Linear Programming: A Diet Problem with Methane Emission

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Abstract: Cattle diet problems are concerned with finding the optimal diet for cattle. Classical diet problems are linear programming problems. This paper considers a complication of diet problem, adding to the traditional setting the extra component of methane emission. We find that under proper assumptions this complicated model is yet another linear programming problem. We then realize our model with empirical data and obtain the optimal diet for cattle via the simplex method. Sensitivity analysis is run against selected parameters. We conclude that our model is mostly successful, yielding many practical considerations.

1 INTRODUCTION

Diet, by its definition, is the food and drink that a person or animal drinks and eats regularly. Researches have been done on optimizing the diet of cattle in a limited range or within a specific area. To generalize, we developed an adaptable model which aims at balancing between the cost of a diet and maintaining the animal's basic conditions. The general goal of this process is getting the maximum nutritional requirements for the least amount of money. A unique feature of our model is the added goal of reducing methane emissions.

In building the model, we have made assumptions, gathered data for nutrition intake required for cattle, formulated functions, and gained results by utilizing linear programming.

2 BACKGROUND AND PROBLEM SET-UP

First we present a quick summary of our problem scenario. Our model starts with a cattle representative which is bought in at an initial weight. This cattle gains weight daily. The cattle is fed with a diet of N types of food. That cattle is also assumed to have a realistic eating habit, where it eats only a certain amount of food every day determined by its weight. This value is known as the dry matter intake of the cattle.

For each type of food, a set of nutrient concentrations are obtained from laboratory and empirical estimations. 4 nutrient requirements are checked every day to ensure that the cattle is growing healthily. The daily growth in weight of the cattle depends on how much food the cattle consumes, and in particular we assume that the growth in weight depends on the total energy intake of the cattle. Ultimately the main source of income of the cattle owner comes from selling the finishing cattle. The sale price is per unit weight.

Finally, IPCC estimated the daily methane emission of cattle as a linear function of its total energy intake (Dong, Mangino, & McAllister 2006). We then imagine that the government imposes environmental policies so that an emission fine is charged for every unit of excessive methane emission (amount that exceeds an emission threshold).

We move on to formulate the above scenario into an optimization problem. First, recall that our objectives are to increase the sale price of cattle, to reduce methane emission, and to reduce the diet cost. We then translate these into the following equation:

$$max.\, p \cdot \Delta W - c^T \cdot x - f \cdot \widehat{M} \tag{1}$$

where ΔW is the daily increase of weight, c^T (Rankin, 2021, Halopka, 2020, Livestock, Poultry, & Grain, University of Missouri Extension, IndexMundi, Alibaba 2021) is a vector of diet food

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cost, f is the unit fine charged on methane emission $\widehat{M}$, p (Ag Decision Maker 2021) is the unit sell price of cattle, and x is a vector of the weight of each food type in the diet. The given question requires us to vary x to achieve the maximum of the objective function of x defined above.

To ensure that this problem is realistic, we set up six constraints with respect to the food vector x. First, obviously entries of x should be positive:

$$x \geq 0 \quad (2)$$

We introduce the term D to represent the daily dry matter intake of a cattle, which is estimated using the linear equation $D = 1.8545 + 0.01937 \cdot W_0$ where W_0 is the initial weight (Ag Decision Maker (2021). D then equals the total weight of the food provided:

$$e^T \cdot x = D, \ \ e = [1, 1, \dots, 1]^T \quad (3)$$

We require that our diet should satisfy the daily nutrient requirements of cattle. We introduce the matrix A (National Academy of Science, Engineering, and Medicine (2000), whose columns correspond to each food type and rows correspond to the different types of nutrients. We further introduce the term b National Academy of Science, Engineering, and Medicine (2000) to represent the minimum nutrients required by the cattle for sustenance. Hence,

$$A \cdot x \geq b \quad (4)$$

We also require that the cattle are always gaining weight. This ensures that the farmer is always making profit. According to NASEM, the daily increase in weight can be estimated as $\Delta W = 13.91 \cdot (E_t - E_m) \cdot W^{-0.6837}$, where W is the daily weight, E_t is the total energy intake, and E_m is the energy required to maintain a cattle's activities. E_t and E_m can further be estimated by $E_t = C^T \cdot x$ and $E_m = s \cdot W^{0.75}$, where C National Academy of Science, Engineering, and Medicine (2000) is an energy vector specifying the energy concentration in each type of food and s National Academy of Science, Engineering, and Medicine (2000) is a scaling factor depending on the cattle's breed, sex, and cattle's nutritional conditions etc. These equations then allow us to write ΔW as a function of x, and we obtain another constraint:

$$\Delta W(x) = 13.91 \cdot (C^T \cdot x - s \cdot W^{0.75}) \cdot W^{-0.6837} \geq 0 \quad (5)$$

According to IPCC, the total methane emission M is estimated as $M = \frac{E_t \cdot k \cdot m}{mec}$, where k is a unit conversion factor, m is a methane conversion factor, and mec is the methane energy constant (Dong, Mangino, & McAllister, 2006). We then set an emission threshold M_0 and define the corresponding excessive methane emission $\widehat{M} = M - M_0$. Our problem is meaningful only if $\widehat{M}$ is positive:

$$\widehat{M} \geq 0 \quad (6)$$

We also want to ensure that $\widehat{M}$ follows IPCC's empirical estimation:

$$\widehat{M} \geq \frac{E_t \cdot k \cdot m}{mec} - M_0 = \frac{m \cdot k}{mec} \cdot C^T \cdot x - M_0 \quad (7)$$

Combining these estimates, we obtain the following optimization problem:

$$(*) \ max.\, p \cdot \Delta W(x) - c^T \cdot x - f \cdot \widehat{M} \quad (8)$$

$$s.t.\, x \geq 0, e^T \cdot x = D, A \cdot x \geq b, \Delta W(x) \geq 0, \widehat{M} \geq 0, \widehat{M} \geq \frac{m \cdot k}{mec} \cdot C^T \cdot x - M_0 \quad (9)$$

A summary of all variables and their symbols can be found in table 1.

Table 1: *Symbols*

Symbol	Unit	Description
x	kg	N-dimensional vector specifying how much kg of each food types a cattle eats per day.
M	*kg*	Estimated kg of methane emitted by a cattle per day
c^T	\$/kg	N-dimensional vector specifying how much a kg of each food type costs
A	unit	MN matrix specifying how much nutrient each food type contains
b	kg	M-dimensional vector specifying daily requirement of a specific nutrient
C	J/kg	N-dimensional vector specifying energy concentration in each food type
e^T	unit	N-dimensional vector of ones

p	\$/kg	Selling price of cattle meat
W	kg	Daily weight of a cattle
f	\$/kg	Amount of capital punishment per kg of excessive methane emission
D	kg	Daily dry matter intake of a cattle
E_m	MCal	Daily energy of maintenance for a cattle
s	J/kg0.75	Correlation factor between Emand W, which depends on breed, nutritional state, and sex etc.
k	MJ/MCal	Unit conversion factor
E_t	MCal	Total energy intake of a cattle per day
m	unit	Methane conversion factor
mec	MJ/kg	Methane energy constant
M_0	kg	Methane emission threshold

3 DATA AND METHODOLOGY

3.1 Data

Our primary data sources are IPCC and NASEM. Other data (mainly market prices of diet food and cattle) are quoted from various Internet sources. We manually assign values to some parameters, such as the fine placed on excessive methane emissions. See table 2-4 for a full presentation of data and their sources.

Table 2: Nutritional/Energy concentration and Cost (A/C and c):

Food Type	NE (Mcal/kg)	CP (%)	C (%)	P (%)	Cost (\$/ton)
ALFALFA Fresh	1.38	18.90	1.29	0.26	167
Hay	1.31	18.6	1.40`	0.28	171
Straw	0.6	4.40	0.30	0.07	70
Gluten meal	2.20	66.3	0.07	0.61	700
Seed	2.24	24.4	0.17	0.62	370
Barley Grain	2.06	13.2	0.05	0.35	114
Sugar beets	1.76	9.8	0.68	0.1	125
Cotton hulls	0.68	4.2	0.15	0.09	230
Wheat midds	1.6	18.7	0.17	1.01	220
Cottonseed Meal	1.79	46.1	0.02	0.02	370
Distiller's grains dried	2.18	30.4	0.26	0.83	271
Oat hulls	0.41	4.1	0.16	0.15	129

Table 3: Nutritional requirement (b): (assume W_0 = 400 kg; breed code "1 Angus"; W_T = 890 kg).

Type	Measurement/Unit (per day)	Value
Energy	Net Energy (NE) / Mcal	6.38

Table 4: Data Sources.

Protein	Metabolizable protein (MP) /kg (CP = MP/0.64)	0.274
C/Calcium	C / kg	9/1000
P/Phosphorus	P / kg	7/1000
Parameter	Estimated Value/Unit	Source

c^t	See table 2	
A	See table 2	NASEM 2000, 134
b	See table 3	NASEM 2000, 106
C	See table 2	NASEM 2000, 134
p	2.9548/\$/kg	
W_0	400/kg	NASEM 2000, 106
f	88644/1000000	30% of sell price
s	0.077	NASEM 2000, 6
k	4184000/1000000	/
m	3%	IPCC 2006, 4 ,10.30
mec	55.65	IPCC 2006, 4, 10.31
M_0	0.125	Assume $E_m = E_t$ then compute IPCC's equation

3.2 Methodology

Simplex Method is an iterative algorithm, which aims to find the optimal solution of x. Its main steps are:

1) find a basic feasible solution;

2) judge whether the solution is the optimal according to the optimality theory;

3) if it is the optimal, then stop the process;

4) if it is not, then try to generate a new feasible solution which better minimizes the objective function; and

5) judge its optimality again.

This loop is continued until an optimal solution is found.

However, in different cases, there are distinct results of the simplex method. In some cases, the LP problem would degenerate. This happens when one of basic feasible variables has zero value. Infinite iterations would appear, and this prevents the algorithm from converging.

If no degeneracy appears, the simplex method would converge after certain iterations. Two results would appear: an optimal feasible solution, which leads to a minimum value of objective; or an infinite amount of feasible solutions where the objective has no lower bound.

Simplex method only works for optimization problems in their canonical forms. To convert (*) into its canonical form, we define the canonical variables as follows:

Table 5: Canonical Variables.

Slack Variable	Definition
x	$\begin{pmatrix} x \\ \widehat{M} \\ v \end{pmatrix}$ (where v is a vector of slack variables)
c	$\begin{pmatrix} c - 13.91pW^{-0.6837} \cdot C \\ f \\ 0 \end{pmatrix}$
A	$\begin{pmatrix} C^T & 0 & (1,0,0,\ldots,0) \\ A & 0 & (0,I,0) \\ e^T & 0 & 0 \\ -\left(\frac{k \cdot m}{mec}\right) \cdot C^T & 1 & (0,0,\ldots,0,1) \end{pmatrix}$
b	$\begin{pmatrix} s \cdot W^{0.75} \\ b \\ D \\ -M_0 \end{pmatrix}$

These canonical variables define an optimization problem as follows:

$$(**)\ \ min.\ \ \hat{c}^T \cdot \hat{x}$$

$$s.t.\ \ \hat{x} \geq 0, \hat{A} \cdot \hat{x} = \hat{b} \qquad (10)$$

And it is easy to check that (**) is equivalent with (*). We then run the simplex method to obtain a solution to (**), from which we obtain a solution to (*).

3.3 Results and Analysis

The algorithm run on the data apparently converges and outputs an optimal diet with two non-zero components: about 8.9 kg of barley grains and 0.67 kg of sugar beets. The program also reports an optimal objective of -12.2564, meaning that the maximum profit for the cattle owner on the first day is about 12 dollars.

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Initial Objective = 23.3262
Simplex Method has converged
Optimal Objective = 0 after 8 iterations
simplex method successful for Phase I
Initial Objective = -2.5258
Simplex Method has converged
Optimal Objective = -12.2564 after 4 iterations
simplex method successful for Phase II
LP optimal reached at objective = -12.2564

x =

         0
         0
         0
         0
         0
     8.936
   0.66647
         0
         0
         0
         0
         0
  0.031647
    12.694
    13.201
   0.81675
```

Figure 1: Simplex Method Status and Optimal $\hat{\boldsymbol{x}}$.

A couple of remarks on the result are in place. First, only two types of food are included in the optimal diet. Usually one should worry when the number of non-zero outputs is small. In particular, it may mean that the problem is degenerate if the number of non-zero outputs is significantly lower than the number of constraints. However, for our problem we only have one equality constraint in our original diet problem:

$$e^T \cdot x = D \qquad (11)$$

Hence if the other inequality constraints are merciful, then potentially we only need to allow one or a few degrees of freedom in the diet vector x to meet all the constraints. This intuition is justified. Observe that $\hat{x}$ has five other non-zero components. One of these corresponds to $\widehat{M}$, and the other four are slack variables. Indeed, $\hat{A}$ in the canonical form is a rank 7 matrix. Hence, it makes sense for us to obtain exactly seven non-trivial components in our solution. The fact that only two components of the diet vector are non-trivial is a direct consequence of the way the constraints are formulated. Therefore, mathematically the result is consistent.

There are two zero slack variables. This indicates that two constraints have reached the boundaries on the feasibility plane. When we plug in our vector x back to the optimization problem, we can see that the requirement of Calcium (C) is exactly reached. If the cattle may require slightly more protein, phosphorus, and energy, the diet given above is still suitable and effective. But if calcium (C) requirement increases, then the boundary is pushed outwards and the optimum diet above no longer satisfies the nutrient requirements.

Another legitimate concern is that compared with the nutrient requirements specified in vector b, some reasonably priced food types are simply too nutritious so that the nutrient requirements are easily met. This intuition can be backed up by a simple sensitivity analysis on the parameter b. Indeed, apparently doubling b only increases the maximum profit by about 0.1 dollars. The new optimal diet is still composed of sugar beets and barley grains where barley grains. This pattern persists as we impose stricter nutrient requirements, until when the nutrient requirements are about four times stricter than the original one and the linear program then becomes infeasible. Therefore, the objective function is apparently insensitive to the nutrient requirements (wherever the problem is feasible). This then supports the intuition that when the cattle owner is choosing between different types of diet food with the hope of maximizing net profit, nutrient requirements turn out to be a rather insignificant factor.

Next, observe that the thirteenth component (of which corresponds to $\widehat{M}$) is closed to 0. This means that the optimal methane emission is very closed to the emission threshold (M_0). Recall that methane emission and growth in weight both depend linearly on energy intake:

$$(***) \qquad f = \frac{13.91 \cdot p \cdot W^{-0.6837} \cdot mec}{m \cdot k} \qquad (12)$$

So, changing x should result in comparable changes in the profit generated by growth in weight $p \cdot W$ and the cost resulted from excessive methane emission $-f \cdot \widehat{M}$. Sensitivity analysis on p and f confirms a part of this intuition. The objective is sensitive to p but quite stable with respect to f, unless f is multiplied to be quite large. This makes sense intuitively, since f is per kg charge on methane emission. As we all know methane is a gas, and hence kg is a rather misleading dimension of weight for methane.

Furthermore, when the profit generated by growth in weight matches with the cost from emission (the exact f that makes this happen can be obtained from solving the equation (***) above) the farmer is not making any real profit out of the cattle business. This is perhaps a useful fact for governors to decide how much fine should be in place (with respect to the sale price of cattle) in order to economically effectively affect the cattle owner's behaviors.

Finally, sensitivity analysis on the food cost c and D are also conducted. The food cost is apparently very low in comparison with sale price p and methane emission fine. Hence, apparently the objective is quite insensitive to the food cost unless the sale price of cattle diet food becomes comparable to the sale price of cattle itself. D acts as an upper bound for the diet vector x. Should D increase drastically, it is expected that under our problem set up the cattle grows in weight indefinitely. The objective indeed shoots off as D increases indefinitely.

4 RESULTS AND ANALYSIS

In this paper, we have considered a revised cattle diet problem with the added component of methane emissions. We formulate the problem as an optimization problem and solve it through the simplex method. In general, our problem is easy to construct and solve. Sensitivity analysis proves that the methane emission fee is an effective way to affect the cattle business. In addition, our model is also adaptable to different species of animals and different nutrition requirements, by simply changing several parameters in our model. Within reasonable range, we expect our method to still converge to an optimal.

Any model has to give way to simplifications. We now turn to point out some issues and suggestions for future improvements. To begin with, there are a couple of assumptions that can be avoided by considering more complicated models. First, in our model we assume that the weight gain of the cattle is a constant. Our model can naturally be extended to a dynamic programming problem by introducing a weight function that evolves over discrete time.

Moreover, we assume that the same nutrient requirements apply to cattle among all age groups. In reality, it requires more nutrients when cattle are young and in the process of growth. Hence the constraint for nutrient requirements should depend on time as well, and this can be incorporated with the growth in weight in a dynamic programming version of our simply model.

Also, note that the methane emission fee we put on is regarded as fixed regardless of the amount of excessive emissions. However, as in the case of taxation capital punishments are typically piecewise functions. This can be potentially a more complex constraint for our problem.

Finally, in our optimal diet only two types of food are selected. This is because our constraints and nutritional estimations are simple and we pay no attention to digestion processes and finer nutritional requirements. For example, our system is based on an assumption that the cattle will take in all the nutrition the food supplies them. However, in the real world it is definitely not the truth. The cattle may require other food to help them fully absorb the nutrients. Future studies of cattle diet problems should take these into the account and consider, for example, a more intricate mechanistic model of cattle's nutrition.

5 CONCLUSIONS

Our group formulates a revised diet problem, which considers sell price of cattle, cost of the diet, the fine charged on methane emission, and the weight growth of cattle. We obtain formulas mainly from NASEM and IPCC and collect data of parameters on the Internet. This could then be solved by Simplex Method in Matlab. We conclude with a suggested optimal diet and suggest for future research that a dynamic version of our problem to be developed.

REFERENCES

Ag Decision Maker (2021). Historical Cattle Prices. https://www.extension.iastate.edu/agdm/livestock/pdf/b2-12.pdf

Alibaba.com (2021). Organic Price Sugar Pellets Beet Pulp. https://ua1335743999cerh.trustpass.alibaba.com/product/62006362618-810718723/Organic_Price_Sugar_Pellets_Beet_Pulp.html

AMS Livestock, Poultry, & Grain Market News. US (2021). Hay Auction Weighted Average Report For June 23, 2021.https://www.ams.usda.gov/mnreports/ams_1725.pdf

Dong, H., Mangino, J., & McAllister, T. (2006). Emissions from Livestock and Manure Management. In: IPCC (Ed.), 2006 Guidelines for National Greenhouse Gas Inventories. The Institutes for Global Environmental

Strategies, Japan. volume 4.10: pp. 31-32.https://www.ipcc-nggip.iges.or.jp/public/2006gl/pdf/4_Volume4/V4_00_Cover.pdf
Halopka, R. US (2020). Hay Market Demand and Price Report for the Upper Midwest For January 13, 2020.https://fyi.extension.wisc.edu/forage/files/2020/01/01-13-20.pdf
IndexMundi (2021). Barley Monthly Price. https://www.indexmundi.com/commodities/?commodity=barley&months=60
National Academy of Science, Engineering, and Medicine (2000). Nutrient Requirements of Beef Cattle: Seventh Edition. National Academy Press. p.6, pp.106-134.
Rankin, M. US (2021). USDA Hay Market. https://hayandforage.com/articles.sec-7-1-Markets.html
University of Missouri Extension. US (2021). By-Product Feed Price Listing. http://agebb.missouri.edu/dairy/byprod/bplist.asp