Formal Concept Analysis Applied to a Longitudinal Study of COVID-19

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Abstract: The COVID-19 pandemic, and consequently the difficulty of obtaining feedback on the effectiveness of contamination prevention methods, has caused an increased need to produce a relevant and consistent analysis from collected data. Through Formal Concept Analysis, applying the triadic approach, called Triadic Concept Analysis (TCA), it is possible to evaluate the correlation between prevention measures and the number of contaminated people by performing concept extraction and implication rules. The advantage of using this method is the possibility of correlating the waves, which allows us to explain and understand the evolution of the data over the collection waves, helping us draw a more assertive conclusion from the data analyzed. This paper uses the data collected from the 2020 National Population Survey of Nigeria to depict how Nigerian society's essential and everyday behaviors impacted the evolution of the COVID-19 pandemic in that country. The results obtained from this research can assist governments, and public entities in developing better public policies to combat highly infectious diseases. Furthermore, it provides practical evidence of how TCA can be applied, bringing benefits to different areas and fields of science.

1 INTRODUCTION

Formal Concept Analysis (FCA) (Wille, 1982) was introduced in 1982 by Rudolf Wille as a derivation of concept hierarchy from a set of objects and their properties. The main objectives are to extract and represent knowledge in an efficient way that aids and corroborates decision-making from database analysis.

On the fundamental theorem of triadic concepts and contexts, Wille (1982), providing considerable contributions to the intelligent and adequate database modeling for the extraction of triadic concepts, developed the Triadic Concept Analysis (TCA).

The developed work has shown that FCA's qualitative concept data analysis capability can be combined with a more quantitative approach, investigating data correlations that are not a priori visible or easy to link.

The main benefit of choosing to use TCA in this research is in the correlation between different periods of data collection, represented by the longitudinal base analyzed here.

As we will further explain in the theoretical foundation, triadic rules consist of a context formed from a quadruple $T = (G,M,B,Y)$ where G,M,B are, respectively, sets of objects, attributes, and conditions belonging to the ternary relation $Y \subseteq (G \times M \times B)$. This relation is interpreted as: *object g has attribute m under condition b*.

Thus, in any longitudinal databases, the advantage of using an analysis such as TCA is to make it possible to explain the evolution of variables between collection waves, clarifying possible causes of change, such as temporal behaviors, impacts of external factors, assisting in a more accurate conclusion of the data analyzed.

In this work, Triadic Formal Analysis is applied to behavioral data collected from a society widely impacted by the COVID-19 pandemic, specifically the society of Nigeria. Through this analysis it is possible to generate knowledge, portraying within the population context of this country, the relationship between basic hygiene and public health actions that can interfere in the contamination and dissemination of the disease.

The analysis of behavioral relations such as washing hands with soap, the use of masks in public, the need for medical treatment, and the frequency of children/adolescents with school activities that can affect the spread of COVID-19 is made, helping in indirect actions that aim at the containment and prevention of contamination.

The data collected and analyzed bring about complex relationships due to the variety of question cat-

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egories and possible answers, the number of interviewees, and the several collection waves carried out. However, after a refinement, they can be explored, through the Formal Analysis of Triadic Concepts (TCA), to obtain results derived from several combinations that optimize actions and health policies adopted to fight the pandemic.

The result intended with this study is the application of the TCA in a questionnaire of basic public hygiene and health behaviors. Thus, this work brings as a result rules (implications) regarding the behavior of society by means of health policies and the evolution of the pandemic.

Such results can direct, corroborate and open possibilities for taking actions that add value, either by optimizing the flow of data in a given network or by increasing confidence in decision making for strengthening or weakening of certain policies to fight pandemics and diseases.

This work is structured by presenting a brief introduction to Triadic Formal Concept Analysis (TCA). After the TCA introduction, the database processing steps are described, alongside the results of the methods applied. At the end of this work, over the conclusion of the analysis, the contributions of the study are revelead as to how critical gaps could be filled on information that can be used by the Nigerian government. This is relevant to assist the development of public policies to mitigate the negative impacts of COVID-19 on its population.

2 BACKGROUND

Formal Concept Analysis (FCA) is a mathematical formalism that deals with data represented in two dimensions, relating objects and their attributes. Triadic Formal Concept Analysis (TCA) extends FCA, transforming the binary relation into a ternary consideration by introducing a dimension called condition or mode, establishing a link between objects, attributes, and conditions.

This approach allows the identification of factors related to object changes over time. Moreover, the triadic approach contributes to databases from longitudinal studies by considering the time factor as a condition for analyzing the variation of the relationship between objects and attributes. In this way, the triadic implication rules also explore the temporal evolution of the relations studied.

Thus, with the results from the TCA, it is possible to indicate points of improvement and failure regarding the impacts from health policies and the discovery of previously unnoticed relationships. Nevertheless, the gains for the increase in the quality of life and health of the population, in general, are countless, even if we consider only the impacts coming from the discoveries of new relations and correlations.

Formal Concept Analysis involves different components such as: Formal Context, formal concept, and rules.

A formal Context K is a triple $K = (G,M,I)$, where G is a set of objects, M is a set of attributes and I is the incidence relation ($I \subseteq G \times M$) that assigns an object certain characteristics.

$$K = (G,M,I) \tag{1}$$

$$I \subseteq G \times M \tag{2}$$

If an object (g $\in$ G) and an attribute (m $\in$ M) are in relation I, this is represented by *((g,m) $\in$ I)* or *gIm* and is interpreted as *the object g has the attribute m*.

From a set of objects $A \subseteq G$, coming from a formal context K, it is interesting to recognize which attributes $B \subseteq M$ are shared by all objects in A. Analogously, determine for a set $B \subseteq M$, which objects $A \subseteq G$ have in common the attributes defined in B. The search for these answers leads to derivation operators, which are formally defined as:

$$A' := \{m \in M \mid gIm \,\forall\, g \in A\} \tag{3}$$

$$B' := \{g \in G \mid gImIm; \forall\, m \in B\} \tag{4}$$

From K, it is possible to extract association rules dependent on elements in a set of M. An association between attributes of M is a pair (X, Y), $X,Y \in M$, receiving the notation X $\rightarrow$ Y. Association rules, in turn, reveal frequent patterns in data.

From FCA, we extend the definition to encompass a third dimension resulting in a triadic context T, which is now a quadruple $T = (G,M,B,Y)$ where G,M,B are, respectively, sets of objects, attributes and conditions pertaining to the ternary relation $Y \subseteq (G \times M \times B)$. This relation is interpreted as: *object g has attribute m under condition b*.

Table 1 presents a triadic context similar to the one analyzed in this paper.

$$T = (G,M,B,Y) \tag{5}$$

$$Y \subseteq (G \times M \times B) \tag{6}$$

Table 1: Example of triadic context.

	Cicle 1			Cicle 2			Cicle 3		
Obj	*p*1	*p*2	*p*3	*p*1	*p*2	*p*3	*p*1	*p*2	*p*3
1	x		x		x	x	x	x	x
2	x		x		x		x	x	
3						x	x		
4		x	x	x	x		x		x
5		x	x				x	x	x
6	x			x	x	x	x		

One can describe two types of triadic association rules that can be extracted from the context

$K := (G,M,B,Y)$ (Biedermann, 1997): *Biedermann Conditional Attribute Association Rule (BCAAR)* and *Biedermann Attributional Condition Association Rule (BACAR)* (Zhuk et al., 2014).

The BCAAR rule is represented by: (R $\rightarrow$ S) C, where $R,S \subseteq M$ and $C \subseteq B$. That is, each object possessing all attributes in R also possesses all attributes in S under a condition C, with one support (sup.) and one confidence (conf.).

The BACAR rule is represented by: P $\rightarrow$ Q N (sup.,conf.), where $P,Q \subseteq B$ and $N \subseteq M$. That is, every object under the conditions in P will also be under the conditions in Q on attribute N, with one support (sup.) and one confidence (conf.).

The support corresponds to the proportion of objects in the subset $g \in G$ that satisfy the implication $P \rightarrow Q$, relative to the total number of objects $|G|$ of the formal context K (Equation 7) where ($'$) corresponds to the derivation operator.

$$Support(P \rightarrow Q) = \frac{|(P \cup \{Q\})'|}{|G|} \tag{7}$$

Confidence corresponds to the ratio of objects $g \in G$ that contain P, which also contain Q, to the total number of objects $|G|$ (Equation 8).

$$Conf(P \rightarrow Q) = \frac{|(P \cup \{Q\})'|}{|P'|} = \frac{Support(P \rightarrow Q)}{Support(P)} \tag{8}$$

3 RELATED WORKS

This paper utilizes the Formal Analysis of Triadic Concepts, and the approach and rationale for this understanding are applied through a practical and theoretical bias. Related work on this research topic is presented below.

Wei et al. (2018) analyzed the Triadic approach of Formal Concept Analysis (TCA), from four aspects: (i) basics of triadic concept analysis, (ii) triadic implications and triadic association rules, (iii) triadic factor analysis, and, (iv) triadic fuzzy concept analysis.

The addition offered by Konecny and Osicka (2010) on the theoretical research and the seek subsidies for applications of TCA, aims to better understand the general approach on TCA and its fundamentals. (Konecny and Osicka, 2010) discusses Triadic Concept Analysis (TCA), presenting it as an extension of Formal Concept Analysis (FCA), dyadic case, through the optic of conditions, besides just weighting on objects and attributes relations.

The work of Ganter and Obiedkov (2004) head towards a more practical bias of the research on Triadic Concept Analysis, considering the various implications of its use in multiple scenarios. The work addresses the various possibilities of defining implications of a triadic formal context. Due to the vast different interests one can have for a given triadic context, the authors aim to present compact descriptions and incorporate them into an algorithm that generates implications from triadic contexts. Examples of this variety of interests in a triadic context is the work presented in Silva et al. (2017), where social networks interactions are in focus, and in Ferreira et al. (2017), in which molecular biology challenges are faced.

Kis et al. (2016) reveals the development of a tool that provides a visualization for dyadic and triadic concepts, focusing on a navigation paradigm for triadic contexts. The authors take into consideration the complex contexts of application of Triadic Concept Analysis, given the importance of the existence of methods and procedures for the development and improvement of analyses.

Biedermann (1997), also presents contexts and ways of applying Triadic Concept Analysis. The work is dedicated to explain different types of information and knowledge that can be read in triadic diagrams. These labeled line diagrams graphically represent the conceptual structure of triadic contexts that can be represented as three-dimensional. The author discusses throughout the paper how one should interpret such diagrams and how such dyadic conceptual structures can be determined within the triadic diagrams.

Bringing fundamentals and subsidies for the determination of Triadic Concept Analysis guidelines in various scenarios and circumstances, similiar to Ananias et al. (2019), Dau and Wille (2000) developed a study on modal applications for the understanding of triadic contexts, having as perspective the triadic context as sets of formal objects, formal attributes and formal conditions, along with the formalization of the ternary relation, indicating when an object has an attribute under a certain condition.

4 METHODOLOGY

4.1 Database

The database used in this paper was obtained from the *COVID-19 National Longitudinal Phone Survey 2020 (Nigeria COVID-19 NLPS)*, a survey applied on individuals residing in Nigeria, conducted by the National Bureau of Statistics (NBS), an affiliate of the Federal Government of Nigeria and produced by the World Bank.

The survey, which collects data monthly, aims to monitor the socio-economic effects of the pandemic COVID-19 with real-time evolution. The study is part of the World Bank's Living Standards Measurement Survey program to provide support to countries to help mitigate the spread and impact of the new coronavirus disease.

The data collection is done through several telephone calls to households located throughout Nigeria. Each month, Nigerian households across the country are asked about topics that are likely to be affected by COVID-19 restrictions. Thus, the Nigeria COVID-19 NLPS intends to monitor the socio-economic effects of the COVID-19 pandemic that will contribute to the development of policies that will mitigate the negative impacts on the population.

This paper performs the triadic analysis applied to this base, using data collected in 3 waves of monthly interviews, which had as areas of interest (i) access to basic services, (ii) behavior, (iii) education, and (iv) access to medical treatment.

The survey was conducted with 1,800 individuals who are components and residents of the population of Nigeria, aged eighteen (18) years and above.

In order to analyze the areas of interest mentioned, four questions were selected related to areas of behavior and status in society, such as: washing hands with soap, use of mask in public, need for medical treatment, and frequency of children and adolescents with school activities.

The resulting analysis will allow us to associate such behaviors with the spread of COVID-19 and the socioeconomic impacts on the country.

The first wave of Nigeria COVID-19 NLPS telephone interviews considered for the present work was conducted between June 2 and 16, 2020. The second took place between July 2-16, 2020, and the third between the dates of August 9-24, 2020.

To bring the intended result in the present paper, relating the behavioral results of the Nigeria COVID-19 NLPS survey to the spread of COVID-19 during June, July, and August 2020 in Nigeria, the base Nigeria: Coronavirus Pandemic Country Profile was used, which determines the estimate of the effective reproduction rate (R) of COVID-19. The reproduction rate represents the average number of new infections caused by a single infected individual.

The aforementioned statistics and survey Nigeria: Coronavirus Pandemic Country Profile, was conducted by Our World in Data. The database provided, in addition to the many linked graphs, uses the complete set of data on confirmed cases and deaths from Johns Hopkins University (JHU) and the European Centre for Disease Prevention and Control (ECDC).

This Johns Hopkins University dataset is maintained by the Center for Systems Science and Engineering (CSSE). Since January 22, 2020, it has published updates on confirmed cases and deaths for all countries and updates its data several times a day. The data used from this database comes from the government, national and subnational agencies in Nigeria.

4.2 Methods

To achieve the goal of the paper, it was necessary to perform data transformation in a triadic context, using the Nigeria COVID-19 NLPS database as input to the concept extraction algorithm. To do this, the following process was carried out:

1) A preprocessing of the base was performed. Relevant and consistent questions were filtered and selected between the collection waves. After selecting the questions, the respective answers were discretized according to each context, so that there was no interference in the configuration and purpose of each question.

2) Grouping the preprocessed data into their waves. After preparing the questions and discretizing the answers, the data from each wave were grouped in the same table, keeping the reference of each wave for later interpretation of the implication rules generated.

3) For the rule generation the Lattice Miner 2.0 tool was used, a data mining prototype developed under the supervision of Professor Rokia Missaoui by the LARIM research laboratory of the Université du Québec (Missaoui and Emamirad, 2017). It is a public domain Java platform whose main functions include all the low-level operations and structures to represent and manipulate input data, structures, and association rules. The platform allows the generation of clusters, called formal concepts, and association rules, including logical implications, given a binary relationship between a collection of objects and a set of attributes or properties.

Among the functions made available by Lattice Miner 2.0, this work focuses on the use of the rules proposed by Biedermann (1997), these are: Biedermann Conditional Attribute Association Rule (BCAAR) and Biedermann Attributional Condition Association Rule (BACAR).

The first, BCAAR, takes the form $(A1 \rightarrow A2)C(sup,conf)$, where $A1,A2 \subseteq M$ and $C \subseteq B$. This rule indicates that every time *A1* occurs in the conditions *C*, then *A2* also occurs, with support *sup* and confidence *conf*. The second rule used is BACAR, which takes the form $(C1 \rightarrow C2)A(sup,conf)$, where $C1,C2 \subseteq B$ and $A \subseteq M$, that is, every time *C1* occurs for all attributes in *A*, then for the condition *C2*, there is an occurrence of the same attributes, with support *sup* and confidence *conf*.

5 RESULTS

From the triadic analysis performed on the database, it was possible to relate how the various basic, everyday behaviors of the Nigerian population can affect the spread of COVID-19.

Considering:
$w1$: Collection wave 1 conducted in June 2020;
$w2$: collection wave 2 conducted in July 2020, and
$w3$: Collection wave 3 carried out in August 2020.
We could obtain the follwing rules:

BCAAR Implication Rules

$1 : (B \to D)\ w1\ [sup = 53,6\%\ conf = 68,5\%]$
$2 : (B \to A)\ w3\ [sup = 70,7\%\ conf = 92,4\%]$
$3 : (B \to D)\ w3\ [sup = 50,5\%\ conf = 66,0\%]$
$4 : (A \to D)\ w2\ [sup = 51,0\%\ conf = 58,8\%]$
$5 : (A \to B)\ w2\ [sup = 74,6\%\ conf = 86,0\%]$
$6 : (D \to B)\ w1\ [sup = 53,6\%\ conf = 84,1\%]$
$7 : (A \to B)\ w3\ [sup = 70,7\%\ conf = 85,7\%]$
$8 : (A \to D)\ w3\ [sup = 52,8\%\ conf = 64,1\%]$
$9 : (D \to A)\ w2\ [sup = 51,0\%\ conf = 89,1\%]$
$10 : (B \to A)\ w2\ [sup = 74,6\%\ conf = 94,0\%]$
$11 : (D \to B)\ w3\ [sup = 50,5\%\ conf = 83,2\%]$
$12 : (D \to A)\ w3\ [sup = 52,8\%\ conf = 87,1\%]$

For this research, the questions asked to the nigerian population were:
A: Washing hands with soap after going in public;
B: Wearing a mask in public;
C: Need for medical treatment in the last 7 days;
D: Children and adolescents with school activities;

The value of 50% was used as lower limits of support and confidence in the generation of the BCAAR implication rules. With the results in hand, for interpretation and exposition in this paper, we kept the 50% for support and considered only the rules above 75% confidence.

From rule 5 it can be seen that during wave 2 (w2, conducted in June), 74.6% of the individuals interviewed who took care to wash their hands with soap in public (A) also took precautions by wearing masks (B). Note that of those who washed their hands in public, 86.06% certainly wore masks.

From rule 9, it is observed that during collection wave 2, in June 2020, 51% of the individuals living with children and adolescents with school activities (D) were careful to wash their hands after going in public (A). Of these respondents, 89.1% were sure to wash their hands with soap after going out in public.

From the generated rule 10, 74.6% of individuals who wore a mask in public (B), 94.5% were associated with washing their hands with soap after going out in public (A), in wave 2.

It is observed by rule 11, through the data collected during wave 3 (August 2020), that 50.5% of the interviewed individuals live with children and adolescents with school activities (D). Of this percentage, it is perceived that 83.2% were certain to have worn masks in public (B).

The aforementionted rules implies a direct relationship between individuals who washed their hands after going out in public, who also used masks in public, and in which families kept children and adolescents with school activities, during July and August.

BACAR Implication Rules

$1 : (w1 \to w3)\ B\ [sup = 64,5\%\ conf = 82,5\%]$
$2 : (w1 \to w2)\ B\ [sup = 63,5\%\ conf = 81,1\%]$
$3 : (w3 \to w1)\ B\ [sup = 64,5\%\ conf = 84,3\%]$
$4 : (w3 \to w2)\ B\ [sup = 61,1\%\ conf = 79,9\%]$
$5 : (w2 \to w3)\ A\ [sup = 71,5\%\ conf = 82,5\%]$
$6 : (w3 \to w2)\ A\ [sup = 71,5\%\ conf = 86,8\%]$
$7 : (w2 \to w1)\ B\ [sup = 63,5\%\ conf = 80,0\%]$
$8 : (w2 \to w3)\ B\ [sup = 61,1\%\ conf = 77,0\%]$
$9 : (w1w3 \to w2)\ B\ [sup = 52,4\%\ conf = 81,2\%]$
$10 : (w1w2 \to w3)\ B\ [sup = 52,4\%\ conf = 82,5\%]$
$11 : (w2w3 \to w1)\ B\ [sup = 52,4\%\ conf = 85,7\%]$

Similarly applied to the BCAAR rules, no rules were generated with support and confidence lower than 50%. Also, as with the previous interpretation, only rules with percentages above 50 support and 75 confidence were taken into account.

From rule 1, it can be seen that among the 64.5% of individuals interviewed who had taken precautions by wearing masks in public (B) in wave 2, in June 2020, 82.5% continued to wear their masks in wave 3, in August of the same year.

It is observed from rule 2 that among the 63.5% individuals who were wearing masks in public (B) in wave 2, in June 2020, 81.1% maintained the same protective behavior in wave 2, the following month, in July.

Also, employing rule 8, it is possible to identify that of the 61.1% of respondents who adhered to

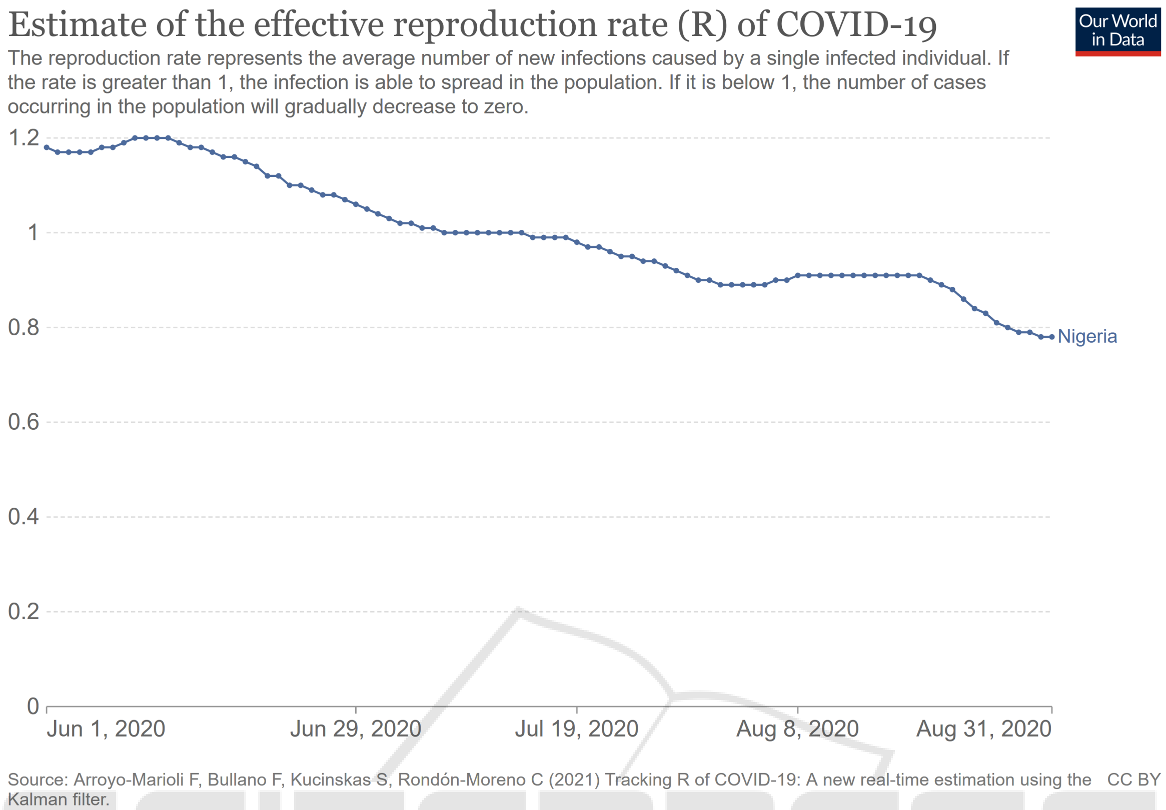

Figure 1: Covid Rate.

wearing masks in July 2020 (wave 2), 77.0% continued wearing the masks in public also in the subsequent month, during wave 3.

Finally, from rule 10, it is noted that 52.4% of the interviewed individuals who were wearing masks in public continuously between the months of June and July 2020 ($w1$ and $w2$), 82.5% continued with the protection method during the month of August of the same year ($w3$).

It is possible to observe that the behavior of individuals who used a mask in June, in the wave $w1$, was maintained during July and August, waves $w2$ and $w3$ respectively. Rule 5 also enforces basic service access behavior:

$5 : (w2 \rightarrow w3)\ A\ [sup = 71.5\%; conf = 82.5\%]$

Using rule 5, it can be seen that of the 71.5% individuals who followed the recommendation to wash their hands with soap after going in public in July 2021 ($w2$), 82.5% maintained this good practice in August ($w3$).

When comparing such implication rules with the results found in Nigeria (Figure 1): Coronavirus Pandemic Country Profile survey on the estimation of the effective reproduction rate (R) of COVID-19, we can observe that during June, July, and August, there was a decrease in R rate:

It is possible to infer, when compiling such databases, that the adherence of individuals in public policies to basic behavioral services, such as wearing masks and washing hands after going public, had an impact in preventing the spread of new cases of infection by an individual contaminated by COVID-19.

Yet another interpretation we can draw from BCAAR rules 5, 9, 10, 11, we find that keeping children and adolescents in school activities may have assisted in the adherence of families to the public health recommendations put forth by the government of Nigeria.

Thus, by analyzing these implications, it is possible to generate practical results, increasingly reliable, to support the construction of mitigatory and preventive actions and policies for the contamination of COVID-19, helping the government to obtain greater adherence of the population in restrictive actions with low socioeconomic impact and that have positive results in combating COVID-19.

6 CONCLUSION AND FUTURE WORK

The intention of the work, after careful selection of the results, is to understand the importance of the extracted knowledge on mortality rates, infection rates, and the number of cases of COVID-19 during the collection period.

In addition, the research seeks to highlight the contrasts of the results obtained with information from the Nigeria: Coronavirus Pandemic Country Profile" database, maintained and updated daily by the organization "Our World In Data".

This database provides statistical information about the progression of COVID-19, with data such as the number of deaths, number of confirmed cases, mortality rate, and number of tests performed.

Thus, in face of the proposed analyses, the present work aimed to expand the applicability of the Triadic Concepts Analysis.

It turned out to be an efficient and useful approach to find aspects not easily identified at first, related to several contexts of knowledge areas, as health and social information related to pandemics, diseases, and social behaviors.

Such results, by bringing correlations between population behaviors and dissemination of diseases, can be very useful in the basis and rationale for making government decisions of great impact on the health of the world population.

The study, therefore, by performing the complex analysis of data not directly linked, finding their correlation, can help governments and public entities to develop better public policies to combat highly infectious diseases and promote sanitation.

Thus, it is expected that several areas of study and scenarios will benefit from this methodology in their investigations and analysis, relying on the data interpretation enabled through TCA.

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