MULTIRATE OUTPUT FEEDBACK BASED DISCRETE-TIME SLIDING MODE CONTROL FOR A CLASS OF NONLINEAR SYSTEMS

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Abstract: The property of certain nonlinear continuous-time systems to be exactly representable in discrete-time is known as finite discretizability. This paper presents a method for the discrete-time sliding mode control for nonlinear systems that are finitely discretizable.

1 INTRODUCTION

The concept of sliding mode control was first introduced by Emelyanov [Emelyanov, 1967] and Utkin [Utkin, 1977]. It is a technique that achieves desired characteristics for the system by confining its states to a specified subset of the state space. This is done by application of a control of variable structure. The main advantage of sliding mode control is its insensitivity to system parameter variations [Hung et al., 1993, Young et al., 1999]. In the recent years, considerable efforts have been put in the study of the concepts of Digital Sliding Mode (DSM) controller design [Furuta, 1990, Gao et al., 1995, Sarpturk et al., 1978]. In case of the DSM design, the control input is applicable only at certain sampling instants and the control effort is constant over the entire sampling period. Moreover, when the states reach the switching surface, the subsequent control would be unable to keep the states confined to the surface. As a result, DSM can undergo only quasi-sliding mode, i.e., the system states would approach the sliding surface but would generally be unable to stay on it. Thus, in general, DSM does not possess the invariance property found in continuous-time sliding mode. In [Gao et al., 1995] a "reaching law" approach for the design of control for DSM using state feedback was introduced. This reaching law ensures that the system trajectory will hit the switching manifold and thereafter undergo a zigzag motion about the switching manifold. The magnitude of each successive zigzagging step decreases so that the trajectory stays within a specified band called the quasi-sliding-mode band.

However, most of the sliding mode control strategies are based on full-state feedback. But, in practice, all the states of the system may not be available for measurement. Since the output is available for measurement, output feedback can be used for the controller design. Few research works are available which deal with SMC design using output feedback [Bag et al., 1997, Diong, 1993, Zak and Hui, 1993]. An output feedback technique that guarantees the closed loop stability for controllable and observable systems has been proposed in [Werner and Furuta, 1995]. This method is termed as " Fast Output Sampling" technique in which the system output is sampled at a rate that is N times faster than the rate at which the control input is given. A fast output sampling feedback based discrete-time sliding mode control strategy for linear systems has been developed

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in [Saaj et al., 2002].

This paper presents a method for the multirate output feedback based discrete-time sliding mode control of a class of nonlinear systems by using the concept of finite discretizability [Chelouah and Petitot, 1995].

2 FINITELY DISCRETIZABLE SYSTEMS

2.1 Definition

Let $x = (x_1, \cdots, x_n)$ be the local coordinates for an open neighborhood of q, defined as $U_q \subset M$. where M is a real analytical connected n-dimensional manifold. Consider the locally-controllable and observable nonlinear system of the form

$$\begin{aligned} \Sigma : \dot{x}(t) &= \sum_{i=1}^{m} u_i(t) X_i\left(x(t)\right) \qquad (1) \\ y(t) &= g\left(x(t)\right) \end{aligned}$$

where $X_1, \cdots, X_m$ are real analytical vector fields on U_q, $u = (u_1, \cdots, u_m) \in \mathbb{R}^m$ and $g : \mathbb{R}^n \to \mathbb{R}^p$ is a polynomial function of x.

The solution of (1) corresponding to a constant control $u(t) = \bar{u}$, for $t \geq 0$, is denoted $\left(\exp tY\right)(I_d)|_{x(0)}$, where $Y = \sum_{i=1}^{m} \bar{u}_i X_i$ and I_d is the identity function.

The nonlinear system Σ is said to be finitely discretizable [Chelouah and Petitot, 1995] at the order $\nu \geq 1$ if the solution of (1), corresponding to a constant control $u(t) = \bar{u}$ for $t \geq 0$ is a polynomial of degree $\nu - 1$ in t, $\forall t \geq 0, \forall \bar{u} \in \mathbb{R}^m$ and $x(t_0) \in U_q$ *i.e.,*

$$\begin{aligned} x(t + t_0) &= \left(\exp tY\right)(I_d)|_{x(t_0)} \\ &= (I_d)|_{x(t_0)} + tY\,(I_d)|_{x(t_0)} \qquad (2) \\ &\quad + \cdots + \frac{t^{\nu-1}}{(\nu-1)!} Y^{(\nu-1)}\,(I_d)|_{x(t_0)} \end{aligned}$$

$$\forall t \geq 0, \forall \bar{u} \in \mathbb{R}^m, \forall x(0) \in U_q$$

In other words, one has

$$Y^{\nu+\mu}\,(I_d)|_{x(t_0)} = 0, \forall \mu > 0 \qquad (3)$$

Thus, if the system is discretized at a sampling interval of τ sec, the discrete-time representation would be

$$\begin{aligned} x((k+1)\tau) &= \left(\exp tY\right)(I_d)|_{x(k\tau)} \\ &= (I_d)|_{x(k\tau)} + tY\,(I_d)|_{x(k\tau)} \qquad (4) \\ &\quad + \cdots + \frac{t^{\nu-1}}{(\nu-1)!} Y^{(\nu-1)}\,(I_d)|_{x(k\tau)} \end{aligned}$$

Remark 1 : *The system is considered to be driftless only for the convenience of notation. All the definitions can be extended to drift systems by setting, say,* $u_{m+1} \equiv 1$ □

2.2 Sufficient Condition of Finite Discretization

We will use the capital letter $I = (i_1, \cdots, i_m)$ to denote multi-indices with $i_\mu \in \mathbb{N}, \mu \leq m$. We also define

$$\begin{aligned} |I| &= i_1 + \cdots + i_m \\ X^I &= X_1^{i_1} \cdots X_m^{i_m} \\ X^{ш I} &= X_1^{i_1} ш \cdots ш X_m^{i_m} \end{aligned}$$

where $XY = \frac{\partial Y}{\partial x} X$, $\frac{\partial Y}{\partial x}$ representing the Jacobian matrix [Khalil, 2002] and " ш" denotes the shuffle product inductively defined on the length as follows

$$\begin{aligned} X ш I_d &= I_d ш X = X \\ X^i ш Y^j &= X\left(X^{i-1} ш Y^j\right) + Y\left(X^i ш Y^{j-1}\right). \end{aligned}$$

The shuffle product is associative and commutative.

Definition 1 : Consider $\mathbb{R}^n$ with the coordinates $x = (x_1, \cdots, x_n)$. A *dilation* is a map $\delta_t : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ is of the form $\delta_t(h(x)) = h\left(t^{r_1} x_1, \cdots, t^{r_n} x_n\right)$, where $h : \mathbb{R}^n \to \mathbb{R}^n$ is a polynomial in x and we assume that $r_i \in \mathbb{N}, i \leq n, r_i \leq r_{i+1}$. □

Definition 2 : A *polynomial* $h : \mathbb{R}^n \to \mathbb{R}^n$ is *homogeneous of degree* $j \in \mathbb{Z}$ with respect to a dilation δ_t if $\delta_t^* = h \circ \delta_t = t^j h$. Let H be the algebra of real polynomial functions in $(x_1, \cdots, x_n)$, we define $H_j = \{h \in H, \delta_t^* h = t^j h\}$ and set $H_j = \{0\}, \forall j < 0$, then $H = \oplus_{j \geq 0} H_j$. We denote by P_j the set of all polynomials homogeneous of degree $\leq j$ *i.e.,* $P_j = \oplus_{l=0}^{j} H_l$ □

Definition 3 : A *polynomial vector field* X is said to be *homogeneous of degree* $-s \in \mathbb{Z}$ with respect to a dilation δ_t if

$$\left((\delta_t)^* X\right)(h) = t^s \delta_t^*\left(X(h)\right), h \in H$$

or equivalently $X(h) \in H_{j-s}$ if $h \in H_j$. □

Theorem 1 : Let $X_1, \cdots, X_m$ be real analytical vector fields, with polynomial coefficients, homogeneous of degree -1 with respect to the dilation $\delta_t(x) = (x_1^{r_1}, \cdots, x_n^{r_n})$, then Σ is finitely discretizable at most of the order $r_n + 1$. □

The proof of the theorem is presented in [Chelouah and Petitot, 1995]

3 MULTIRATE OUTPUT SAMPLING

Consider the nonlinear system (1). Let Σ is controllable,observable, and finitely discretizable. Let the system input is given with a sampling interval of τ sec and the outputs y_i are sampled at intervals $\Delta_i = \frac{\tau}{/}N_i, N_i \in \mathbb{N}, i = 1, \cdots, p$. It can then be shown that the system states can be expressed as a function of past N_i samples of outputs y_i and immediate past control input.

Proof: Since $y = g(x)$ is a polynomial function in x, using the result that the finite discretization property is preserved under polynomial transformation [Chelouah and Petitot, 1995], it can be said that y would also be finitely discretizable. $y_i(t+\tau), i \in \mathbb{N}, i \leq p$ would therefore be of the form

$$\begin{aligned} y_i(t_0+\tau) &= y_i(t_0) + \tau y_i^{(1)}(t_0) + \cdots \\ &\quad + \frac{\tau^{N_i-1}}{(N_i-1)!} y_i^{(N_i-1)}(t_0) \end{aligned} \tag{5}$$

for some $N_i \in \mathbb{N}, \{\tau, t\} \in \mathbb{R}^+, u(t) = \bar{u}, \forall t \in [t_0, t_0+\tau)$. Due to the assumption that the system Σ is observable, the system states can be expressed as

$$\begin{aligned} x(t_0) &= f\left(y, \dot{y}, y^{(2)}, \cdots, y^{(N_{\max})-1}, \bar{u}\right) \\ N_{\max} &= \sup_{i=1,\cdots,p} N_i \end{aligned} \tag{6}$$

where N_i is the highest order derivative of y_i appearing in the nonlinear continuous-time observer. Now if the system input u is applied and held constant for every τ sec interval and each of the system outputs y_i is sampled at a rate $\Delta_i = \frac{\tau}{N_i}$, then using (5), and using $y_i^{(k)}$ to denote $y_i^{(k)}(k\tau)$

$$\begin{aligned} y_i(k\tau) &= y_i^{(0)} \\ y_i(k\tau + \Delta_i) &= y_i^{(0)} + \Delta_i y_i^{(1)} \\ &\quad + \cdots + \frac{\Delta_i^{N_i-1}}{(N_i-1)!} y_i^{(N_i-1)} \\ &\vdots \\ y_i((k+1)\tau - \Delta_i) &= y_i^{(0)} + \cdots \\ &\quad + \frac{((N_i-1)\Delta_i)^{N_i-1}}{(N_i-1)!} y_i^{(N_i-1)} \end{aligned} \tag{7}$$

The left hand side of the above set of N_i equations constitute the multirate output samples for y_i. The equations are independent in the N_i variables $y_i, y_i^{(1)}, \cdots, y_i^{(N_i-1)}$ and hence the output derivatives can be obtained by solving (7).

$$\begin{bmatrix} y_i^{(0)} \\ y_i^{(1)} \\ \vdots \\ y_i^{(N_i-1)} \end{bmatrix} = A_i^{-1} \begin{bmatrix} y_i(k\tau) \\ y_i(k\tau+\Delta_i) \\ \vdots \\ y_i((k+1)\tau - \Delta_i) \end{bmatrix} \tag{8}$$

$$A_i = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & \Delta_i & \cdots & \Delta_i^{N_i-1} \\ \vdots & & \ddots & \vdots \\ 1 & (N_i-1)\Delta_i & \cdots & ((N_i-1)\Delta_i)^{(N_i-1)} \end{bmatrix} \tag{9}$$

This along with the observability condition in (6) and the discrete state equation (4) would mean that one can derive the system state information at $t = (k+1)\tau$ by measuring the outputs y_i for the period $t \in [k\tau, (k+1)\tau)$ with a sampling interval Δ_i respectively and holding the input $u = \bar{u}$ constant during the same period. Hence, the observability of the continuous time system along with the finite discretization property ensures that a multirate output sampling interval of $\frac{\tau}{N_i}$ for each output y_i for the input sampling interval of τ is *a sufficient condition* for the discrete-time observability of the nonlinear system.

4 DISCRETE-TIME SLIDING MODE CONTROL

4.1 State based Control

The application of the sliding mode control strategy to nonlinear system representations has received considerable attention in the recent years [Khan and Spurgeon, 2001,Munoz and Sbarbaro, 2000,Sira-Ramirez et al., 1997,Zhou et al., 2001].

Using a strategy similar to that discussed in [Gao et al., 1995], we first design stable sliding surfaces $s_i(t) = 0, i = 1, \cdots, m$ by finding the relationship between the states so that a chosen candidate Lyapunov function $V(x)$ has $\dot{V}(x) < 0$. Since, the system stability is conserved on discretization, the same sliding surfaces $s_i(k\tau) = s_i(k) = 0$ would also be stable for the discrete-time system representation. For the reminder of the paper, the notation $x(k)$ is used instead of $x(k\tau)$ for brevity. Now applying the reaching condition

$$s_i(k+1) - s_i(k) = -q_i \tau s_i(k) - \epsilon\tau \mathrm{sgn}(s_i(k))$$

ans substituting the value of $x(k+1)$ from the discrete system representation (4), one can solve for $u_i(k)$ and obtain the control inputs that would guide the system along the chosen sliding surfaces.

4.2 Multirate Output Feedback Control

As discussed in Section. 1, the above algorithm may not be always implementable because all the states may not be measurable, or even physical variables. The existing output feedback control strategies for sliding mode control either require the sliding surface to be an explicit function of the outputs [Khan and Spurgeon, 2001, Sira-Ramirez et al., 1997], which restricts the scope of possible sliding manifolds. Even in case of a output based sliding surface is successfully constructed, it cannot ensure that the system as a whole would be stabilized [Thomas and Bandyopadhyay, 1997]. However, it has been shown in Section. 3, the observability and finite discretizability of the system ensures that each of the system states can be represented as a function of the past N_i multirate samples of the output and the past input $u_i(k-1)$.

Thus, the state based control derived in Section. 4.1 can now be easily translated to one that is based on past output samples and the immediate past control signal, whenever the finitely discretizable system is observable in continuous time by using the procedure described in Section. 3.

5 ILLUSTRATIVE EXAMPLE

The above said multirate output feedback based discrete-time sliding mode control technique has been illustrated in the following example.

Consider the following continuous time system representation defined in the manifold U_p : $\left(x_1 > -1, \{x_2, x_3\} \in \mathbb{R}^2\right)$

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 \\ \dot{x}_3 &= x_1 x_2 + x_2 \\ \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} &= \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} \end{aligned} \tag{10}$$

The system has vector fields $X_1 = \partial_1, X_2 = \partial_2$ and the drift vector field $Y = (x_1x_2 + x_2)\,\partial_3$, where ∂_i denotes the partial derivative with respect to x_i. It can be verified that relative to the dilation $\delta_t = \left(tx_1, tx_2, t^3x_3\right)$, the vector fields are homogeneous of degree -1. The verification for X_1 has been shown here.

$$\begin{aligned} \left((\delta_t)^* X_1\right)(x) &= X_1\left(\delta_t(x)\right) \\ &= X_1\left(tx_1, tx_2, t^3x_3\right) \\ &= \frac{\partial}{\partial x_1}\left(tx_1, tx_2, t^3x_3\right) \\ &= (t, 0, 0) \\ \delta_t\left(X_1(x)\right) &= \delta_t\left(\frac{\partial}{\partial x_1}(x_1, x_2, x_3)\right) \\ &= \delta_t\left((1,0,0)\right) \\ &= \delta_t\left(\left(x_1^0, 0, 0\right)\right) \\ &= \left((tx_1)^0, 0, 0\right) \\ &= (1,0,0) \end{aligned}$$

Hence, it can be seen

$$\left((\delta_t)^* X_1\right)(x) = t^1\left(\delta_t^*\left(X_1(x)\right)\right)$$

Therefore, the vector field X_1 is homogeneous of degree -1 with respect to the dilation δ_t. The property can be easily verified for the other vector fields also. Therefore, the system (10) is finitely discretizable. For a sampling time of $\tau\,\mathrm{sec}$ the discrete-time representation can be given as

$$\begin{aligned} x_1(k+1) &= x_1(k) + \tau u_1(k) \\ x_2(k+1) &= x_2(k) + \tau u_2(k) \\ x_3(k+1) &= x_3(k) + \tau\left(x_1(k)x_2(k) + x_2(k)\right) \\ &\quad + \frac{\tau^2}{2}\left(x_2(k)u_1(k) + \left(1 + x_1(k)\right)u_2(k)\right) \\ &\quad + \frac{\tau^3}{3}u_1(k)u_2(k) \end{aligned} \tag{11}$$

5.1 Design of Sliding Surfaces

The system is a multi-input system and hence requires the design of two sliding surfaces. Dividing the system (10) into two coupled sub-systems with states (x_1) and (x_2, x_3), it can be observed that the only possible sliding surface for the former system would be

$$s_1 = x_1 = 0 \tag{12}$$

and in order to obtain the sliding surface for the second subsystem we use the candidate Lyapunov function $V = \frac{x_3^2}{2}$, which would give

$$\dot{V} = x_3\left(x_1x_2 + x_2\right)$$

and thus a stable sliding surface for this sub-system would be

$$s_2 = x_3 + x_1x_2 + x_2 = 0 \tag{13}$$

5.2 Multirate Output Sampling based Nonlinear Observer

From the discrete model (11), it can be said that if the outputs have multiplicities as $N_1 = 2, N_2 = 4$, then it would be a *sufficient condition* for the system states to be computable through multirate output sampling. However,by choosing $N_1 = 1, N_2 = 2$, i.e., $\Delta_1 = \tau, \Delta = \Delta_2 = \frac{\tau}{2}$, the discrete-time observer can be derived as

$$\begin{aligned} x_1(k) &= y_{11}(k) + \tau u_1(k-1) && (14)\\ x_2(k) &= \frac{1}{3\Delta}\frac{f_1(k)}{(2y_{11}(k) + u_1(k-1)\Delta + 2)} && (15)\\ x_3(k) &= \frac{1}{3}\frac{f_2(k)}{(2y_{11}(k) + u_1(k-1)\Delta + 2)} && (16) \end{aligned}$$

where

$$\begin{aligned} f_1(k) &= 6\left(y_{22}(k) - y_{21}(k)\right) && (17)\\ &\quad +9\Delta^2 u_2(k-1)(y_{11}(k)+1)\\ &\quad +4\Delta^3 u_1(k-1)u_2(k-1)\\ f_2(k) &= 6\left(y_{11}^2(k)+1\right)\Delta^2 u_2(k-1) && (18)\\ &\quad +12y_{11}(k)y_{22}(k) && (19)\\ &\quad +12y_{11}(k)\Delta^3 u_1(k-1)u_2(k-1)\\ &\quad +12y_{22}(k)\\ &\quad +12\Delta^3 u_1(k-1)u_2(k-1)\\ &\quad +12(y_{11}(k)+1)\Delta^2 u_2(k-1)\\ &\quad -6(y_{11}(k)+1)y_{21}(k)\\ &\quad +3u_1(k-1)\Delta(4y_{22}(k) - 3y_{21}(k))\\ &\quad -4\Delta^2 u_2(k-1)\left(u_1^2(k-1)\Delta^2 - 3\right) \end{aligned}$$

$$\begin{aligned} y_{11}(k) &= y_1(k-1) && (20)\\ y_{21}(k) &= y_2(k-1) && (21)\\ y_{22}(k) &= y_2(k\tau - \Delta) && (22) \end{aligned}$$

Thus, the system states can be derived using the past N_i multirate output samples and the immediate past control signals.

Remark 2 : *It is to be noted here that during the estimation of the states $x_2(k)$ and $x_3(k)$ a singularity would occur whenever $(2y_{11}(k) + u_1(k-1)\Delta + 2) = 0$. Therefore, the control signal u_1 should be computed in such a manner that this condition is avoided.* □

5.3 Controller Design

5.3.1 Computation of $u_1(k)$

Using the Gao's [Gao et al., 1995] reaching law for $s_1(k)$,the control signal $u_1(k)$ can be derived as

$$\begin{aligned} s_1(k+1) - s_1(k) &= -q_1\tau - \epsilon_1\tau\,\mathrm{sgn}(s_1(k))\\ \tau u_1(k) &= -q_1\tau - \epsilon_1\tau\,\mathrm{sgn}(s_1(k))\\ u_1(k) &= -q_1 - \epsilon_1\mathrm{sgn}(s_1(k)) && (23) \end{aligned}$$

with the restrictions on q_1, ϵ_1 as

$$\begin{aligned} q_1, \epsilon_1 &> 0 && (24)\\ 1 - q_1\tau &> 0 && (25) \end{aligned}$$

This control would ensure that the state $x_1(k)$ converges monotonically to within the quasi-sliding mode band of width given by

$$\delta_1 = \frac{\epsilon_1\tau}{2 - q_1\tau} \quad (26)$$

The condition

$$\delta_1 < 1 \quad (27)$$

is imposed so that the sliding mode control u_1 has a quasi-sliding mode band completely inside U_p. If the value of $u(k)$ is substituted from (23) into $(2y_{11}(k) + u_1(k-1)\Delta + 2)$ and then equated to zero, we get the disallowed state as follows.

1. For $x_1 > 0$,

$$\begin{aligned} \left((x_1(k)+1) + \frac{\Delta}{2}(-q_1 x_1(k) - \epsilon_1)\right) &= 0\\ x_1(k)\left(1 - \frac{\Delta}{2}q_1\right) &= \left(\frac{\Delta}{2}\epsilon_1 - 1\right)\\ x_1(k) &= \frac{(\epsilon_1\Delta - 2)}{(2 - q_1\Delta)} \end{aligned}$$

Since the Gao's reaching law stipulates $1 - q_1\Delta > 0$, the denominator would always be positive, thus if it is ensured that

$$(\epsilon_1\Delta - 2) < 0, \quad (28)$$

the above case can be completely ignored.

2. For $x_1 < 0$,

$$\begin{aligned} \left((x_1(k)+1) + \frac{\Delta}{2}(-q_1 x_1(k) + \epsilon_1)\right) &= 0\\ x_1(k)\left(1 - \frac{\Delta}{2}q_1\right) &= -\left(\frac{\Delta}{2}\epsilon_1 + 1\right)\\ x_{d_1} &= -\frac{(\Delta\epsilon_1 + 2)}{(2 - \Delta q_1)} \end{aligned}$$

This case can also be ignored provided it is ensured that the disallowed states falls outside the manifold U_p. That is by imposing the condition

$$\frac{(\Delta\epsilon_1 + 2)}{(2 - \Delta q_1)} > 1 \tag{29}$$

Since the initial state $x_1(0)$ would be inside the manifold U_p and the control u_1 would take it monotonically to a band of width $\delta_1 < 1$, the disallowed state $x_1(k) = x_{d_1}$ would not be encountered.

3. And the special case of $x_1(k) = 0$, In this case, the system becomes of a reduced order and hence it is the observer that has to be modified (and not the control input, which would obviously be $u_1(i) = 0, i \geq k$). The new discrete state equations would be

$$\begin{aligned} x_2(k+1) &= x_2(k) + \tau u_2(k) \\ x_3(k+1) &= x_3(k) + \tau x_2(k) + \frac{\tau^2}{2} u_2(k) \end{aligned}$$

Hence, in this case, $x_{(2,3)}(k+1)$ are estimated as

$$\begin{aligned} x_2(k+1) &= \frac{\left(y_{22}(k) - y_{21}(k) + \frac{3}{2}u_2(k)\Delta^2\right)}{\Delta} \\ x_3(k+1) &= 2y_{22}(k) - y_{21}(k) + u_2(k)\Delta^2 \end{aligned}$$

5.3.2 Computation of $u_2(k)$

$$\begin{aligned} s_2(k+1) - s_2(k) &= -q_2\tau s_2(k) - \epsilon_2\tau \text{sgn}(s_2(k)) \\ &(x_1(k)x_2(k) + x_2(k)) \\ &+ \left(\frac{\tau}{2} + 1\right) \\ &x_2(k)u_1(k) \\ +u_2(k)\left(\tau x_1(k) + \frac{\tau}{2}(1 + x_1(k))\right) & \\ +u_2(k)\left(\left(\frac{\tau^2}{3} + \tau\right)u_1(k)\right) &= -q_2(s_2(k)) \\ &-\epsilon_2 \text{sgn}(s_2(k)) \end{aligned}$$

$$\begin{aligned} u_2(k) &= -\frac{(q_2 s_2(k) + \epsilon_2 \text{sgn}(s_2(k)))}{f_3(k)} \\ &-\frac{(x_1(k)x_2(k) + x_2(k))}{f_3(k)} \\ &-\frac{\left(\frac{\tau}{2} + 1\right)x_2(k)u_1(k)}{f_3(k)} \end{aligned} \tag{30}$$

$$f_3(k) = \frac{3\tau}{2}x_1(k) + \Delta + \left(\frac{\tau^2}{3} + \tau\right)u_1(k)$$

with the inequality conditions

$$q_2, \epsilon_2 > 0 \tag{31}$$

$$1 - q_2\tau > 0 \tag{32}$$

Here too, there would be a singularity encountered in the computation of the control u_2 whenever the denominator vanishes. In this case the disallowed state would be

1. If $x_1 > 0$

$$\begin{aligned} 0 &= \tau x_1(k) + \frac{\tau}{2}(1 + x_1(k)) \\ &+ \left(\frac{\tau^2}{3} + \tau\right)(-q_1 x_1(k) - \epsilon_1) \\ x_{d_2} &= \frac{\left(\frac{\tau^2}{3} + \tau\right)\epsilon_1 - \Delta}{3\Delta - \left(\frac{\tau^2}{3} + \tau\right)q_1} \end{aligned}$$

2. If $x_1 < 0$

$$\begin{aligned} 0 &= \tau x_1(k) + \frac{\tau}{2}(1 + x_1(k)) \\ &+ \left(\frac{\tau^2}{3} + 1\right)(-q_1 x_1(k) + \epsilon_1) \\ x_{d_3} &= \frac{-\left(\left(\frac{\tau^2}{3} + 1\right)\epsilon_1 + \Delta\right)}{3\Delta - \left(\frac{\tau^2}{3} + 1\right)q_1} \end{aligned}$$

Both these cases would be avoided if q_1 and ϵ_1 are chosen such that

$$\left(\frac{\tau^2}{3} + \tau\right)\epsilon_1 - \Delta < 0 \tag{33}$$

$$3\Delta - \left(\frac{\tau^2}{3} + \tau\right)q_1 < 0 \tag{34}$$

In this case, $x_1 = 0$ does not cause any singularity in $u_2(k)$

When the states in control law (23,30) are substituted from the nonlinear multirate observer constructed in (14), the control law would now be translated into one based on multirate output feedback.

5.4 Simulation Study

A simulation of the response of the system (10) under the designed control, was studied. The control inputs u_1 and u_2 were designed according to (23) and (30). The sampling time was chosen as $\tau = 0.1\,\text{sec}$, and the controller parameters were chosen as $q_1 = q_2 = 2, \epsilon_1 = \epsilon 2 = 0.1$ so as to satisfy the inequality conditions in equations (24,25,27-29,31-34).

The simulation results for $X(0) = [\ 2.5 \quad 5 \quad 0\ ]^T$ are shown in Figs. (1-3). Fig. (1) gives the time-response of the system states when the designed control is applied to the system. The phase portrait of the system is shown in Fig. (2). The evolution of the sliding surfaces s_1 and s_2 and the plots of the control inputs are given in Fig. (3).

It can be seen from the plots (Fig. (3)) that the sliding surfaces decrease monotonically in magnitude to within the quasi-sliding mode band. The response of the system states and the applied control inputs are also found to be satisfactory.

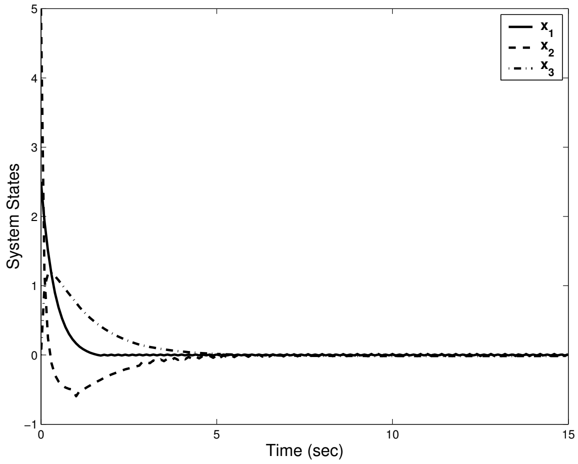

Figure 1: Response of System States.

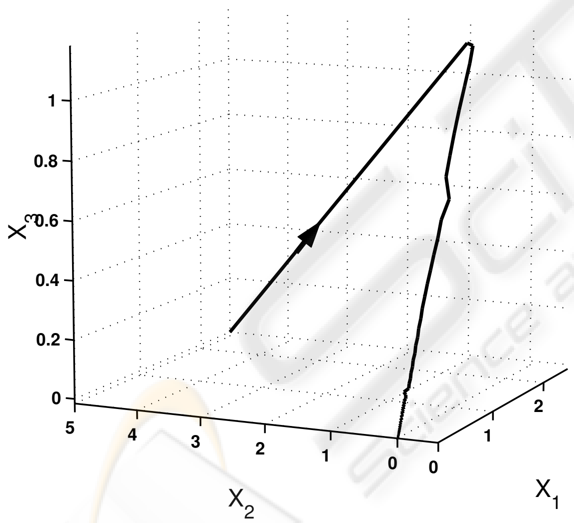

Figure 2: Phase Portrait of the System

6 CONCLUSION

A procedure for the design of discrete-time sliding mode controller for a class of nonlinear systems viz. finitely discretizable nonlinear systems has been proposed in the paper. The technique uses the concept of multirate output sampling to realize the behavior of a state feedback based nonlinear control law. It has an advantage that it would be applicable to a larger class of nonlinear systems as many of the physical systems are, in fact, finitely discretizable under appropriate coordinate transformation. Moreover, the technique is practical as it is able to translate a state based control law into one based on system outputs and past input samples. Further, it does not impose any restrictions on the choice of the sliding surfaces.

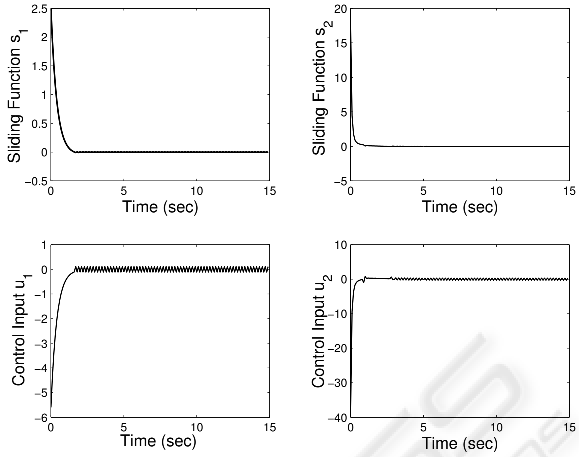

Figure 3: Evolution of Sliding Surfaces and Control Inputs.

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