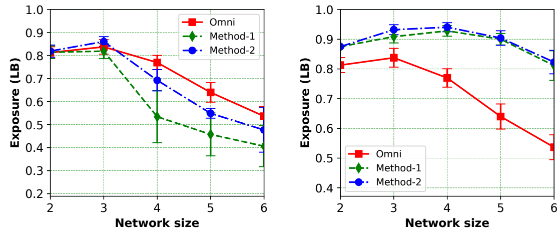


(a) Number of links (b) Exposure

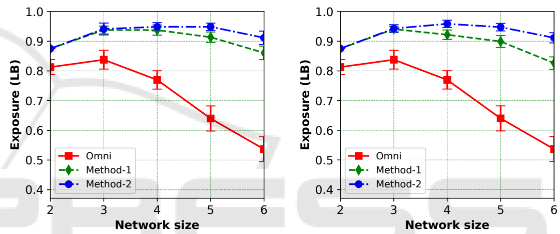
Figure 5: Links and exposure versus network size.



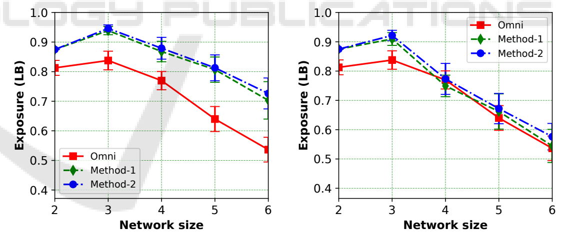
(a) $\Theta_{mid} = 0°$ (b) $\Theta_{mid} = 45°$



(c) $\Theta_{mid} = 90°$ (d) $\Theta_{mid} = 135°$



(e) $\Theta_{mid} = 180°$ (f) $\Theta_{mid} = 225°$



(g) $\Theta_{mid} = 270°$ (h) $\Theta_{mid} = 315°$

Figure 6: Obtained bounds versus network size.

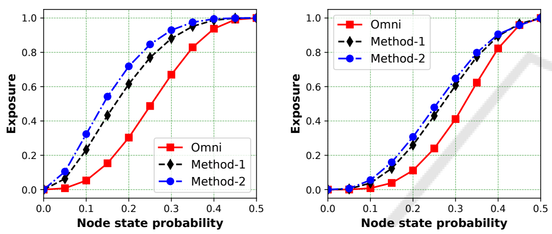
7.3 Identifying Good Working Direction Θ_{mid}

Configuring the direction of the transmission beam center (determined by the angle Θ_{mid}) of nodes in the network is critical for obtaining good performance. In cases when this parameter is misconfigured for a node, it is hoped that by adapting the beam width the node can still deliver acceptable performance.

In this set of experiments, we examine this aspect when using $W \times W$ x-grids and varying W in the range $[2,6]$. Since the sink node is placed at the (x, y)-

coordinate $(0,0)$, the direction of the line between a node x and the sink varies in the range $[180°, 270°]$ ($180°$ for nodes on the x-axis, and $270°$ for nodes on the y-axis).

Fig. 6 shows detailed obtained results when changing Θ_{mid} in the range $[0°, 360°]$. The results show that directional settings outperform the omni-directional setting when all nodes are oriented so that $\Theta_{mid} \in [180°, 270°]$. The results also show that even when $\Theta_{mid} = 90°$ (a setting that can be viewed as a misconfiguration, given the sink position), the working of Method-1 and Method-2 have been able to adjust the angle α of each node and the obtained LBs are comparable with the omnidirectional case. The results also point to the importance of adjusting the beam-width 2α based on the quality of the obtained routes to the sink (as considered in Approach 2).



(a) $k_{req} = 1$ and $\Theta_{mid} = 180°$ (b) $k_{req} = 3$ and $\Theta_{mid} = 180°$

Figure 7: Exposure versus node state probability.

7.4 Exposure versus Node State Probability

Here, we compare directional transmission with omnidirectional transmission as we set $p_{full}(x) = p_{red}(x) = p$ for each node x, and vary p in the range $[0.0, 0.5]$. The probability p here can be viewed as the node's operating (either in the *full* or *reduced* sates) probability. We note that low p values correspond to cases where the fraction of time when nodes in the network operate in the full or reduced energy states is small. This can happen, e.g., because nodes can not harvest enough energy, or nodes decide to conserve power.

The experiments use a 6×6 x-grid with $k_{req} = 1$ (Fig. 7a), and $k_{req} = 3$ (Fig. 7b). In all cases, for any value of p, directional transmission achieves higher average LB on $\text{Expo}(G, p)$ than omnidirectional transmission. The results show the advantage of utilizing and properly configuring directional EH-WSNs. The curves also show that directional networks are capable of having an exposure reliability that exceeds the operating probability of any single node in the network.

8 CONCLUSIONS

In this paper, we consider a fundamental problem on configuring the transmission beams of nodes in a WSN that employs energy harvesting (EH) to achieve prolonged operating time. We take the overall network reliability for a path exposure problem as an objective function that we seek to maximize. The proposed approaches have shown the advantages of using directional transmission over omnidirectional transmission. For future work, we propose investigating the design of more comprehensive dynamic configuration mechanisms of such networks for a variety of WSN reliability problems.

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