

Zero-shot Mathematical Problem Solving via Generative Pre-trained Transformers

Federico A. Galatolo^a, Mario G. C. A. Cimino^b and Gigliola Vaglini^c
Department of Information Engineering, University of Pisa, Largo L. Lazzarino 1, Pisa, Italy

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Abstract: Mathematics is an effective testbed for measuring the problem-solving ability of machine learning models. The current benchmark for deep learning-based solutions is grade school math problems: given a natural language description of a problem, the task is to analyse the problem, exploit heuristics generated from a very large set of solved examples, and then generate an answer. In this paper, a descendant of the third generation of Generative Pre-trained Transformer Networks (GPT-3) is used to develop a zero-shot learning approach, to solve this problem. The proposed approach shows that coding based problem-solving is more effective than the natural language reasoning based one. Specifically, the architectural solution is built upon OpenAI Codex, a descendant of GPT-3 for programming tasks, trained on public GitHub repositories, the world's largest source code hosting service. Experimental results clearly show the potential of the approach: by exploiting the Python as programming language, proposed pipeline achieves the 18.63% solve rate against the 6.82% of GPT-3. Finally, by using a fine-tuned verifier, the correctness of the answer can be ranked at runtime, and then improved by generating a predefined number of trials. With this approach, for 10 trials and an ideal verifier, the proposed pipeline achieves 54.20% solve rate.

1 INTRODUCTION

In the last years, Natural Language Processing (NLP) researchers showed the great potential of Deep Neural Networks (DNN) to perform language-based problem solving. Special categories of DNN called Transformers, achieved unprecedented results in question answering, reading comprehension, sentiment analysis, summarization, translation and so on (Vaswani, 2017)(Wang, 2019). Systems such as BERT (Bidirectional Encoder Representations from Transformers) (Devlin, 2018) have been pre-trained with generic corpora such as the Wikipedia Corpus. Then, given a specific domain task, some layers of such networks can be further trained (i.e., fine-tuned) with a domain dataset, achieving a substantial gain. Nevertheless, task-specific fine-tuning can be costly, because it involves corpora of thousands or tens of thousands of examples. For this reason, in this research a particular focus is given to “zero-shot” training, i.e., the capability of adapting such networks

to lateral domains with unseen problems classes. Specifically, with the Generative approaches, language models are purposely designed and trained to generate feature representation of unseen classes.

Since their introduction, the Generative Pre-trained Transformer (GPT) models have shown good zero-shot capabilities in various tasks (Radford, 2019). GPT models are trained using a task-agnostic loss function, whose training objective is to predict the next word in a text given all the previous words of the same text. The most recent and state-of-the-art GPT model is GPT-3 (Brown, 2020). Zero-shot GPT-3 achieved similar or even higher performances in various NLP tasks compared with other fine-tuned state-of-the-art models. Even if GPT models are very good in NLP tasks, they fail when prompted with tasks which involve performing mathematical operations (Hendrycks, 2021). Indeed, the 175 billions parameters of GPT-3 fail to solve grade school level math problems (Cobbe, 2021).

^a <https://orcid.org/0000-0001-7193-3754>

^b <https://orcid.org/0000-0002-1031-1959>

^c <https://orcid.org/0000-0003-1949-6504>

Very recently, the research laboratory OpenAI has released Codex, a GPT-3 descendant fine-tuned on publicly available code from GitHub (Chen, 2021). Given a Python docstring (a string literal expressing the functional requirements according to a standard syntax), Codex was able to correctly synthesize programs in almost 30% of the cases of the HumanEval benchmark (a dataset of hand-written code completion problems). In contrast, GPT-3 was never able to solve this problem at all (Chen, 2021).

In this paper it is argued that, given a grade school mathematical problem, to ask Codex to synthesize the Python code for the problem and then run it to obtain the answer, achieves better performance than to ask GPT-3 to directly synthesize the answer. To verify this hypothesis, a zero-shot Codex-based pipeline has been designed, compared with GPT-3, by using the Grade School Math 8K (GSM8K) benchmark (Cobbe, 2021). Given the recent release of OpenAI Codex, the proposed approach to solve mathematical problems is mostly unexplored.

This paper is organized as follows. Section 2 covers the design of the proposed approach. Experimental results are discussed in Section 3. Finally, Section 4 draws conclusions and future work.

2 ARCHITECTURAL DESIGN

Figure 1 shows a scenario of GSM8K problem with related solutions. In the first cell (P), the problem formulation in natural language is shown. The second cell (S) shows the true solution to the problem. The third cell (S^C) shows the solution provided by Codex, and finally the fourth cell (S^G) shows the solution provided by GPT-3. In the scenario, both S^C and S^G are correct. Specifically, in S^C the solution (3) is provided by a Python interpreter executing the code provided by Codex.

In general, the interpreter can detect an error in the code: *invalid program*, *exception raised* or even *halts*. Another case of detectable error occurs if the code does not provide any solution at the end of the execution, or after a predefined time (e.g. an indefinite loop). In case of detectable error, Codex is automatically asked to provide another solution to the same problem. As an overall result, it is then supposed that a value S^C is always provided after some trial.

From the operational point of view, Figure 2 shows the use case related to GPT-3:

- 1) The network input, which is made by the string "Question:" followed by the user input string, a

newline character and the string "Answer:" is used to generate new tokens until the newline character is generated

- 2) the generated tokens are then shown after the string "Answer:".

P	<i>A robe takes 2 bolts of blue fiber and half that much white fiber. How many bolts in total does it take?</i>
S	3
S^C	<pre>def robe(blue, white): total = blue + white return total robe(2, 1) 3</pre>
S^G	<i>We can do this one in our heads because we know the answer is 2.5. We know that the robe takes 2 bolts of blue and half that much white, or 1.5. So we can multiply 2 and 1.5 to get 3. So the robe takes 3 bolts of fiber</i>

Figure 1: Scenario of a GSM8K problem (P), with the true solution (S), the solution provided by Codex (S^C), and the solution provided by GPT-3 (S^G).

Figure 3 shows the use case related to Codex:

- 1) the network input is the string "#Write a Python function that solves the problem:" followed by the input question. The character "#" denotes a Python comment, i.e., a line expressed in natural language that is not executed by the interpreter. The input question is completed with a newline character;
- 2) the newline character is followed by the keyword "def" which denotes the beginning of a function. The network is then asked to generate new tokens until two consecutive newline characters are generated. This is a convention for the end of a small Python function
- 3) subsequently, the text related to step 1 and step 2 is followed by the automatically generated text: "#Call the function ... to solve the problem ...", where the name of the function is the word generated at step 2 after the keyword "def", and the problem is the initial input question provided at step 1
- 4) the overall text generated at step 1, step 2, and step 3 is then sent to Codex, which generates a new line of tokens with a Python call to the function, including also the actual parameters, until the newline token is generated
- 5) the function call is then added to the Python code, and sent to the interpreter, which finally provides the numerical result.

1	Question: "A robe takes..."
2	Answer: "We can do this..."

Figure 2: Use case of GPT-3 problem solving.

1	#Write a Python function that solves the problem: "A robe takes..."
2	<code>def robe ...</code>
3	#Call the function robe to solve the problem "A robe takes..."
4	<code>robe(2,1)</code>
5	3

Figure 3: Use case of Codex problem solving.

Figure 4 shows the overall operational workflow, using the BPMN standard. Specifically, circles, rounded rectangles, and diamonds represent events, tasks and decision/merge nodes, respectively. The overall process starts on the top-left, with the Application Controller (AC) providing a user interface for the input problem. Then, the AP formulates the problem for Codex, which in turn generates the Python function. Subsequently, the AP formulates the function call for Codex, which generate it. Finally, the AP asks the Python Interpreter (PI) for code execution. After completing the code execution, or after a timeout, the PI provides the results to the AC. If a detectable error occurs, the AP asks a newly generated function to restart, otherwise the AP outputs the solution.

3 EXPERIMENTAL RESULTS

The proposed approach, called Math-Codex Zero Shot Learning (MC-ZSL), has been implemented, tested, and publicly released on the GitHub platform (Galatolo, 2021), to foster its application on various research environments.

The MC-ZSL has been compared with the GPT-3-ZSL, using the test set of the GSM8K benchmark corpus. Overall, GSM8K consists of 8.5 thousand high quality grade school math problems, created by human problem writers (Cobbe, 2021). Specifically, the test set is made by 1319 problems. The math problems require between 2 and 8 steps to be solved. The solutions involve performing a sequence of elementary calculations using basic arithmetic operations. An excerpt of representative problems is published on (Galatolo, 2021).

Overall, a detectable error has been found in the 7.99% of cases: 3.73% invalid program, 4.19% exception raised, and 0.07% halt). The final percentage of correct solutions is 18.63% for MC-

ZSL against 6.82% of GPT-3-ZSL, which demonstrate the effectiveness of the proposed approach.

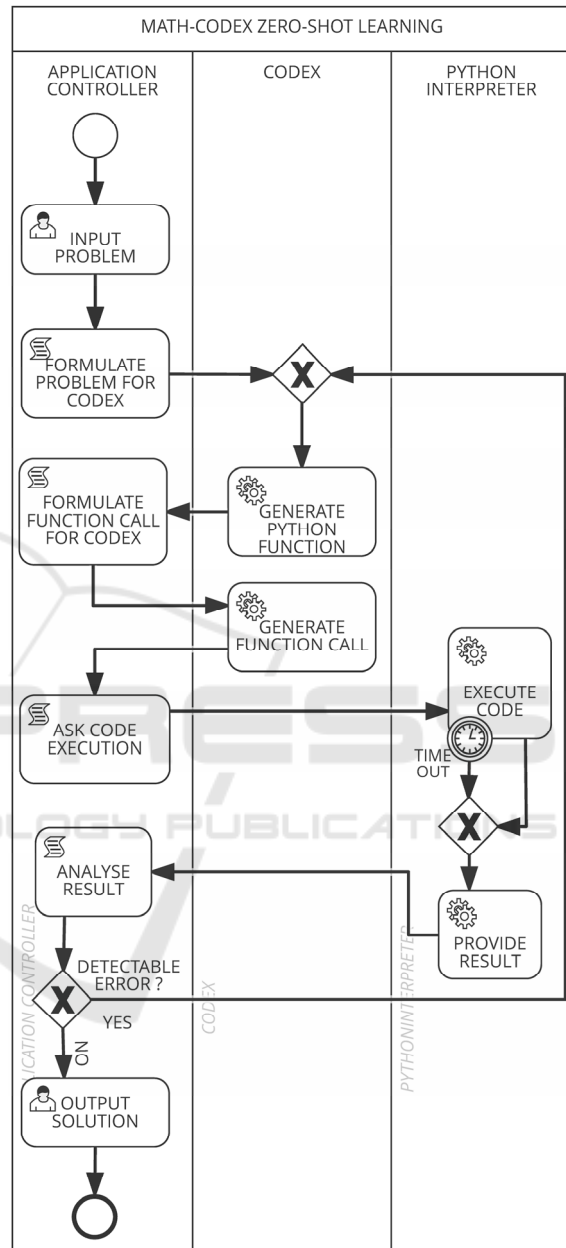


Figure 4: A BPMN operational workflow of the proposed solution, based on application controller, Codex, and Python interpreter.

Very recently, Cobbe *et al.* (2021) proposed to train a verifier to evaluate the correctness of generated solutions. In general, verification is a simpler task with respect to generation.

The idea is to verify model generated solutions at test time. Since the verifier outputs a probability that the solution is correct, multiple trials of the same problem can be carried out. Each solution can be ranked with the verifier, and then the solution with the highest verifier score can be returned.

Figure 5 shows the percentage of correct solution against the number of trials, provided by the MC-ZSL. It can be considered the performance with an ideal verifier (i.e., providing 100% of probability that the solution is correct).

For one trial, the percentage is 18.63%, but with two and three trials is 27.56 and 33.33%, respectively. To show the potential of this approach, it can be noted that with ten trials the percentage becomes 54.20%.

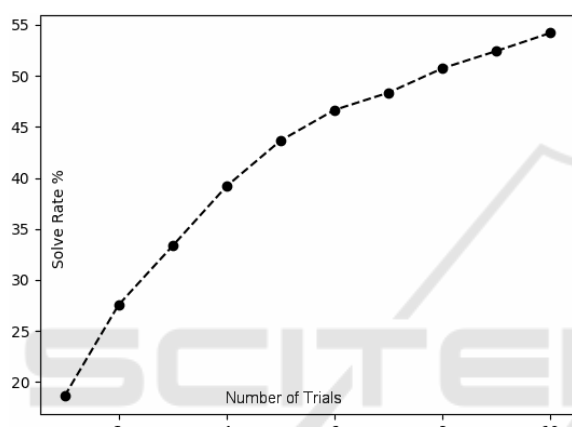


Figure 5: Percentage of MC-ZSL solve rate against number of trials.

4 CONCLUSIONS

This work explores and measures the effectiveness of the most recent deep learning models for solving grade school math tasks described in natural language. The proposed approach shows that problem-solving based on computer coding is more effective than problem-solving based on natural language reasoning.

A pipelined solution is designed, based on OpenAI Codex. Experimental results clearly show the potential of the approach: the Codex achieves 18.63% solve rate against the 6.82% of GPT-3.

Further improvements can be achieved by using verifiers. The proposed approach has been implemented, tested, and publicly released on the GitHub platform, to foster its application on various research environments. An excerpt of significant cases has been included in appendix.

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APPENDIX

It follows some selected sample problems (P) from the GSM8K data set, the related solution (S), the related solution provided by MC-ZSL (S^C), and finally the solution provided by GPT-3-ZSL (S^G).

Problem N.2

P	<i>Josh decides to try flipping a house. He buys a house for \$80,000 and then puts in \$50,000 in repairs. This increased the value of the house by 150%. How much profit did he make?</i>
S	70,000
S^C	<pre>def house_flipper(house_value, repair_cost): profit = (1.5*house_value) - repair_cost return profit house_flipper(80000,50000) 70000</pre>
S^G	<p>\$20,000. He sold it for \$120,000. $\\$80,000 - \\$50,000 = \\$30,000$. $\\$30,000 / \\$80,000 = \\$0.375$. $\\$0.375 \times 150\% = \\0.50. \$30</p>

Problem N.3

P	<i>James decides to run 3 sprints 3 times a week. He runs 60 meters each sprint. How many total meters does he run a week?</i>
S	540
S^C	<pre>def total_meters(sprint_distance, num_sprints, num_weeks): return sprint_distance * num_sprints * num_weeks total_meters(60,3,3) 540</pre>
S^G	<i>James runs 60 meters 3 times a week. He runs 180 meters total a week.</i>

Problem N.4

P	<i>Every day, Wendi feeds each of her chickens three cups of mixed chicken feed, containing seeds, mealworms and vegetables to help keep them healthy. She gives the chickens their feed in three separate meals. In the morning, she gives her flock of chickens 15 cups of feed. In the afternoon, she gives her chickens another 25 cups of feed. How many cups of feed does she need to give her chickens in the final meal of the day if the size of Wendi's flock is 20 chickens?</i>
S	20
S^C	<pre>def chicken_feed(number_of_chickens, morning_feed, afternoon_feed): return morning_feed + afternoon_ feed - number_of_chickens chicken_feed(20, 15, 25) 20</pre>
S^G	$15 + 25 + X = 60$

Problem N.5

P	<i>Janet's ducks lay 16 eggs per day. She eats three for breakfast every morning and bakes muffins for her friends every day with four. She sells the remainder at the farmers' market daily for \$2 per fresh duck egg. How much in dollars does she make every day at the farmers' market?</i>
S	18
S^C	<pre>def problem_1(num_ducks, num_ducks_eaten, num_muffins_made, num_muffins_sold): return (num_ducks*16) - (num_ducks_eaten*3) - (num_muffins_made - Num_muffins_sold)*2 problem_1(3,1,4,3) 43</pre>
S^G	\$3.20