# Using Machine Learning and Finite Element Modelling to Develop a Formula to Determine the Deflection of Horizontally Curved Steel I-beams

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Abstract: The use of curved I-beams has been increasing throughout the years as the steel forming industry continues to advance. However, there are often design limitations on such structures due to the lack of recommendations and design code formulae for the estimation of the expected deflection of these structures. This is attributed to the lack of understanding of the behaviour of curved I-beams that exhibit extreme torsion and bending. Thus, currently, there are no formulae readily available for practising engineers to use to estimate the deflection of curved beams. Since the design of light steel structures is often governed by serviceability considerations, this paper aims to analyse the properties of curved steel I-beams and their impact on deflection as well as develop an accurate formula that will be able to predict the expected deflection of these beams. By using a combination of an experimentally validated finite element modelling approach and machine learning. Numerous formulae are developed and tested for the needs of this research work. The final proposed formula, which is the first of its kind, was found to have an average error of 4.11% in estimating the midspan deflection on the test dataset.



## 1 INTRODUCTION

A beam is a structural element that has been studied extensively for numerous years and is used widely, not only in the field of civil engineering but mechanical engineering as well. The use of curved steel I-beams have been increasing steadily throughout the years due to the aesthetically pleasing designs that they produce in buildings as well as for industrial applications. Horizontally curved steel I-sections can be found in various applications ranging from girders in modern highway bridges, interchange facilities as well as in industrial buildings where they are used as crawl beams. Due to advancements in steel forming technology, almost any section can be curved with minimum limitations that relate to the beam's section size or length. Numerous studies have been conducted in efforts to understand the behaviour of curved beams based on different materials as discussed in (Al-Hassaini, 1962), (McManus et al., 1969), (Hsu et al., 1978, Mansur and Rangan, 1981), (Al-Hashimy and Eng, 2005), (Chavel, 2008) and (Lee et al., 2017). It is widely known that, unlike straight beams, horizontally curved beams subject to gravity loads experience a complex system of combined effects, namely, shear, flexure and torsion. This loading effect causes a complex structural response such as warping due to non-uniform torsion.

The study of curved girders began with Umanskii (1948) who obtained solutions for curved beams based on several loading conditions by assuming initial parameters in the solution procedure (Liew et al., 1995). Numerous other studies and experiments have been conducted from the 1960s to the present day, however, these studies mainly focused on the ultimate limit states of the beam and not on the serviceability limit states. Those that did focus on serviceability limit states design, did not analyse the effect various parameters have on deflection, whereas those that did analyse the effect on deflection did not determine an appropriate formula without using Castigliano's second theorem. Currently, most design codes (SANS, AISC, Eurocode 3, etc.) do not have a section detailing deflection estimation of curved steel I-beams. AISC however does state that the usage of finite element modelling (FEM) is appropriate when



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deflection results are of importance (Dowsell, 2018). The issue arises when Engineers conduct a linear static analysis on the structure as a whole and check deflections in that manner. The issue with that approach is that curved beams experience large deflections, which leads to nonlinear strain-displacement and curvature-slope relations, a phenomenon that necessitates accounting for geometric nonlinearities. Another approach would be to use Castigliano's second theorem which is numerically tedious (Dahlberg, 2004). Nevertheless, researchers have attempted to provide a formula for deflection of curved steel I-beams based on Castigliano's second theorem, however, this formula is often not readily available to practising engineers and is only applicable for specific support and loading conditions.

This research work attempts to derive an easy to use formula to determine the midspan deflection of real cantilever types of curved steel I-beams with loads applied only at the midspan. A machine learning algorithm that uses nonlinear regression is implemented herein to create a closed-form equation based on midspan deflection results, a dataset developed through the use of Reconan FEA (2020) (Mourlas and Markou, 2020). Prior to the development of the models that were used to train the machine learning algorithm, the software was validated through the use of experimental results found in (Shanmugam et al., 1995). It should be noted herein that the final curved beam properties must be used because typically beams are curved through a cold process known as pyramid roll bending which alters the material properties of the steel (Dowsell, 2018). After developing different formulae through training the machine learning algorithm on the developed dataset, an investigation on the prediction ability of each formula was performed. The factors considered for developing the formulae that foresaw different numbers of features, were: section area (A), curved length of the beam (L), the radius of the beam (R), yielding strength ($f_y$), Young's modulus (E).

## 2 MACHINE LEARNING ALGORITHMS

Machine learning has grown in popularity in recent years in numerous scientific fields, including civil engineering. The power of machine learning algorithms is that even though complex nonlinear models seem to present no correlation in small sample spaces, by considering a large enough sample a pattern emerges, which can then be exploited to generate a formula to predict out-of-sample data. Various machine learning and artificial intelligence methods are applied for engineering problems such as Random Forests, Gradient Boosting and Artificial Neural Networks, amongst others. The issue with such models is that their results often cannot be interpreted in practical cases unless integrated into some software. In this work, a higher-order nonlinear regression modelling framework was utilized (Gravett et al., 2021) due to its ability to provide an explicit closed-form formula. The model is based on the creation of nonlinear terms based on the independent variables up to the third degree. The algorithm automatically selects the nonlinear features which would correspond to the minimum error. The algorithm found in Table 1 represents the procedure that would lead to the development of the desired formula:

Table 1: Higher-Order Nonlinear Regression Algorithm (Gravett et al., 2021).

| | |
|---|---|
| Input: | *XX* (matrix of Independent Variables), *YY* (Vector of Dependent Variable), *nlf* (number of nonlinear features to be kept in the model) |
| Output: | Prediction Formulae |
| | 1. Create all nonlinear features (*anlf*) |
| | 2. For *i* from 1 to *nlf*, do: |
| | 3. For *j* from 1 to *anlf*, do: |
| | 4. Add $j^{th}$ feature to the model |
| | 5. Calculate Prediction Error, $MAPE_j$ |
| | 6. END |
| | 7. Keep in the model the $j^{th}$ feature which yields the minimum prediction error |
| | 8. END |
| Return: | Prediction Formula |

## 3 NUMERICAL CAMPAIGN

This section presents the procedure adopted to generate the dataset that will be presented thereafter. Initially, a mesh sensitivity analysis was performed to determine the optimum mesh size, where the ability of Reconan FEA (2020) to calculate the mechanical response of these types of beams is investigated. Then, beams with varying geometrical and material properties were developed and analysed for constructing the dataset that was used to train the machine-learning algorithm presented in the previous section. Finally, the predictive models that were developed as closed-form formulae, were parametrically tested on additional data to determine their ability to predict the deformation at the midspan of the curved steel I-beams.

### 3.1 Mesh Sensitivity and Validation Analysis

A mesh sensitivity analysis was conducted to determine a mesh size that derives the optimal results of an experiment found in Shanmugam et al. (1995). It was decided to compare 8 noded and 20 noded hexahedral elements with element sizes of 20, 30, 50, and 100 mm hexahedral element sizes for this investigation. The developed models can be seen in Figure 1. The numerically obtained curves were compared with the experimental curve found in Shanmugam et al. (1995).

Figure 2 shows the experimental setup of the beam tested by Shanmugam et al. (1995) and investigated numerically herein. As it can be seen, the curved length of the beam (L) is 5 m, the point of load application ($L_1$) 3.8 m from the left support and the radius (R) is 20 m.

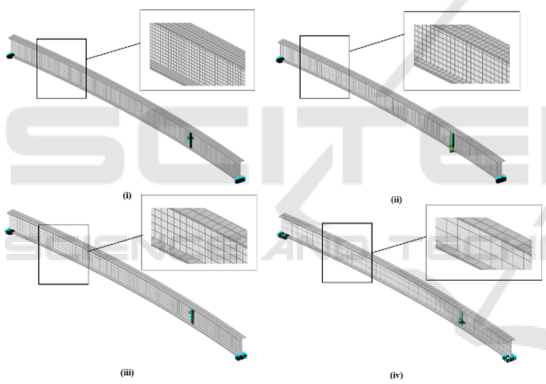


Figure 1: Mesh sensitivity analysis models with mesh sizes, (i) 20mm, (ii) 30mm, (iii) 50mm, (iv) 100mm.

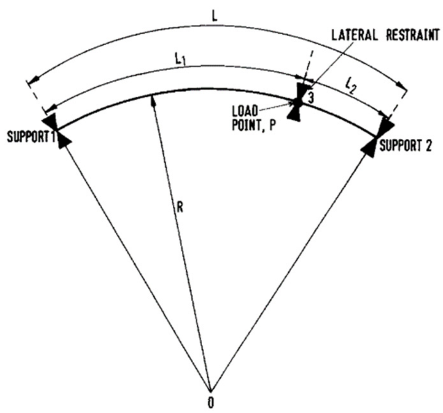


Figure 2: Experimental setup (Shanmugam et al., 1995).

Nonlinear analyses were performed using Reconan FEA (2020) that foresaw 100 load increments and an energy convergence tolerance of $10^{-5}$. It is important to note that only material nonlinearities were accounted for in the developed models through adopting the von Mises yielding criterion. The final results of the mesh sensitivity analysis can be seen in Figure 3 and Table 2. It is easy to depict that most beams managed to come close to the ultimate load as found in the experimental data, however, the beams with the coarse mesh failed to accurately model the ductility of the steel and in general reproduce the overall mechanical response of the curved beam.

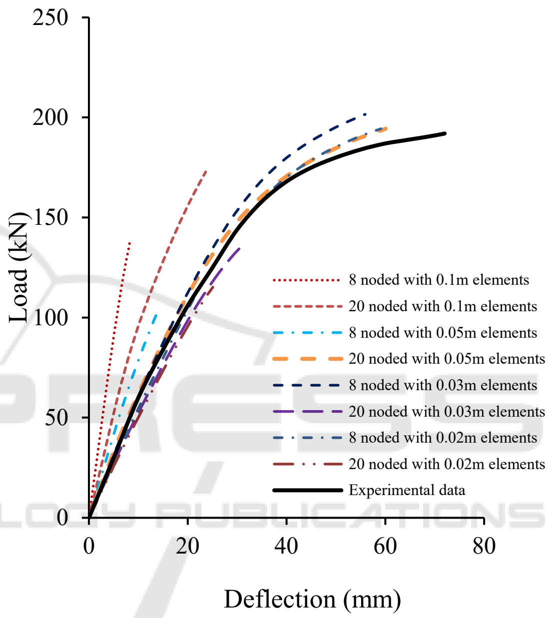


Figure 3: Graphically comparing load-deflection results of various mesh sizes against experimental data.

Table 2: Numerical comparison of the various mesh sizes against experimental data.

| Model | $P_{exp}/P_{NL}$ | $\Delta_{exp}/\Delta_{NL}$ | Analysis time (hh:mm:ss) |
|---|---|---|---|
| 8 noded with 0.1m elements | 1.376 | 8.588 | 00:21.9 |
| 20 noded with 0.1m elements | 1.108 | 3.049 | 01:22.6 |
| 8 noded with 0.05m elements | 1.813 | 5.040 | 01:04.5 |
| 20 noded with 0.05m elements | 0.985 | 1.199 | 05:16.3 |
| 8 noded with 0.03m elements | 0.950 | 1.288 | 06:17.0 |
| 20 noded with 0.03m elements | 1.425 | 2.371 | 15:12.3 |
| 8 noded with 0.02m elements | 0.985 | 1.218 | 16:20.9 |
| 20 noded with 0.02m elements | 1.662 | 2.874 | 03:06:05 |

The 50 mm, 20-noded isoparametric hexahedral finite elements derived the best results with a 1.5%

error in predicting the ultimate failure load, less than 20% error for capturing the max deflection at failure and 6.33% error in estimating deflection at 50% of the total load. It is evident that this experiment did not develop any local or global buckling prior to ultimate failure, thus the nonlinear detailed model was able to capture the overall mechanical response with acceptable accuracy.

## 3.2 Finite Element Models

Figure 4 shows the 20-noded hexahedral mesh of one of the beams that were analysed in this paper for different material properties, while other beams were developed with different geometric properties. It was chosen to place a fixed support on the left side of the beam and a roller support on the right side of the beam which closely represents the boundary conditions as experienced by crawl beams in industrial buildings. The load was placed at the centre span of the beam and were divided into 100 load increments to accurately determine the midspan deflection. The loads were placed on the web of the member to avoid local failure of the flange.

Numerous beams were created to make sure the formula encompasses a large variety of parameters. The parameters considered in this study included the curved length of the beam (L), the radius of the curve (R), the Young's modulus of steel (E), and the section sizes. It is also noted that specific combinations of these input parameters are of significance in analysing curved beams, specifically, the R/L ratio which is an indication of the amount of curvature a beam experiences. This shall be considered in the sensitivity analysis later but will lead to complications when developing the formula. A total of 270 beams were initially created, where, 3 I-beam sections were considered, namely a 305x165x46UB, 533x210x82UB and a 203x133x25UB. These beam sections represent the extreme ends of I-beam sections that are commercially available in South Africa as well as an intermediate beam size. All beams were analysed until failure, where the ultimate load was recorded. Various points along the elastic load-deflection diagrams were considered which lead to a total number of 1890 unique data points to train the machine learning algorithm. To ensure that the points were on the elastic region, the points that were selected foresaw a maximum of 50% load level of the maximum load level as computed from the analysis.

A statistical summary of the various properties can be seen in Table 3. Where A is the section area in $mm^2$ and $I_{xx}$ is the second moment of area of the cross-section about the x-axis in $mm^4$. A large number of radii were considered as this was hypothesised to be the parameter to have a significant impact on the deflection.

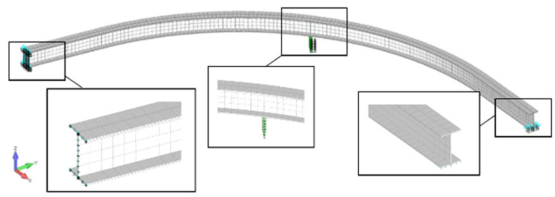

Figure 4: General beam undeformed mesh.

Table 3: Statistical summary of independent variables.

| Metric | $I_{xx}$ ($mm^4$) | A ($mm^2$) | L (m) | R (m) | E (GPa) |
|---|---|---|---|---|---|
| mean | $2.02 \times 10^8$ | $6.58 \times 10^3$ | 5.02 | 58.99 | 205.10 |
| std | $1.99 \times 10^8$ | $3.02 \times 10^3$ | 1.64 | 47.55 | 8.38 |
| median | $9.93 \times 10^7$ | $5.88 \times 10^3$ | 5.00 | 60.00 | 205.10 |
| min | $2.35 \times 10^7$ | $3.22 \times 10^3$ | 3.00 | 3.50 | 194.85 |
| max | $4.75 \times 10^8$ | $1.05 \times 10^4$ | 7 | 140.00 | 215.36 |

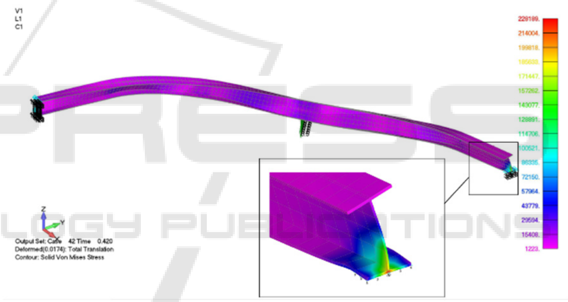

Figure 5: General beam deformed shape and Von Mises stress contour.

# 4 RESULTS AND VALIDATION

Figure 5 shows the general deflected shape that the beam experienced as well as the von Mises stresses developed throughout the beam at 50% of the ultimate load. It is easy to observe that torsion dominates the beam while controlling the type of failure. This was expected due to the eccentricity of the load and shows that the mechanical behaviour of the beam is as expected. The maximum stress experienced was close to the pin-like support, where the failure occurred at the bottom flange of the section due to excessive stresses. This was a mechanical response that was noted in all understudy curved steel I-beams. The beams that had a low R/L ratio (beams with lower curvature) failed close to the pin-like

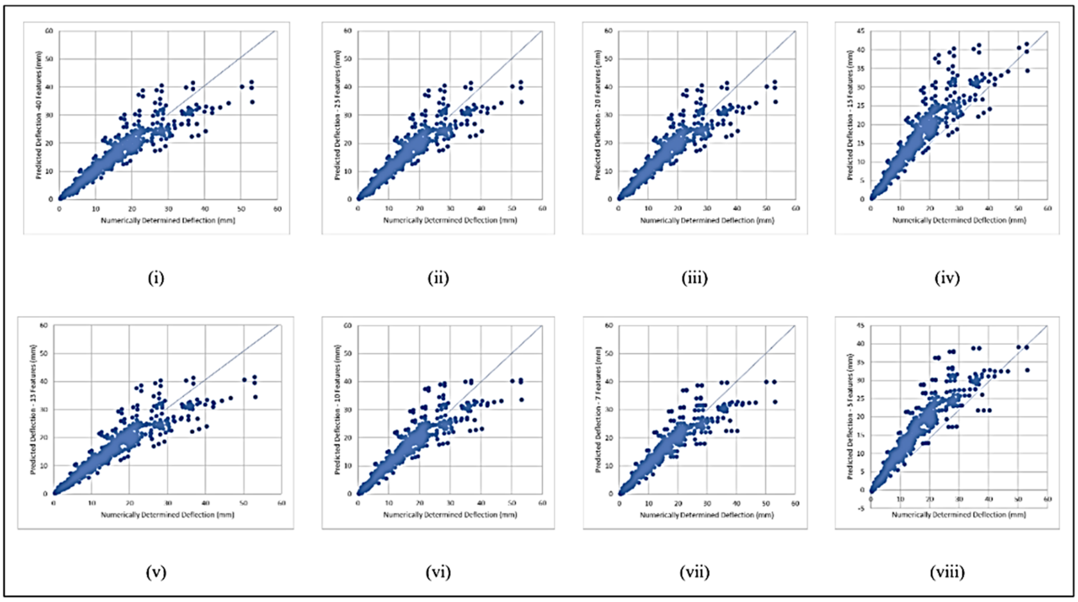


Figure 6: Correlation between numerically determined deflection and (i) 40 term, (ii) 25 term, (iii) 20 term, (iv) 15 term, (v) 13 term, (vi) 10 term, (vii) 7 term and (viii) 5 term formulae.

support, while failure occurred at the web and not within the flange. It should, however, be noted that failure analysis was not the subject of this research work but rather, the deflection experienced during the loading within the elastic region, which is what interests Civil Engineering designers.

Once the models were analysed, a numerical database was created that contained the parameters of each beam and the respective midspan deflections (one row per deflection point). This numerical database was then used to train the machine learning algorithm, which was able to provide formulae as well as other descriptive statistics about the training and testing process (sensitivity analysis that will be presented in an extended version of this manuscript).

When generating the formulae, as was seen in the machine learning section of this manuscript, the beam features are considered as input parameters. A range of 5 to 40 features were considered when training the formulae and the various results were compared to determine the most accurate formula in terms of predictability. Figure 6 shows the graphs comparing the generated formulae to the numerically computed deflection. As can be seen, there is a strong correlation between the numerically determined deflections and the deflections predicted by the various formulae.

In order to quantify the accuracy of the proposed formula on the training and test set, several error metrics were used, namely, the correlation coefficient ($r^2$), alpha metric ($\alpha$), the root mean square error (RMSE), the mean absolute error (MAE), the mean absolute percentage error (MAPE), the max absolute percentage error (MAXPE), and the quotient error (SR).

It was seen that as the number of features decreases, the error increases, a numerical response that is in line with the finding reported by Gravett et al. (2021). Some error metrics, such as the $r^2$, barely change as the number of features changes (the $r^2$ error was seen to be approximately 0.95 regardless of the number of features), however, other metrics such as MAXPE change drastically as the number of features increase (MAXPE ranged from 1.9607 when 5 features were used to 0.7850 when 40 features were used). This shows the importance of considering numerous error metrics in determining the optimum formula that will show advanced predictive capabilities. It must be noted here that the dataset was divided into 85% and 15%, training and testing data, respectively.

## 4.1 Validation

After training and testing the numerous formulae that were discussed in the previous section, the proposed formulae were then further validated by creating an out of sample model and estimating the midspan deflection. The beam considered was a 305x127x42 UB with a curved length of 5 m and a radius of 20 m. The Young's modulus of the beam was also varied (205.1 GPa, 200 GPa and 210 GPa). Various ultimate load percentages were also considered in this validation process, where a total of 15 new data points were developed for the needs of this investigation. It was found that the function that consisted of 10 features derived the lowest average error, of only 4.11% in estimating the deflection of 15 out-of-sample data points, while the model with 40 features resulted in the largest error of 13.05%. This is a numerical phenomenon that is usually attributed to overfitting during the training and testing procedure.

The proposed formula used to estimate the deflections that consisted of 10 terms can be seen in Equation 1. The various independent variables are E which is the Young's modulus in GPa, L is the curved length of the beam in metres, A is the section area in $mm^2$, $f_y$ is the yielding stress, $I_{xx}$ is the second moment of area about the strong axis in $mm^4$ and Q is the percentage of ultimate loading applied on the beam as a number (50% = 50). The resulting deflection from the formula is in mm.

$$
\begin{aligned}
Deflection = {} & 1.08128 * 10^{-1} * Q * L - 8.41124 \\
& * 10^{-06} * Q * L * A - 9.81969 \\
& * 10^{-06} * Q * E * R + 1.82604 \\
& * 10^{-05} * Q * R * R - 2.71029 \\
& * 10^{-04} * Q * R * L + 6.22991 \\
& * 10^{-03} * Q * L * L + 2.55963 \\
& * 10^{-11} * Q * L * Ixx + 2.99150 \\
& * 10^{-06} * Q * Q * fy + 4.19580 \\
& * 10^{-07} * Q * fy * fy - 1.07754 \\
& * 10^{-04} * Q * E * L - 8.95792 \\
& * 10^{-02}
\end{aligned} \tag{1}
$$

## 5 CONCLUSIONS

A formula was successfully developed for the prediction of the deflection of curved steel I-beams. When comparing the proposed formula with the out-of-sample data, it was found that the formula containing 10 features was the most accurate, having an average error of 4.11%, while the formula with 40 features was the least accurate having an error of 13.05%. The lack of accuracy in the 40 feature equation was attributed to an over-fitting phenomenon but can also be attributed to another phenomenon known as the "interaction effect", which can greatly increase the effect of the independent variables on the dependent variable.

Based on the parametric and sensitivity investigation, it was concluded that the variables with the largest impact on deflection are the curved length and radius of the beams. Due to the page limitations of this manuscript, the results of the in-depth sensitivity analysis could not be shared, however, these will be published at a later stage. Even though the results of this study are positive seeing as very low error metrics were observed, the study has to be expanded in the future by developing additional models with a larger spectrum in terms of geometries. Various boundary conditions, as well as different yield strengths of steel, will also be considered. Experimental curved steel I-beams will also be tested to validate the proposed formula developed in this study. Future research work will foresee the development of similar formulae on curved concrete beams. Finally, the long-run objective is to develop machine learning models that will be able to evaluate the response of full scale-structures

## REFERENCES


Al-hashimy, M. A. & Eng, P. Load distribution factors for curved concrete slab-on-steel I-girder bridges. Masters Abstracts International, 2005.

Al-hassaini, M. M. F. 1962. *Graphs and tables for the analysis and design of curved concrete beams.* Virginia Tech.

Chavel, b. W. 2008. *Construction and detailing methods of horizontally curved steel I-girder bridges.* University of Pittsburgh.

Dahlberg, T. 2004. Procedure to calculate deflections of curved beams. *International journal of engineering education,* 20, 503-513.

Dowsell, B. 2018. *Design Guide 33: Curved Member Design,*, American Institute of Steel Construction,.

Gravett, D. Z., Mourlas, C., Taljaard, V.-L., Bakas, N., Markou, G. & Papadrakakis, M. 2021. New fundamental period formulae for soil-reinforced concrete structures interaction using machine learning algorithms and ANNs. *Soil Dynamics and Earthquake Engineering,* 144, 106656.

Hsu, T. T., Inan, M. & Fonticiella, L. Behavior of reinforced concrete horizontally curved beams. Journal Proceedings, 1978. 112-123.

Lee, K., Davidson, J. S., Choi, J. & Kang, Y. 2017. Ultimate strength of horizontally curved steel I-girders with equal end moments. *Engineering Structures,* 153, 17-31.

Liew, J. R., Thevendran, V., Shanmugam, N. & Tan, L. 1995. Behaviour and design of horizontally curved steel beams. *Journal of Constructional Steel Research,* 32, 37-67.

Mansur, M. A. & Rangan, B. V. Study of design methods for reinforced concrete curved beams. Journal Proceedings, 1981. 226-254.

Mcmanus, P. F., Nasir, G. A. & Culver, C. G. 1969. Horizontally curved girders-state of the art. *Journal of the Structural Division,* 95, 853-870.

Mourlas, C. & Markou, G. 2020. *ReConAn v2.00 Finite Element Analysis Software User's Manual.*

Reconan FEA v2.00, User's Manual. 2020. https://www.researchgate.net/publication/342361609_ReConAn_v200_Finite_Element_Analysis_Software_User's_Manual

Shanmugam, N., Thevendran, V., Liew, J. R. & Tan, L. 1995. Experimental study on steel beams curved in plan. *Journal of Structural Engineering,* 121, 249-259.

Umanskii, A. 1948. Spatial structures. *Moscow (in Russian).*