# DESIGN OF LOW DELAY BANDPASS FIR FILTERS WITH MAXIMALLY FLAT CHARACTERISTICS IN THE PASSBAND AND THE TRANSMISSION ZEROS IN THE STOPBAND

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Abstract: The large group delay of the high order FIR filters is unacceptable in some applications. Therefore, recently, how to reduce the group delay of FIR filters has been studied intensively. To reduce the ringing in the time domain and to maximize the stopband attenuation, it is useful to design FIR filters with maximally flat characteristics in the passband and transmission zeros in the stopband. We present a mathematically closed form transfer function of low delay bandpass FIR filters with maximally flat amplitude in the passband and the transmission zeros in the stopband. Because of the mathematically closed form transfer function, the designing filters are very simple. Moreover, we propose a design method of low delay bandpass FIR filters with maximally flat amplitude in the passband and equiripple in the stopband by using an iterative method of a closed form transfer function and Remez algorithm.

## 1 INTRODUCTION

FIR digital filters realized nonrecursively can always be stable. FIR digital filters with exactly linear phase characteristics can be easily designed and are important for applications such as waveform transmission and image processing. In (McClellan et al., 1973), an excellent program to design the transfer function of FIR filters with equiripple characteristics in both the passband and the stopband haa been presented. These filters have the disadvantages of having echoes in the impulse response and ringing in the step response due to their sharp cutoff frequency responses. The amplitude of these echoes is proportional to the amplitude of the passband ripples. Consequently, FIR filters with maximally flat characteristics in the passband are required (Herrmann, 1971). However, the roll off property of these filters is not steep in the frequency domain. Accordingly, to reduce echoes and ringing and to maximize the stopband attenuation, it is important to design FIR filters with maximally flat characteristics in the passband and transmission zeros in the stopband. The design method of such FIR filters by Remez algorithm has been proposed (Selesnick and Burrus, 1996; Aikawa and Sato, 2000). However, since the delay of linear-phase filters is half of filter length, the delay of linear-phase filters will become large when high-order filters are required.

Recently, how to reduce the delay of FIR filters have been studied intensively (Fukae et al., 1997; Karm and McClellan, 1995; Selesnick and Burrus, 1998; Samadi et al., 2000). In (Samadi et al., 2000; Ogata et al., 2000), the design method of the transfer function of low delay FIR lowpass filters with flat characteristics in both the passband and the stopband has been presented. In (Samadi et al., 2000), the transfer function is given in a mathematically closed form. However, the roll off property of the filter is not steep in the frequency domain. In (Ogata et al., 2000), the design method of low delay FIR filters with maximally flat characteristics in the passband and equiripple characteristics in the stopband by using successive projections method has been proposed. This filter can be reduced echoes and ringing, and maximized the stopband attenuation. However, the mathematically closed form transfer function of low delay bandpass FIR filters with maximally flat characteristics in the passband and the transmission zeros in the stopband is not proposed.

We propose a mathematically closed form transfer function of low delay bandpass maximally flat FIR filters with prescribed transmission zeros in the stopband. This method can be easily realized the transfer function of the filter with arbitrary center frequency


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due to its closed form regardless of case of filter. Moreover, we propose a design method of low delay bandpass FIR filters with maximally flat amplitude in the passband and equiripple in the stopband by using an iterative method of a closed form transfer function and Remez algorithm. Finally, the usefulness of the proposed method is verified through the examples.

## 2 A CLOSED FORM TRANSFER FUNCTION OF LOW FELAY FIR FILTERS

In generally, a frequency response of FIR filters with $N$ order becomes

$$H(e^{j\omega}) = \sum_{n=0}^{N} h(n)e^{-jn\omega} = A(\omega)e^{j\theta(\omega)} \quad (1)$$

where $A(\omega)$ is amplitude response and $\theta(\omega)$ is phase response. It is necessary to satisfy the following conditions so that the frequency response in (1) has the flat response in the passband, the transmission zeros in the stopband, and the group delay response $\tau$ at $\omega = \pm\omega_0$.

$$A(\omega)|_{\omega=\pm\omega_0} = 1 \quad (2a)$$

$$\left.\frac{d^m A(\omega)}{d\omega^m}\right|_{\omega=\pm\omega_0} = 0,\ m = 1, 2, \cdots, M \quad (2b)$$

$$A(\omega_l) = 0,\ l = 1, 2, \cdots, L_1 + L_2 \quad (2c)$$

$$G(\omega)|_{\omega=\pm\omega_0} = -\left.\frac{d\theta(\omega)}{d\omega}\right|_{\omega=\pm\omega_0} = \tau \quad (2d)$$

$$\left.\frac{d^m G(\omega)}{d\omega^m}\right|_{\omega=\pm\omega_0} = 0,\ m = 1, 2, \cdots, D \quad (2e)$$

where $M$ and $D$ are parameters to decide the degree of flatness of amplitude response and group delay response at $\omega = \pm\omega_0$ , respectively. Moreover, $L_1$ and $L_2$ are number of the transmission zeros in the low stopband and in the high stopband, respectively. One of a closed form transfer function which satisfies conditions (2) is obtained as

$$H(e^{j\omega}) = P(e^{j\omega}) - R(e^{j\omega})S(e^{j\omega}). \quad (3)$$

We decide functions $P(e^{j\omega})$, $R(e^{j\omega})$ and $S(e^{j\omega})$ according to the following methods.

Consider $P(e^{j\omega})$ with $K$ degrees of flatness

$$P(z) = \sum_{i=0}^{2K} p_i z^{-i} = e^{\alpha(\omega)+j\beta(\omega)} \quad (4)$$

where $z = e^{j\omega}$. Thus, we can be written as

$$\phi(z) = z\frac{P'(z)}{P(z)} = \tau_d(\omega) + j\nu(\omega) \quad (5)$$

where $\tau_d(\omega)$ and $\nu(\omega)$ are differentiation the amplitude and the group delay of $P(z)$, respectively. Then, owing to simultaneous the amplitude and the group delay having maximally flat characteristics at $\omega = \pm\omega_0$, it should be

$$\phi(z) = \tau_P - \frac{B_K\left(z^2 - 2x_0z + 1\right)^K}{P(z)} \quad (6)$$

where $\tau_p$ is the delay of $P(e^{j\omega})$ and $x_0 = \cos\omega_0$. From (5) and (6), we obtain

$$-zP'(z) + \tau_P P(z) = B_K\left(z^2 - 2x_0z + 1\right)^K. \quad (7)$$

By differentiating (7) with respect to $z$ and eliminating $B_k$, we obtain

$$\begin{aligned}&P''(z)\left\{z\ \left(z^2 - 2x_0z + 1\right)\ \right\}\\&+P'(z)\left\{(1-\tau_P)\left(z^2 - 2x_0z + 1\right) - 2Kz(z - x_0)\right\}.\\&+P(z)\left\{\ 2K\tau_P(z - x_0)\right\} = 0\end{aligned} \quad (8)$$

From (8), the coefficient $p_i$ of $P(z)$ can be obtained as eigen value problem shown by following equation.

$$\mathbf{Ap} + \lambda\mathbf{p} = \mathbf{0} \quad (9)$$

where $\mathbf{p}$ is the column vector of coefficient $p_i$ and $\mathbf{A}$ is $(2K + 1) \times (2K + 1)$ square matrix given by

$$\left.\begin{aligned}&a_{i,i-1} = (\tau_P - i + 1)(2K - i + 1)\\&a_{i,i} = -x_0\left\{(\tau_P - i)(2K - i) + (-\tau_P + i)i\right\}\\&a_{i,i+1} = (-\tau_P + i + 1)(i + 1)\\&a_{i,j} = 0 \quad ; j \neq i - 1, i, i + 1\\&\qquad\quad ; i = 0, 1, \cdots, 2K\end{aligned}\right\}. \quad (10)$$

Moreover, $S_{L-1}(z)$ in (3) is Lagrange interpolation polynomial shown by the following equation.

$$S_{L-1}(z) = \sum_{i=1}^{L_1+L_2} F_i \prod_{j=1 j\neq i}^{L_1+L_2} \frac{z^{-1} - z_j^{-1}}{z_i^{-1} - z_j^{-1}} \quad (11)$$

where

$$F_i = \frac{P(e^{j\omega_i})}{R(e^{j\omega_i})} \quad (12)$$

$R(e^{j\omega})$ in (3) is

$$R(e^{j\omega}) = \left\{(e^{j\omega} - e^{-j\omega_0})(e^{j\omega} - e^{j\omega_0})\right\}^{K+\frac{1}{2}}. \quad (13)$$

In (11), $z_k = e^{j\omega_k}$ is the transmission zero in the stopband. Thus, there are typical zero positions for each of the four cases to obtain FIR filter with real coefficient. To obtain filter of case 1, we need to select complex conjugate pairs for all $z_k$. In the case 2, we need to select a $z_k = -1$ and remainder zeros are complex conjugate pairs. Similarly, in the case 3 and type 4, we need to select $z_k = 1$ and remainder zeros are complex conjugate pairs and $z_k = -1$ , $z_k = 1$

and remainder zeros are complex conjugate pairs, respectively. Thus, the relationship $N$ , $M$, $L_1$ and $L_2$ is given by

$$N = M + L_1 + L_2. \tag{14}$$

In (3), the flatness parameter of the amplitude response, $M$ , and group delay response, $D$, become

$$M = \begin{cases} K & K : even \\ K+1 & K : odd \end{cases} \tag{15a}$$

and

$$D = \begin{cases} K & K : even \\ K-1 & K : odd \end{cases} \tag{15b}$$

In addition, group delay at $\omega = \pm\omega_0$ is given by

$$\tau = 2K - \tau_P. \tag{16}$$

It is clear from (3)-(13) that we will easily realize the transfer function by deciding flatness in the passband and transmission zeros in the stopband because of its closed form function regardless of the four cases of filter.

## 3 DESIGN METHOD OF LOWDELAY FIR FILTER WITH EQUIRIPPLE STOPBAND

In this section, we present a design method of low delay maximally flat FIR filter with equiripple characteristics in the stopband by using Remez algorithm for the closed form transfer function in the section 2.

Using $V(e^{j\omega})$ composed of zeros, $z_k = e^{j\omega_k}$ $(k = 1, 2, \cdots, L_1 + L_2)$, in the stopband and $U(e^{j\omega})$, composed of other zeros, the transfer function in (3) can be rewritten as

$$H(e^{j\omega}) = V(e^{j\omega}) \cdot U(e^{j\omega}) \tag{17}$$

where

$$V(e^{j\omega}) = \prod_{l=1}^{L_1+L_2} \left(e^{j\omega} - e^{j\omega_l}\right). \tag{18}$$

A zero phase transfer function doesn't exist because $H(e^{j\omega})$ in (18) is nonlinear phase characteristics. Then, to apply the Remez algorithm, we define the error function as

$$E(e^{j\omega}) = W(e^{j\omega}) \left\{ D(e^{j\omega}) - \bar{H}(e^{j\omega}) \right\} \tag{19}$$

where $W(\omega)$ and $D(\omega)$ are a weight function and an ideal function, respectively. Moreover, $\bar{H}(e^{j\omega}$ in 19 is given by

$$\bar{H}(e^{j\omega}) = H(e^{j\omega}) \cdot H^*(e^{j\omega}) \tag{20}$$

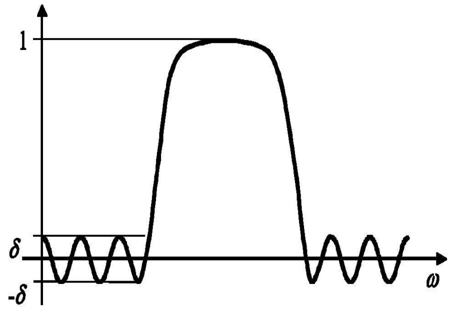


Figure 1: Amplitude response of $\left|U(e^{j\omega})\right|^2 \tilde{V}(e^{j\omega})$

where $H^*(e^{j\omega})$ is a complex conjugate transfer function of $H(e^{j\omega})$. Substituting (17), (18) and (20) into (19) yields

$$E(e^{j\omega}) = \hat{W}(e^{j\omega}) \left\{ \hat{D}(e^{j\omega}) - \left|V(e^{j\omega})\right|^2 \right\} \tag{21}$$

where $\hat{W}(e^{j\omega})$ and $\hat{D}(e^{j\omega})$ are

$$\hat{W}(e^{j\omega}) = W(e^{j\omega}) \cdot \left|U(e^{j\omega})\right|^2 \tag{22}$$

and

$$\hat{D}(e^{j\omega}) = \frac{D(e^{j\omega})}{\left|U(e^{j\omega})\right|^2}, \tag{23}$$

respectively.

However, (21) cannot be solved directly by Remez algorithm. Thus, we define a new error function using $\tilde{V}(e^{j\omega})$ as

$$E(e^{j\omega}) = \hat{W}(e^{j\omega}) \left\{ \hat{D}(e^{j\omega}) - \tilde{V}(e^{j\omega}) \right\}. \tag{24}$$

We solve (24) by Remez algorithm. Using $\tilde{V}(e^{j\omega})$, the amplitude response of $\left|U(e^{j\omega})\right|^2 \tilde{V}(e^{j\omega})$ is shown in Fig. 1. In Fig. 1, let

$$\tilde{A}(e^{j\omega}) = \frac{1}{1+\delta} \left\{ \left|U(e^{j\omega})\right|^2 \tilde{V}(e^{j\omega}) + \delta \right\}, \tag{25}$$

where $\delta$ is ripple. Then, because of $\tilde{A}(e^{j\omega}) \geqslant 0$, $\tilde{A}(e^{j\omega})$ can be rewritten as

$$\tilde{A}(e^{j\omega}) = \left|U(e^{j\omega})\right|^2 \left|V_M(e^{j\omega})\right|^2. \tag{26}$$

The zeros of $V_M(z)$ correspond to the zeros in the stopband when $H(e^{j\omega})$ in (17) has equiripple characteristics in the stopband. It is clear from (25) and (26) that those roots are obtained from the frequency of the minimum value of $\left|U(e^{j\omega})\right|^2 \tilde{V}(e^{j\omega})$.Therefore, factoring is not needed in the proposed algorithm. However, the zeros of $V_M(z)$ do not correspond to the zeros $U(e^{j\omega})$ in (17). Thus, we assume $v_k$ $(k = 1, 2, \cdots, L_1 + L_2)$ which are roots of $V_M(e^{j\omega})$ to be new transmission zeros and calculate (3) again. Namely, in calculation, the filter with a complete equiripple characteristic is not obtained once. Therefore, the calculation of the above-mentioned is repeated until the transmission zeros do not vary.

Table 1: The specifications of varying $\omega_0$

| $w_0$ | $0.15\pi$ | $0.40\pi$ | $0.65\pi$ |
|---|---|---|---|
| $K$ | 4 | | |
| $\tau$ | 18 | | |
| $\omega_s$ | $0.05\pi, 0.25\pi$ | $0.30\pi, 0.50\pi$ | $0.55\pi, 0.75\pi$ |
| $L_1$ | 2 | 12 | 20 |
| $L_2$ | 28 | 18 | 10 |
| $N$ | 38 | | |

# 4 DESIGN EXAMPLES

In this section, the usefulness of the proposed method is verified through the examples.

We shall design all cases of bandpass maximally flat low delay FIR filter with the prescribed transmission zeros in the stopband as following specifications. Here, The transmission zeros in the stopband are arranged at equal intervals.

[Filter of case 1]
$N=40, K=10, \omega_0=0.6\pi, \tau=20, 18, 16, 14$
$\omega_s=0.3\pi, 0.9\pi$[rad/s]$, L_1=14,\ L_2=6$

[Filter of case 2]
$N=39, K=10, \omega_0=0.6\pi, \tau=19.5, 17.5, 15.5, 13.5$
$\omega_s=0.3pi, 0.9\pi$[rad/s]$,\ L_1=14,\ L_2=5$

[Filter of case 3]
$N=39, K=10, \omega_0=0.6\pi, \tau=19.5, 17.5, 15.5, 13.5$
$\omega_s=0.3\pi,\ 0.9\pi$ [rad/s]$,\ L_1=13,\ L_2=6$

[Filter of case 4]
$N=38, K=10, \omega_0=0.6\pi, \tau=19, 17, 15, 13$
$\omega_s=0.3\pi, 0.9\pi$ [rad/s]$,\ L_1=13,\ L_2=5$

The amplitude responses and the group delay responses of the obtained filter are shown in Fig. 5 from Fig. 2. Notice from figures (a) that the obtained filters have the prescribed transmission zeros in the stopband. Likewise, note from figures (b) that the obtained filters have the prescribed group delay in the passband. Moreover, when $\tau = 20$ in case 1, the resulting filter has a linear phase characteristics because the group delay characteristic is flat response in all frequency as shown in Fig. 2. The same is true for case 2, case 3 and case 4.

Next, we shall design the bandpass filters of case 1 with the prescribed transmission zeros in the stopband for some center frequency as specifications in the table 1. The amplitude response and the delay response of the obtained filters are shown by (a) and (b) in Fig. 6, respectively. It is clear from (a) in Fig. 6 that bandpass filter with the prescribed transmission zeros in the stopband for the arbitrary center frequency can be designed in the proposed method. In addition, the obtained filter also has the prescribed group delay in the passband.

Finally, we shall design a bandpass maximally flat

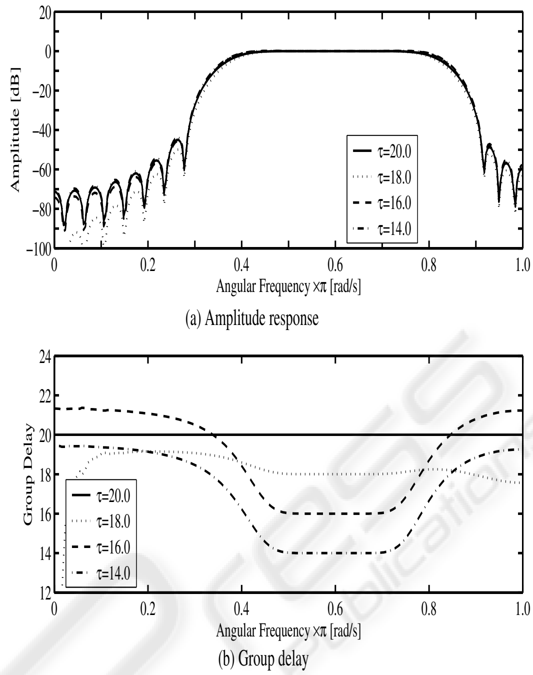


Figure 2: The amplitude response and the group delay of filter of case 1

FIR filter of case 1 with equiripple stopband as following specifications.

[Specifications]
$N = 40, K = 8, \omega_0 = 0.6\pi, \tau = 14$
$\omega_s = 0.3\pi, 0.9\pi$[rad/s]$,\ L_1 = 14, L_2 = 6$

This specification is the same as case 1 of the first example. Therefore, we assume the filter of case 1 that has $\tau = 14$ to be a initial value. That is, we design a low delay bandpass maximally flat FIR filters with prescribed transmission zeros in the stopband by using 3 at first. The obtained amplitude response and group delay response are shown by dashed line in Fig. 2. Next, we design a filter with equiripple characteristics in the stopband by the proposed algorithm shown in chapter 3. Here, the convergence condition of the proposed algorithm is that the difference of angle between $z_k$ and $v_k$ is smaller than $10^{-3}$. In this example, the number of iteration is three. Therefore, convergence of the proposed algorithm is very fast. Moreover, because the transfer function of filter with the prescribed transmission zeros in the stopbands can be realized the closed from and the transfer functiuon of filter with equiripple characteristics is obtained by Remez algorithm, its designing filter is also very simple. The amplitude response and the group delay response of the filter obtained are shown in Fig. 7 (a) and (b), respectively. It is clear from Fig. 7 (a) and (b) to obtain filter with the equiripple characteristics in the stopband and the prescribed group delayin the

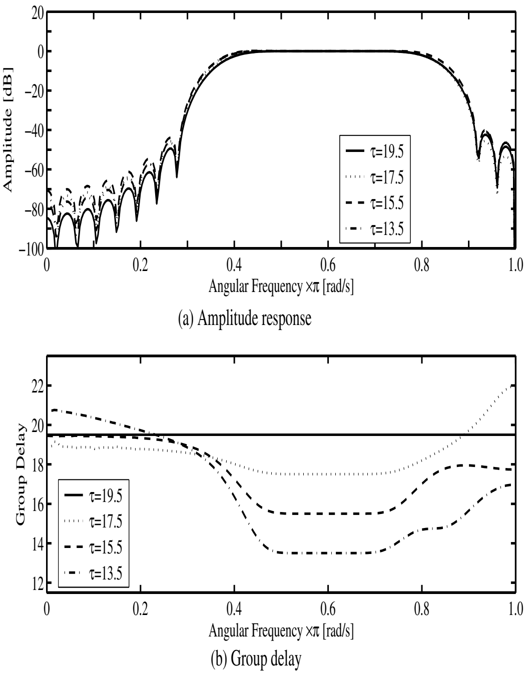


Figure 3: The amplitude response and the group delay of filter of case 2

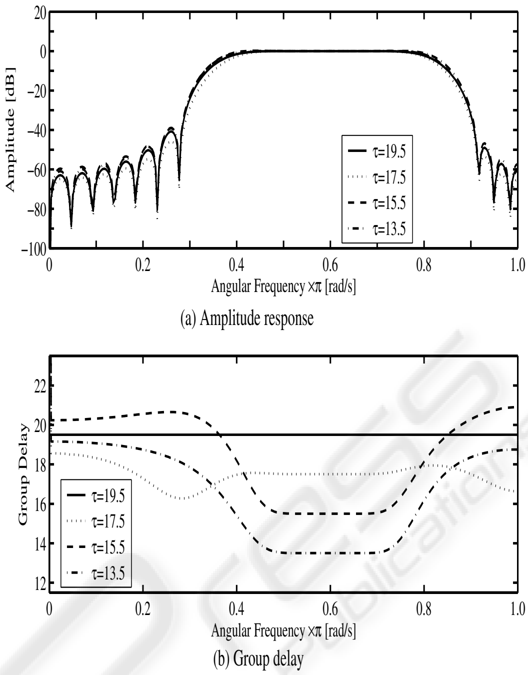


Figure 4: The amplitude response and the group delay of filter of case 3

passband. Note that the filter obtained by this design method differs from the filter described by chapter 2, and can decide the stopband edge frequency, because the Remez algorithm can decide the stopband edge frequency.

## 5 CONCLUSION

We proposed a mathematically closed form transfer function of low delay bandpass maximally flat FIR filters with prescribed transmission zeros in the stopband. This method can be easily realized the transfer function due to its closed form regardless of case of filter. Moreover, the proposed filter has arbitrary center frequency regardless of the even order or the odd order. Next, we proposed a design method of low delay bandpass FIR filter with maximally flat amplitude in the passband and equiripple characteristics in the stopband. This method is used an iterative method of a closed form transfer function and Remez algorithm. Therefore, the designing filter is also very simple. Moreover, the proposed algorithm converges very quickly. Finally, the usefulness of the proposed method was verified through the examples.

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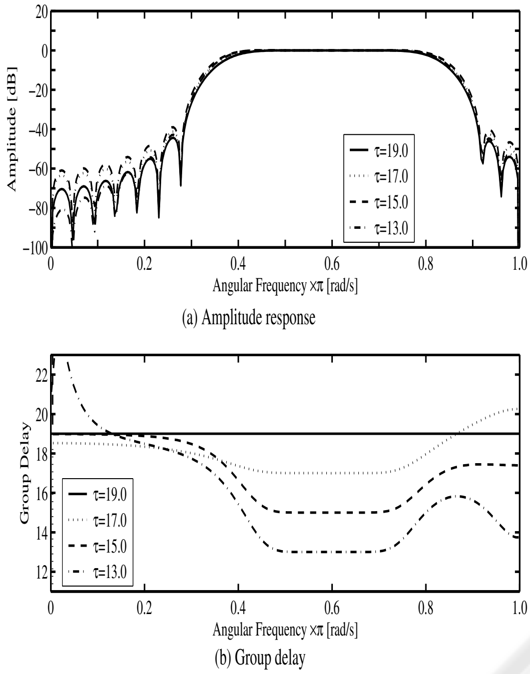


Figure 5: The amplitude response and the group delay of filter of case 4

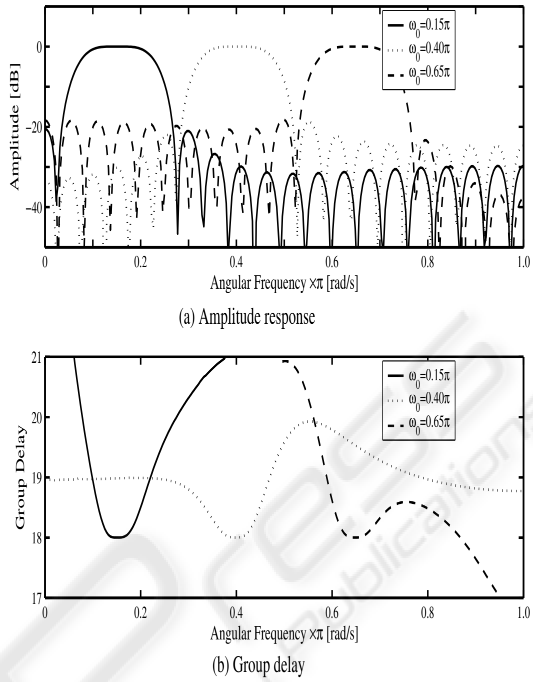


Figure 6: The amplitude response and the group delay response of the bandpass filters for some center frequencies

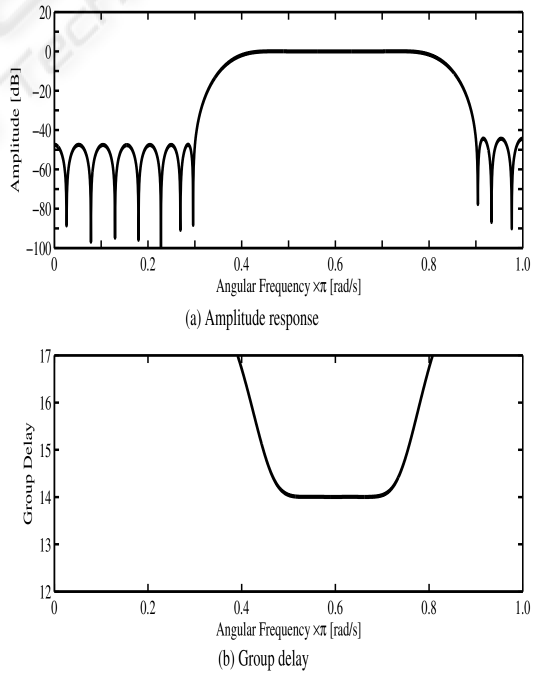


Figure 7: The amplitude response and the group delay response of the bandpass filter with equiripple characteristics in the stopband