PARAMETER CONVERGENCE IN ADAPTIVE FUZZY CONTROL

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Abstract: In this paper, the convergence of parameter estimates and the interactions among the two adaptive fuzzy systems constituting an indirect adaptive fuzzy controller are studied, both analytically and by means of simulations with a second-order nonlinear system. The analytical results and the simulations, performed with various initial conditions and learning rates, highlight how the interactions affect the behavior of the adaptive control scheme with regard to the control performance in terms of a tracking error, accuracy and relevance of the identified fuzzy models.

1 INTRODUCTION

The main goals of adaptive control are (i) to adjust the on-line controller such that the required closed-loop performance (stability in the first place) is preserved in the presence of unforeseen parameter variations and/or (ii) to learn a suitable control law when *a priori* information on the controlled plant is lacking (e.g., the plant parameters are partly or completely unknown).

Adaptive fuzzy control (AFC) combines results from modern control theory, fuzzy systems and adaptation techniques. The most common stable AFC schemes are based on *feedback linearization*, employing fuzzy systems (mostly of the singleton type) as *universal function approximators*. They can be used either to approximate the unknown plant dynamics (*indirect schemes*) or directly the control law (*direct schemes*). With respect to other universal interpolators, such as neural networks, fuzzy systems offer the possibility to interpret the input-output relations by means of linguistic rules. This feature allows one to incorporate *a priori* knowledge in the initial model and/or control law and, at least in principle, to gather useful insights about the controlled process at the end of the learning stage.

The parameter learning laws of AFC are often derived by using *Lyapunov synthesis* (Wang, 1994) and are basically guided by the tracking error with respect to some reference trajectory. The earliest controllers of this type have been introduced for SISO systems in the controllable canonical form (Wang, 1993; Wang, 1996). Since then, considerable efforts have been devoted to improving the performance and extending the applicability to wider classes of systems. For instance, extensions to systems with unmeasurable states are proposed in (Tong et al., 2000), MIMO systems are considered in (Ordonez and Passino, 1999; Tong and Chai, 1999), while in (Wang et al., 2002; Tsay et al., 1999; Spooner and Passino, 1996) first-order Takagi-Sugeno fuzzy systems are used as approximators. Adaptive fuzzy control in the presence of uncertainties is realized by adding a *sliding mode* term to the control law (Su and Stepanenko, 1994; Han et al., 2001; Fishle and Schroder, 1999), or by using H_∞ performance indices (Chen et al., 1996; S. Tong and Wang, 2000).

Despite numerous studies on AFC, some basic problems arising even in the case of simple nonlinear SISO systems have not been addressed in the literature. Examples of these problems are:

- dependence of the AFC performance (learning time, stability of the adapted parameters and quality of the identified models) on *user-defined parameters*;
- inherent conflict between the *control goal* (reduction of the tracking error) and the *identification goal* (reduction of the identification error with respect to the unknown system dynamics);

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- *interactions* between the fuzzy systems' parameters and their convergence during the learning process.

The lack of a proper understanding of these basic aspects makes it difficult to properly exploit the transparency and interpretability of fuzzy systems, which are in fact neglected in a considerable part of the recent literature on AFC.

In this paper, we address the issue of interaction between the fuzzy systems' parameters and their convergence in the case of indirect AFC based on feedback linearization. First, we determine analytical relations between the rates of change of the tracking error and the parameter errors. This analysis highlights the occurrence of conflicts between the identification of unknown system dynamics and the tracking control. Then, we show, by means of simulations with a second-order system, that the AFC performance critically depends on user-defined parameters such as the learning rates or initial values of the adapted parameters. We consider the scheduling of learning rates as a possible solution to overcome the above-mentioned problems.

The remainder of this paper is structured as follows. In Section 2, the basic elements of indirect adaptive fuzzy control are described. In Section 3, an analysis of interactions is carried out. Section 4 describes a case-study with a second-order nonlinear system. The scheduling of the learning rates and its effects on the performance are discussed in Section 5. Finally, in Section 6, conclusions are given.

2 INDIRECT AFC

2.1 Structure of the Controller

Indirect adaptive fuzzy control (Wang, 1996) is suitable for systems in *controllable canonical form*

$$\begin{aligned} x^{(n)} &= f(\boldsymbol{x}) + g(\boldsymbol{x})u &(1)\\ y &= x\,. &(2)\end{aligned}$$

where $\boldsymbol{x} = \left[x, \dot{x}, \ldots, x^{(n-1)}\right]^T \in \mathbb{R}^n$ is the state vector. The control goal is to track a desired trajectory y_m while keeping all the signals bounded in the closed-loop. Define the tracking error $e = y_m - y$ as the difference between the reference trajectory and the output of the system. Further, introduce the vector of the tracking error and its $n-1$ derivatives $\boldsymbol{e} = \left[e, \dot{e}, \ldots, e^{(n-1)}\right]^T$ and the vector of the feedback gains $\boldsymbol{k} = [k_n, \ldots, k_1]^T$. If the functions $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are known and if the gains k_i are chosen such that the roots of the polynomial $h(s) = s^n + k_1 s^{n-1} + \ldots + k_n$ are in the open left-half complex plane, *the feedback linearizing control law*

$$u^* = \frac{1}{g(\boldsymbol{x})}\left[-f(\boldsymbol{x}) + y_m^{(n)} + \boldsymbol{k}^T\boldsymbol{e}\right] \tag{3}$$

produces the desired linear *error dynamic*:

$$e^{(n)} + \boldsymbol{k}^T\boldsymbol{e} = 0 \tag{4}$$

or equivalently

$$\dot{\boldsymbol{e}} = \boldsymbol{\Lambda}_c\boldsymbol{e} \tag{5}$$

where the matrix $\boldsymbol{\Lambda}_c \in \mathbb{R}^{nxn}$ is given by

$$\boldsymbol{\Lambda}_c = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \ldots & 0 & 1 \\ -k_n & -k_{n-1} & \ldots & \ldots & -k_1 \end{bmatrix}. \tag{6}$$

The ideal control law (3) guarantees that $\lim_{t\to\infty} e(t) = 0$. The basic idea of Wang's indirect adaptive control is to approximate the unknown functions $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ in the control law (3) by using two singleton fuzzy systems:

$$\begin{aligned} \hat{f}(\boldsymbol{x}) &= \boldsymbol{\theta}_f^T\boldsymbol{\xi}_f(\boldsymbol{x}) &(7)\\ \hat{g}(\boldsymbol{x}) &= \boldsymbol{\theta}_g^T\boldsymbol{\xi}_g(\boldsymbol{x})\,. &(8)\end{aligned}$$

where $\boldsymbol{\theta}_f$ and $\boldsymbol{\theta}_g$ are the *consequent* parameters to be adapted, $\boldsymbol{\xi}_f(\boldsymbol{x})$ and $\boldsymbol{\xi}_g(\boldsymbol{x})$ are the *normalized degrees of fulfillment* of the (fixed) fuzzy rules. If we replace the functions $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ in (3) with their fuzzy approximations, we have a control law

$$u = \frac{1}{\hat{g}(\boldsymbol{x})}\left[-\hat{f}(\boldsymbol{x}) + y_m^{(n)} + \boldsymbol{k}^T\boldsymbol{e}\right] \tag{9}$$

that is an approximation of the ideal law (3). To properly isolate the effects of interactions between the learning systems from other phenomena related to the *minimal approximation error*, we choose $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ that can be exactly approximated by the two fuzzy systems $\hat{f}(\boldsymbol{x})$ and $\hat{g}(\boldsymbol{x})$: $f(\boldsymbol{x}) = {\boldsymbol{\theta}_f^*}^T\boldsymbol{\xi}(\boldsymbol{x})$ and $g(\boldsymbol{x}) = {\boldsymbol{\theta}_g^*}^T\boldsymbol{\xi}(\boldsymbol{x})$ (where $\boldsymbol{\theta}_f^*$ and $\boldsymbol{\theta}_g^*$ represent the optimal parameters). Adding and subtracting $\hat{g}(\boldsymbol{x})u$ at the right-hand side of (1), substituting the control law (9) only in the term $\hat{g}(\boldsymbol{x})u$ and substituting equations (7), (8) with the actual and optimal parameters, we have, after some manipulations, the error dynamic

$$\dot{\boldsymbol{e}} = \boldsymbol{\Lambda}_c\boldsymbol{e} + \boldsymbol{b}_c\left[\boldsymbol{\phi}_f^T\boldsymbol{\xi}_f(\boldsymbol{x}) + \boldsymbol{\phi}_g^T\boldsymbol{\xi}_g(\boldsymbol{x})u\right] \tag{10}$$

where $\boldsymbol{b}_c = [0, ..., 0, 1]^T$; the parameter error $\boldsymbol{\phi}_f = \boldsymbol{\theta}_f - \boldsymbol{\theta}_f^*$ is the difference between the actual parameters $\boldsymbol{\theta}_f$ of the fuzzy system $\hat{f}(\boldsymbol{x})$ and the optimal parameters $\boldsymbol{\theta}_f^*$; the same holds for the parameter error $\boldsymbol{\phi}_g = \boldsymbol{\theta}_g - \boldsymbol{\theta}_g^*$.

2.2 Adaptive Laws

The adaptive laws are derived using Lyapunov synthesis. The considered Lyapunov function

$$\begin{aligned} V &= \frac{1}{2}\,\boldsymbol{e}^T\mathbf{P}\boldsymbol{e} + \frac{1}{2\gamma_f}\boldsymbol{\phi}_f^T\,\boldsymbol{\phi}_f + \frac{1}{2\gamma_g}\boldsymbol{\phi}_g^T\boldsymbol{\phi}_g \\ &= V_e + V_f + V_g\,. \end{aligned} \tag{11}$$

is the sum of three contributions depending respectively on the tracking error $\boldsymbol{e}$ and the parameter errors $\boldsymbol{\phi}_f$ and $\boldsymbol{\phi}_g$. Matrices $\mathbf{P} \in \mathbb{R}^{nxn}$ and $\mathbf{Q} \in \mathbb{R}^{nxn}$ are positive-definite matrices that fulfill the *Lyapunov equation*

$$\boldsymbol{\Lambda}_c^T\mathbf{P} + \mathbf{P}\boldsymbol{\Lambda}_c = -\mathbf{Q}\,. \tag{12}$$

This choice for V is made with the goal of guaranteeing simultaneously the boundedness of tracking error and of parameter errors. The time-derivative of V is obtained by differentiating (11), substituting for $\dot{\boldsymbol{e}}$ from (10) and using (12)

$$\begin{aligned} \dot{V} &= -\frac{1}{2}\,\boldsymbol{e}^T\mathbf{Q}\boldsymbol{e} + \frac{1}{\gamma_f}\boldsymbol{\phi}_f^T\left[\dot{\boldsymbol{\theta}}_f + \gamma_f e_s\boldsymbol{\xi}_f(\boldsymbol{x})\right] \\ &\quad + \frac{1}{\gamma_g}\boldsymbol{\phi}_g^T\left[\dot{\boldsymbol{\theta}}_g + \gamma_g e_s\boldsymbol{\xi}_g(\boldsymbol{x})\,u\right] \end{aligned} \tag{13}$$

where γ_f and γ_g are the *learning rates*, $e_s = \boldsymbol{e}^T\boldsymbol{p}_n$ and $\boldsymbol{p}_n$ is the last column of $\mathbf{P}$. If the parameters $\boldsymbol{\theta}_f$ and $\boldsymbol{\theta}_g$ are adapted according to the following laws

$$\dot{\boldsymbol{\theta}}_f = -\gamma_f\, e_s\boldsymbol{\xi}_f(\boldsymbol{x}) \tag{14}$$

$$\dot{\boldsymbol{\theta}}_g = -\gamma_g\, e_s\boldsymbol{\xi}_g(\boldsymbol{x})\,u \tag{15}$$

the terms in the brackets in (13) are zero and the derivative $\dot{V}$ is negative-definite. This guarantees that V is decreasing and asymptotically converges to zero.

3 ANALYSIS OF INTERACTIONS

This section focuses on the interactions between the two adaptive fuzzy systems (7) and (8) and the effects of these interactions on the evolution of the tracking error and the parameter errors. In (13) we can separate the time derivatives of the terms V_f, V_g and V_e:

$$\dot{V}_f = \frac{1}{\gamma_f}\boldsymbol{\phi}_f^T\dot{\boldsymbol{\theta}}_f \tag{16}$$

$$\dot{V}_g = \frac{1}{\gamma_g}\boldsymbol{\phi}_g^T\dot{\boldsymbol{\theta}}_g \tag{17}$$

$$\dot{V}_e = -\frac{1}{2}\,\boldsymbol{e}^T\mathbf{Q}\boldsymbol{e} + e_s\boldsymbol{\phi}_f^T\boldsymbol{\xi}_f(\boldsymbol{x}) + e_s\boldsymbol{\phi}_g^T\boldsymbol{\xi}_g(\boldsymbol{x})u \tag{18}$$

If we substitute the expression of $\dot{\boldsymbol{\theta}}_f$ given by (14) in (16), remembering the definition of $\boldsymbol{\phi}_f$ we get

$$\dot{V}_f = -e_s\boldsymbol{\phi}_f^T\boldsymbol{\xi}_f(\boldsymbol{x}) = -e_s\left(\hat{f}(\boldsymbol{x}) - f(\boldsymbol{x})\right)\,. \tag{19}$$

In the same way, substituting $\dot{\boldsymbol{\theta}}_g$ from (15) into (17) and recalling the definition of $\boldsymbol{\phi}_g$, we obtain

$$\dot{V}_g = -e_s\boldsymbol{\phi}_g^T\boldsymbol{\xi}_g(\boldsymbol{x})u = -e_s\left(\hat{g}(\boldsymbol{x}) - g(\boldsymbol{x})\right)u\,. \tag{20}$$

Substituting the control law (9) into $\hat{g}(\boldsymbol{x})u$ and deriving from (1) the product $g(\boldsymbol{x})u$, we can recast the last equation in the following form (after some manipulations):

$$\dot{V}_g = -\dot{V}_f - e_s\left[e^{(n)} + \boldsymbol{k}^T\boldsymbol{e}\right]\,. \tag{21}$$

In the same way, we can prove that

$$\dot{V}_e = -\frac{1}{2}\boldsymbol{e}^T\mathbf{Q}\boldsymbol{e} + e_s\left[e^{(n)} + \boldsymbol{k}^T\boldsymbol{e}\right]\,. \tag{22}$$

Equations (19), (21) and (22) represent the contributions to $\dot{V}$ due to the parameter errors and the tracking error, respectively. They highlight how the adaptive systems interact and how such interactions affect the identification task (reduction of the parameter errors) and the control task (reduction of the tracking error). The term in square brackets in equations (21) and (22) represents the error dynamic if the control law is the ideal feedback linearizing control law u^*. Hence, in the ideal case, it is zero. Since we only have an approximation of this ideal control law, this term is not zero. However, we can expect that if the approximation of the ideal control law is good enough, the error will approximatively fulfill the ideal error dynamic and the term in square brackets will be small compared to $-\dot{V}_f$ (see the simulations results in Section 4). As a result, we can see from equations (19) and (21) that, the variations of V_f and V_g would be of opposite sign: an improvement in the identification of $f(\boldsymbol{x})$ would be compensated by a deterioration in the identification of $g(\boldsymbol{x})$ and vice-versa. Equation (22), on the other hand, shows the interference between identification and control tasks.

Remark: Note that we have not considered an approximation error. In the presence of a minimal approximation error, an additional term appears in the time-derivative of the Lyapunov equation, but the relationship between $\dot{V}_f$ and $\dot{V}_g$ remains unchanged.

4 SIMULATION EXAMPLE

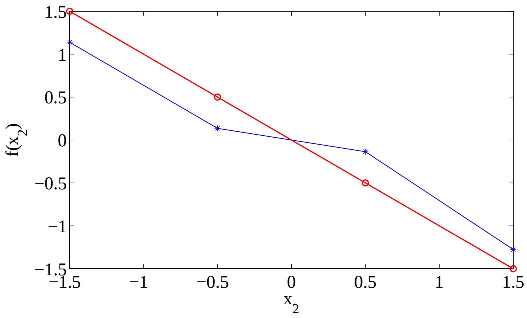
Consider the following second-order nonlinear system:

$$\dot{x}_1 = x_2 \tag{23}$$

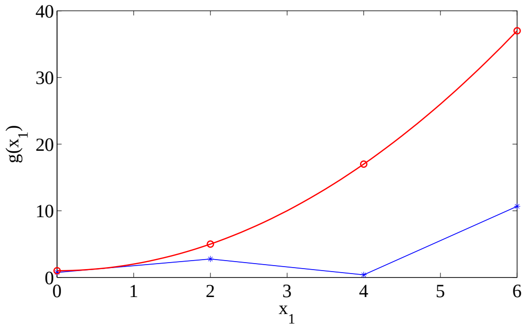
$$\dot{x}_2 = -x_2 + g(x_1)\,u \tag{24}$$

$$y = x_1\,. \tag{25}$$

The functions $f(x_2) = -x_2$ and $g(x_1)$ are chosen such that they can be exactly represented by fuzzy



(a) Final approximation for $f(x_2)$



(b) Final approximation for $g(x_1)$

Figure 1: Final approximations for $f(x_2)$ and $g(x_1)$: 'o' represent the true parameters; '*' represent the actual parameters.

systems with a finite number of rules. This means that there is no minimal approximation error. In fact, $f(x_2)$ is linear and $g(x_1)$ is a singleton fuzzy system (with four singleton consequents and four antecedent triangular membership functions centered at $x_1 = 0, 2, 4, 6$). The consequents are the values of the parabolic function ${x_1}^2 + 1$ for $x_1 = 0, 2, 4, 6$. The fuzzy system $\hat{g}(x_1)$ has the same structure as $g(x_1)$ and hence it can exactly approximate $g(x_1)$. Another distinct feature of the above system is that $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ only depend on one component of the state vector each. This makes it possible to study the interactions between adaptive fuzzy systems by using two simple fuzzy systems with one premise variable.

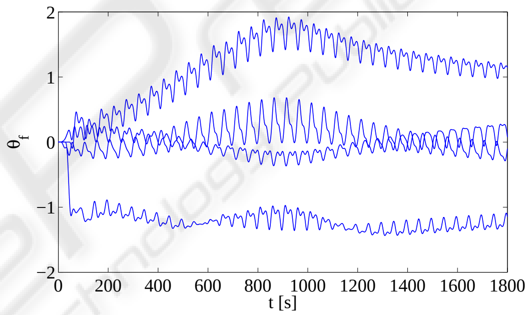
The simulations have been performed with the Matlab/Simulink implementation of the ode45 Dormand-Prince method and the relative tolerance of 0.001. Thanks to the absence of inherent approximation error in our case-study, *supervisory control* is not necessary for stability assurance and is therefore omitted. The fuzzy systems $\hat{f}(x_2)$ and $\hat{g}(x_1)$ have four triangular membership functions equally distributed in their respective domains, $[-1.5, 1.5]$ and $[0, 6]$. The *reference model* is a second order linear system with natural frequency $\omega_n = 1\,\text{rad}\,\text{s}^{-1}$ and damping ratio $\delta = 0.9$. The *reference signal* $\boldsymbol{r}_m$ is represented by a repeating sequence with values in the range $[0, 6]$.

The feedback gains k_i were set to $k_1 = 1.8$ and $k_2 = 1$. The positive-definite symmetric matrices $\mathbf{P}$ and $\mathbf{Q}$ satisfying equation (12) were obtained by numerically solving this linear matrix inequality (LMI).

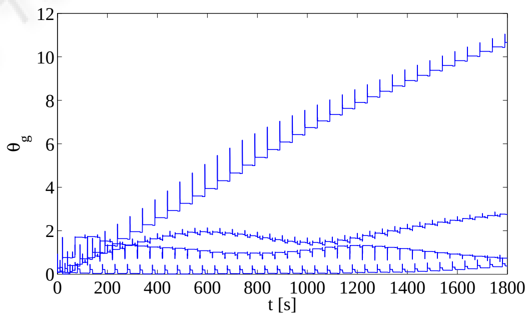
4.1 Standard Setting

An attempt to train $\hat{f}(x_2)$ and $\hat{g}(x_1)$ simultaneously (with equal learning rates $\gamma_f = \gamma_g = 100$) results in poor approximations of the true functions $f(x_2)$ and $g(x_1)$, see Fig. 1. This is despite the fact that an important piece of prior knowledge was used, namely the fact that each of the functions only depends on one component of the state.

It can also be seen in Fig. 2 that in this case the parameters are not converging at all. The analysis of



(a) Filtered singletons for $f(x_2)$



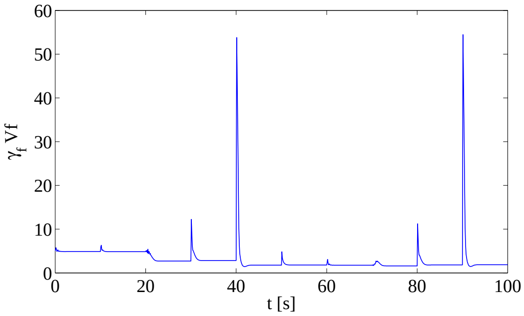
(b) Singletons for $g(x_1)$

Figure 2: Singletons for $f(x_2)$ and $g(x_1)$.

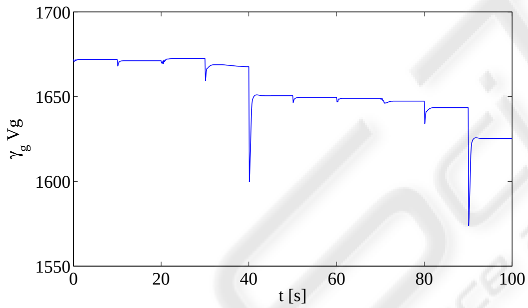
interactions carried out in Section 3 offers an explanation of this simulation results. In Fig. 3, we can see that $\gamma_f V_f$ and $\gamma_g V_g$ exhibit similar complementary variations (the interplay between the two identification tasks): if $\gamma_f V_f$ decreases, $\gamma_g V_g$ increases and vice-versa. The simultaneous identification of the unknown system dynamics cannot be accomplished most of the time. Moreover, in Fig. 4 we can see the interplay between identification and control: an improvement in the parameter errors corresponds to higher tracking error and vice-versa. Hence, we can see that the adaptation is negatively affected by these interactions, in terms of learning time, oscillation of the parameters and accuracy of the final fuzzy models.

Remark 1: For sake of clear presentation, the parameters shown in Fig. 2(a) were filtered by a low-pass Butterworth filter with the cut-off frequency of $0.3\,\mathrm{rad\,s^{-1}}$. Without this filtering, the plot is rather confusing due to large overlapping parameter oscillations.

Remark 2: We have plotted $\gamma_f V_f$ and $\gamma_g V_g$ rather than V_f and V_g because we are interested in the evolution over time of the parameter errors and the learning rates represent just scaling factors.



(a) $\gamma_f V_f$



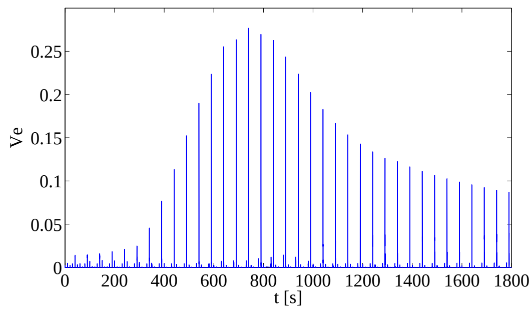
(b) $\gamma_g V_g$

Figure 3: Zoom on the evolution over time of $\gamma_f V_f$ and $\gamma_g V_g$.

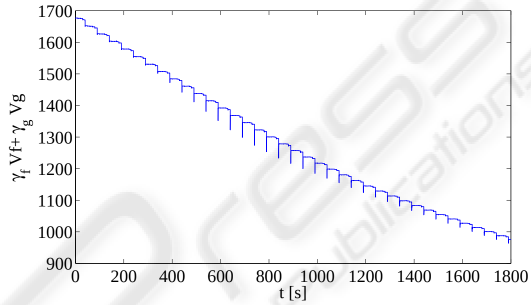
4.2 Alternative Settings

The simulations have been also carried out in different settings, in order to see how the phenomena highlighted in the previous section depend on the particular settings and to what extent they can be generalized.

First, one of the two functions $f(x_2)$ and $g(x_1)$ is assumed to be known and the other one is learnt. This allows us to study what happens when there are no interactions between the learning processes. Second, $f(x_2)$ and $g(x_1)$ are learnt simultaneously, but in this case with different learning rates and with different



(a) V_e



(b) $\gamma_f V_f + \gamma_g V_g$

Figure 4: Evolution over time of V_e and $\gamma_f V_f + \gamma_g V_g$.

initial conditions for their parameters. Finally, simulations have been carried out in which the two fuzzy systems have both states as premise variables (less prior knowledge on the system structure is used).

4.2.1 Separate Adaptation of $\hat{f}(x_2)$ and $\hat{g}(x_1)$

First, we adapt only $f(x_2)$, assuming that $g(x_1)$ is known. The learning rate is $\gamma_f = 100$. At the end of the learning, after $1800s$, we get a good approximation of $f(x_2)$ and the output tracks the reference with a very small tracking error. The parameters show some oscillations but their amplitude is quite small (Fig. 5). With no interacting fuzzy systems, the AFC works more efficiently and the learning time decreases.

When adapting only $g(x_1)$, assuming that $f(x_2)$ is known, one obtains similar results: the final approximation for the unknown function $g(x_1)$ is good and the convergence of the adapted parameters is good with only small oscillations.

4.2.2 Different Learning Rates

If we set $\gamma_f = 1$ (one hundredth of γ_g), the parameters get close to their optimal values and show only reasonable oscillations (Fig. 6). Also the final approximations are very good. Basically, what we have done is decreasing the level of interactions of

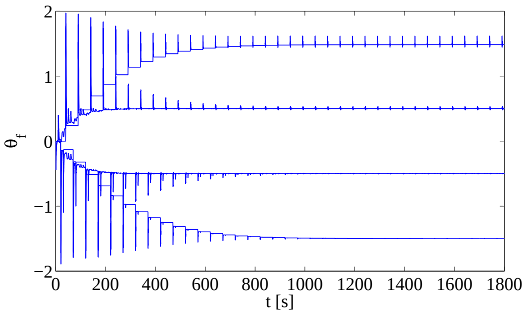
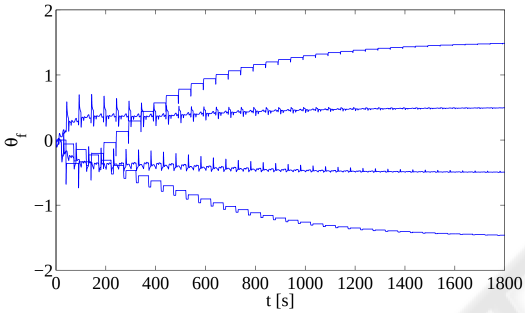
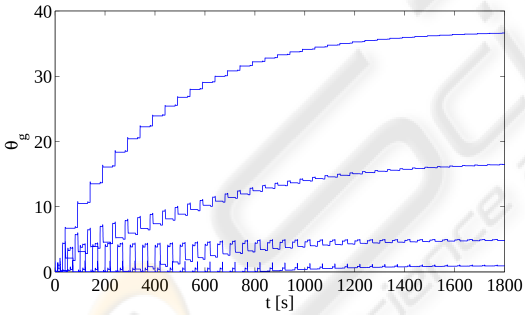


Figure 5: Singletons for $f(x_2)$ when $g(x_1)$ is known.

the fuzzy systems. The adaptive processes become *decoupled* and do not hamper each other.



(a) Final approximation for f



(b) Final approximation for g

Figure 6: Singletons for $f(x_2)$ and $g(x_1)$ with different learning rates ($\gamma_f = 1$ and $\gamma_g = 100$).

4.2.3 Initial Conditions

In all previous experiments, the singletons of $f(\boldsymbol{x})$ were initialized to 0 and the singletons of $g(\boldsymbol{x})$ to 0.1. The adaptive scheme should work whatever the initial conditions are. However, the initial values for the singletons of $\hat{g}(x_1)$ are quite far from the true values. Of course, this makes the learning more difficult. Further simulations indicate that if the initial values of the singletons of both the functions are close to their optimal positions, the learning works properly and the parameters converge (this is equivalent to say that some kind of prior knowledge is embedded). If the initial singletons of $\hat{g}(x_1)$ are far from the optimal values and the initial parameters for $\hat{f}(x_2)$ are exactly optimal, we have an undesirable phenomenon: the adaptation initially changes the parameters of $\hat{f}(x_2)$ and after a while it recovers the optimal parameters. The adaptation is making a kind of *bootstrapping*: in the attempt of learning the control law it changes the parameters of $\hat{f}(x_2)$ for compensating the error on $\hat{g}(x_1)$. Generally speaking, an important requirement for learning systems is the *monotonicity* of the learning process. We would like to have a kind of smooth adaptation that goes straight to the solution without wandering around it. In this particular case, the learning is evidently non-monotonous.

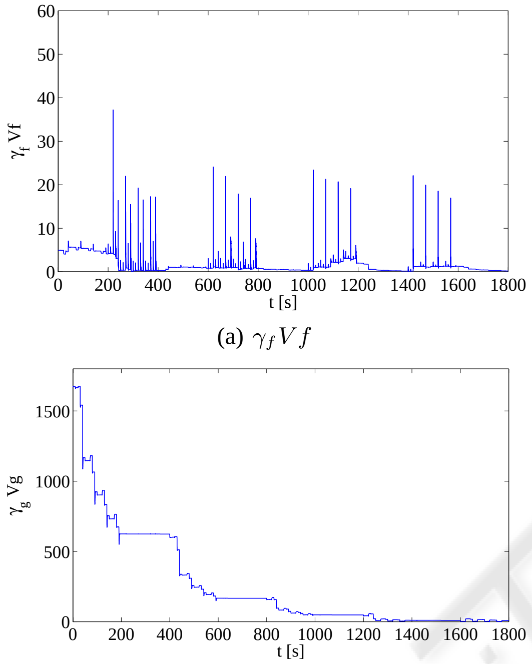
4.2.4 Two Premise Variables

If we assume that no prior knowledge is available on the functions $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$, we have to consider fuzzy interpolators with two premise variables (both state variables). In this case, the above mentioned issues (interactions of fuzzy systems, conflict between identification and control) are still present. However, it should be noted that in this case, there are more parameters involved and the learning task is more difficult. The best performing adaptive fuzzy systems (obtained with learning rates $\gamma_f = 10$ and $\gamma_g = 100$) are thus not able to approximate the unknown functions with the same degree of accuracy as we have fuzzy systems with only one premise variable.

5 SCHEDULING OF LEARNING RATES

The analytical developments and the simulation results with regard to the interactions of adaptive fuzzy systems suggest that it maybe useful to decouple the adaptation processes of the fuzzy systems. One way to accomplish this is to schedule the learning rates, thus allocating a time slot to the learning of $f(\boldsymbol{x})$ and another slot to the learning of $g(\boldsymbol{x})$. Several different values of the learning rates were used: $100/10$, $100/1$, $100/0$ and vice-versa. Moreover, also different scheduling times ($T = 200s, 100s, 50s, 10s$) were tested. The best setting found allocates $200s$ (equivalent to four periods of the reference signal) alternatively to the learning of one of the two functions and assigns the values 1 and 100 to the learning rates. Although the adaptation is not completely switched off for any of the two adaptive systems, the significantly different learning rates ensure a reduced interference between the adaptive systems.

One can see in Fig. 7 that the parameter errors quickly converge to zero. Also the tracking error converges to zero as it can be seen in Fig. 8, but in the first phase of the adaptation it is higher than in the previous cases. We can say that the price for better identification is a worse control performance in terms of tracking error, at least in the initial phase.



(a) $\gamma_f V f$

(b) $\gamma_g V g$

Figure 7: Evolution over time of the $\gamma_f V f$ and $\gamma_g V g$ with scheduling of the learning rates.

6 DISCUSSION

From the mathematical analysis and from the simulations results presented, one can conclude that in an indirect AFC scheme with adaptation laws derived through Lyapunov synthesis, the tracking control and the identification of the system dynamics are most of the time conflicting goals. There are strong interactions between the two fuzzy systems. As a consequence, the adaptation works through successive adjustments: the changes in one of the fuzzy system try to compensate the changes in the other. This interplay can hamper the learning process, which becomes non-monotonous. Some works related to this topic can also be found in the standard system-identification literature.

In the context of ‘closed-loop system identification’, and ‘identification for control’ (Landau, 1999; Hof and Schrama, 1995) it is stated that with standard identification and control design methods it is not possible to simultaneously optimize both the system model and the controller. Hence, in identification for control, system identification and control design are not simultaneous; instead they are temporally separated. System dynamics are identified in closed-loop in the presence of a fixed controller and then, based on the identified model, a new controller is designed. These steps can be repeated. The analysis carried out in this paper suggests that the separation of identification and control may be beneficial in the context of AFC. In (Hojati and Gazor, 2002), it is proven that adaptive laws driven by two sources of information, namely tracking error and prediction error (defined with regard to a *series-parallel identification model*) outperform adaptive laws based only on tracking error.

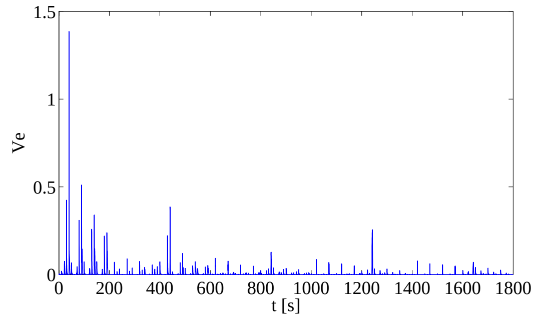


Figure 8: Evolution of Ve.

In this paper, the learning rates are abruptly switched. With regard to stability, it may be better to have a smooth transition of the learning rates. Moreover, if the plant parameters are unknown but fixed, the learning rate should decrease over time (only plants with time-varying parameters require constant learning rates). In the literature on reinforcement learning and neural networks, some heuristics for the choice of learning rates are provided (Sutton and Barto, 1998; Jacobs, 1988).

7 CONCLUSIONS

The main contribution of this paper is the analysis of interactions between parameter updates of the two fuzzy systems constituting an indirect adaptive fuzzy controller based on feedback linearization. First, it has been shown analytically what are the relationships among the time-derivative of the norm of the tracking error and of the parameter errors with regard to the unknown system dynamics. The analysis highlights the existence of conflicting goals: the identification of the unknown system dynamics and tracking control cannot be simultaneously optimized. Then by means of simulations with a second order system, under different scenarios, the mathematical developments are validated. The links with the related literature have been explored and finally some possible improvements were suggested. In particular, we propose the scheduling of the learning rates as possible means to overcome some parameter convergence problems, simultaneously achieving the control goal while performing a proper identification of fuzzy models, which are fully transparent and amenable to off-line interpretation.

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