POSITION AND ORIENTATION CONTROL OF A TWO-WHEELED DIFFERENTIALLY DRIVEN NONHOLONOMIC MOBILE ROBOT

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Keywords: Nonholonomic robots, robot control, stabilization control, linear modeling of robots.

Abstract: This paper addresses the dynamic stabilization problem of a two-wheeled differentially driven nonholonomic mobile robot. The proposed strategy is based on changing the robot control variables from x, y and θ to s and θ, where s represents the robot linear displacement. Using this model, the nonholonomic constraints disappear and we show how the linear control theory can be used to design the robot controllers. This control strategy only needs the robot localization (x, y, θ), not requiring any velocity measurement or estimation. The complete derivation of the control strategy and some simulated results are presented.

1 INTRODUCTION

There are many feedback controllers proposed in the literature (Aicardi et al., 1995; d'Andrea Novel et al., 1995; Lizarralde, 1998; Samson, 1993; Tanner and Kyriakopoulos, 2002; Yang and Kim, 1999) for nonholonomic wheeled mobile robots. Most of these strategies only deal with the problem of kinematic compensation (Aicardi et al., 1995; d'Andrea Novel et al., 1995; Samson, 1993). Pure kinematic controllers lie on the simplification that the generated control signal is instantaneously applied to the robot actuators, not taking into account the dynamic effects.

Recently, some control strategies have been proposed to deal with dynamic compensation of mobile robots (Lages and Hemerly, 2000; Lizarralde, 1998; Tanner and Kyriakopoulos, 2002). Most of them are derived via Lyapunov techniques and do not present a correspondence between the controller parameters and the robot dynamic behavior. Many of the dynamic control laws also requires the measurement of the robot velocities, not always accurate or available.

The control strategy proposed on this paper addresses the dynamic compensation of mobile robots and only requires information about the robot localization. The problem classification is presented on section 2 and the kinematic and dynamic model of the considered robot, on section 3. The control system design is presented on section 4 and some results and final considerations, on sections 5 and 6.

2 PROBLEM CLASSIFICATION

There are two main problems in mobile robots control: the trajectory tracking problem and the stabilisation problem.

The stabilisation problem states that the robot must reach a desired configuration (x_d, y_d and θ_d) starting from a given initial configuration (x_0, y_0 and θ_0) (Luca et al., 1998). This control problem is also known as a parking problem. There are several feedback controllers proposed in the literature for the stabilisation problem (Aicardi et al., 1995; Lizarralde, 1998; Tanner and Kyriakopoulos, 2002), some of them with the previously presented limitations.

In the trajectory tracking problem, the robot must reach and follow a trajectory in the Cartesian space starting from a given initial configuration (Luca et al., 1998). There are several feedback controllers proposed in the literature that address only the trajectory tracking problem (Oliveira and Lages, 2001; Samson, 1993; Yang and Kim, 1999).

The trajectory tracking problem is simpler than the stabilisation problem because there is no need to control the robot orientation: it is automatically compensated as the robot follows the trajectory, provided that the specified trajectory respects the non-holonomic constraints of the robot. As the control strategy presented in this paper is concerned with the stabilisation problem, it can also be applied to the trajectory tracking problem.

Vieira F., Medeiros A., Alsina P. and Araújo Jr. A. (2004).
POSITION AND ORIENTATION CONTROL OF A TWO-WHEELED DIFFERENTIALLY DRIVEN NONHOLONOMIC MOBILE ROBOT.
In *Proceedings of the First International Conference on Informatics in Control, Automation and Robotics*, pages 256-262
DOI: 10.5220/0001138702560262

3 MODELLING

A schematic diagram of the considered robot is presented on figure 1. The robot configuration is represented by its position on the Cartesian space (x and y, that is the position of the robot-body center with relation to a referential frame fixed on the workspace), and by its orientation θ (angle between the robot orientation vector and the reference axis – X, fixed on the workspace).

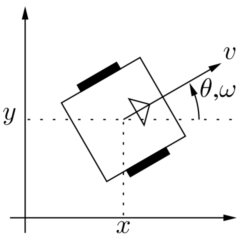

Figure 1: Schematic diagram of a two-wheeled nonholonomic robot.

The kinematic model represents the movements constraints of the robot body. For the considered robot, the kinematic model is given by equation 1.

$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v \\ \omega \end{bmatrix} = \mathbf{G}(\theta)\cdot\mathbf{v} \quad (1)$$

The vector $\mathbf{q} = [\; x \quad y \quad \theta \;]^T$ represents the linear and angular positions and the vector $\mathbf{v} = [\; v \quad \omega \;]^T$ represents the linear and angular velocities. The main feature of this model for wheeled mobile robots is the presence of nonholonomic constraints, due to the rolling without slipping condition between the wheels and the ground. The nonholonomic constraints impose that the system generalized velocities ($\dot{x}$, $\dot{y}$ and $\dot{\theta}$) cannot assume independent values. It can be observed that the kinematic model in equation 1 does not include the dynamic effects of the robot body and actuators.

The dynamic model is derived from the actuators dynamics and the robot dynamics parameters, like mass, inertia momentum and friction coefficients. The final dynamic model for a robot with two DC motors directly connected to the wheels (Yamamoto et al., 2003) is given by equation 2:

$$\mathbf{Ku} = \mathbf{M}\dot{\mathbf{v}} + \mathbf{Bv} \quad (2)$$

The vector $\mathbf{u} = [\; u_l \quad u_r \;]^T$ represents the input signals, usually currents or tensions applied to the left and right electrical motors of the robot. $\mathbf{K}$ is a gain matrix that transforms the input signals $\mathbf{u}$ into forces to be generated by the robot wheels. $\mathbf{M}$ is the generalized inertia matrix and $\mathbf{B}$ is the generalized viscous friction matrix.

The model in equation 2 is a multi-variable linear system and a simple control law for the dynamic stabilisation problem could be designed. However, two drawbacks can be highlighted: the measurement of the state variables (v and ω) is usually inaccurate or unavailable, and velocities references are not well suited for the mobile robot stabilisation control problem, where the references are usually coordinates on the Cartesian space and an orientation angle.

Models in equations 1 and 2 can be rearranged into a single state space representation, by defining the matrices $\tilde{\mathbf{A}} = -\mathbf{M}^{-1}\mathbf{B}$ and $\tilde{\mathbf{B}} = \mathbf{M}^{-1}\mathbf{K}$

$$\begin{bmatrix} \dot{\mathbf{v}} \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{A}} & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{G}(\theta) & \vdots & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{q} \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{B}} \\ .. \\ \mathbf{0} \end{bmatrix} \mathbf{u} \quad (3)$$

The outputs in equation 3 are x, y and θ. Although this model allows the use of Cartesian coordinates and orientation angles as references to the mobile robot, it is a multi-variable non-linear model and the development of control laws based on such model is not trivial.

In order to reduce the model complexity, one could rewrite it in terms of the robot linear and angular displacement, s and θ, so that $\dot{s} = v$ and $\dot{\theta} = \omega$. Defining a vector $\mathbf{p} = [\; s \quad \theta \;]^T$:

$$\begin{bmatrix} \dot{\mathbf{v}} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{A}} & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{I} & \vdots & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{p} \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{B}} \\ .. \\ \mathbf{0} \end{bmatrix} \mathbf{u} \quad (4)$$

The model in equation 4 is linear, with outputs s and θ. One could easily design a control system based on the block diagram on figure 2, if s and θ are measurable and s_{ref} and θ_{ref} are defined. This controller can be based on any of the classic design techniques for linear systems where the controller receives the error signal and generates the input to the plant (a PID, for example).

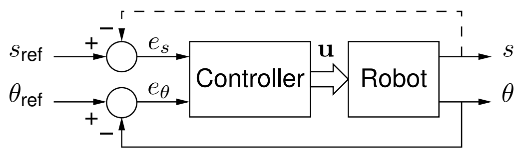

Figure 2: Control system block diagram.

As the design of such a controller is simple, this model has been used for the control system design, despite of two problems that still hold: the linear displacement s along a trajectory is practically unmeasurable and s_{ref} is meaningless. However, these problems can be contoured, as will be shown on the next section.

4 CONTROL SYSTEM DESIGN

The robot stabilisation problem can be divided into two different control problems: robot positioning control and robot orientating control. The robot positioning control must assure the achievement of a desired position $(x_{\text{ref}}, y_{\text{ref}})$, regardless of the robot orientation. The robot orientating control must assure the achievement of the desired position and orientation (x_d, y_d, θ_d).

4.1 Robot positioning control

Figure 3 illustrates the positioning problem, where Δl is the distance between the robot and the desired reference $(x_{\text{ref}}, y_{\text{ref}})$ in the Cartesian space. The robot positioning control problem will be solved if we assure $\Delta l \rightarrow 0$. This is not trivial since the l variable do not appear in the model of equation 4.

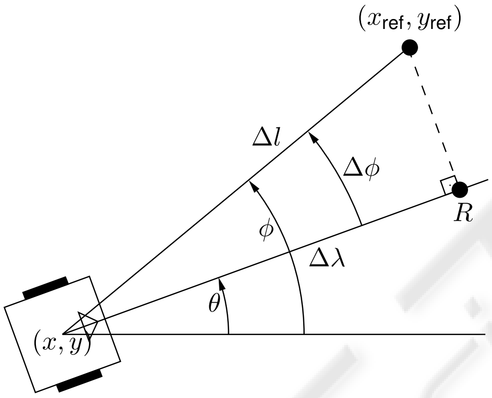

Figure 3: Robot positioning problem

To overcome this problem, we can define two new variables, $\Delta\lambda$ and ϕ. $\Delta\lambda$ is the distance to R, the nearest point from the desired reference that lies on the robot orientation line; ϕ is the angle of the vector that binds the robot position to the desired reference. We can also define $\Delta\phi$ as the difference between the ϕ angle and the robot orientation: $\Delta\phi = \phi - \theta$.

We can now easily conclude that:

$$\Delta l = \frac{\Delta\lambda}{\cos(\Delta\phi)} \tag{5}$$

So, if $\Delta\lambda \rightarrow 0$ and $\Delta\phi \rightarrow 0$ then $\Delta l \rightarrow 0$. That is, if we design a control system that assures the $\Delta\lambda$ and $\Delta\phi$ convergence to zero[1], then the desired reference, x_{ref} and y_{ref}, is achieved. Thus, the robot positioning control problem can be solved by applying any control strategy that assures such convergence.

[1]It is not even necessary to assure the convergence of $\Delta\phi$ to zero: the convergence to any $\Delta\phi$ value where $\cos(\Delta\phi) \neq 0$ will be acceptable.

The block diagram in figure 2 suggests that the system can be controlled using linear and angular references, s_{ref} and θ_{ref}, respectively. We will generate these references in order to ensure the converge of $\Delta\lambda$ and $\Delta\phi$ to zero, as required by equation 5. In other words, we want $e_s = \Delta\lambda$ and $e_\theta = \Delta\phi$. Thus, if the controller assures the errors convergence to zero, the robot positioning control problem is solved.

To make $e_\theta = \Delta\phi$, we just need to define $\theta_{\text{ref}} = \phi$, so $e_\theta = \theta_{\text{ref}} - \theta = \phi - \theta = \Delta\phi$. For this, we make:

$$\theta_{\text{ref}} = \tan^{-1}\left(\frac{y_{\text{ref}} - y}{x_{\text{ref}} - x}\right) = \tan^{-1}\left(\frac{\Delta y_{\text{ref}}}{\Delta x_{\text{ref}}}\right) \tag{6}$$

To calculate e_s is generally not very simple, because the s output signal cannot be measured and we cannot easily calculate a suitable value for s_{ref}. But if we define the R point in figure 3 as the reference point for the s controller, only in this case it is true that $e_s = s_{\text{ref}} - s = \Delta\lambda$. So:

$$\begin{aligned} e_s = \Delta\lambda = \Delta l \cdot \cos(\Delta\phi) = \\ \sqrt{(\Delta x_{\text{ref}})^2 + (\Delta y_{\text{ref}})^2} \cdot \cos\left[\tan^{-1}\left(\frac{\Delta y_{\text{ref}}}{\Delta x_{\text{ref}}}\right) - \theta\right] \end{aligned} \tag{7}$$

The complete robot positioning controller, based on the diagram of figure 2 and the equations 6 and 7, is presented on figure 4. It can be used as a stand-alone robot control system if the problem is just to drive to robot to a given position $(x_{\text{ref}}, y_{\text{ref}})$, regardless of the final robot orientation.

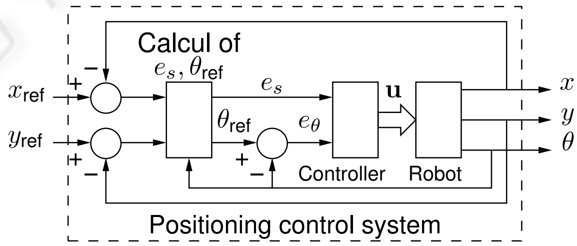

Figure 4: Robot positioning controller

4.1.1 Practical aspects

The Brockett's theorem (Brockett, 1983; Stern, 2002) proved that no continuous control law can completely stabilize a system with the non-holonomic restriction as in equation 1. The best any such control law can do is to drive the system to a region delimited by a circle of radius as small as posible around the desired final position. This restriction naturally appears in the control law proposed in this article: it is impossible to calculate the angular reference θ_{ref} in equation 6 when the robot reaches the position reference $(x_{\text{ref}}, y_{\text{ref}})$. To deal with this limitation, we define, based on the precision of the position measurement sensors, an acceptable distance error (ϵ). When the robot enters this circular region ($\Delta l < \epsilon$), the angular error e_θ and the linear error e_s are assumed to be zero.

The angular error e_θ should be normalized to the interval $-\pi/2 \leq e_\theta \leq \pi/2$. When the robot is near the sign change border ($e_\theta \approx \pm\pi/2$), it is sometimes better to slightly exceed the $\pm\pi/2$ limit to maintain the sign of the angular error the same of the previous iteration. This avoids unnecessary changes in the direction of the angular movement.

Only allowing values between $-\pi/2$ and $\pi/2$ for the angular error does not favor forward movements, as illustrated by the first trajectory in figure 5. But if the robot cannot move as better in the backward direction as in the forward one, the second trajectory in figure 5 would be preferable in some situations. To obtain this behavior, the angular error e_θ should be restricted to the interval $-\pi \leq e_\theta \leq \pi$. The $\pm\pi$ limits for the angular error can only be applied when the robot is far enough from the desired reference point. If they are applied when the robot is near the final position, small overshots can make the robot indefinitely turn around the reference point.

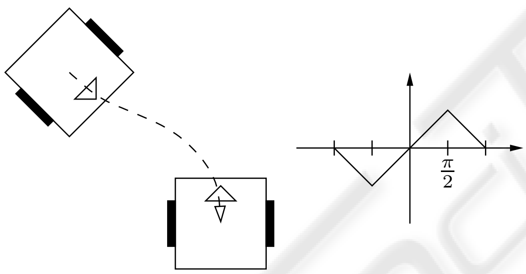

Figure 5: Forward and backward movements

4.2 Robot orientating control

The main idea behind the proposed orientating control strategy is that when we want to move to a final position with a fixed orientation, it is not usually a good idea to go straight to this position, as illustrated by figure 6. Generally, we drive as if we wanted to go to another place until a moment where, if we go straight to the final position, we will reach it with the desired orientation.

In order to attend the robot orientating control problem, an external loop with a moving reference scheme has been designed. The external loop generates Cartesian references, x_{ref} and y_{ref}, for the internal loop (the positioning control scheme), such that the robot reaches the desired position (x_d and y_d) with the desired orientation (θ_d). This approach is illustrated on figure 7: the positioning controller block can be the one presented on figure 4 or any other one.

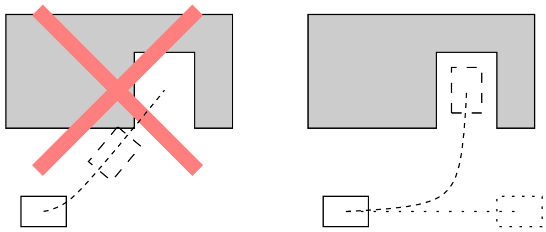

Figure 6: The orientating control idea

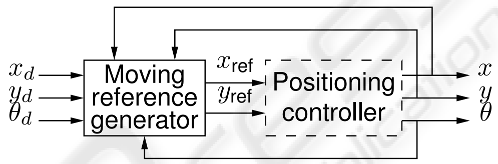

Figure 7: Dynamic stabilisation controller

The strategy to calculate the internal reference is presented on figure 8. The reference $(x_{\text{ref}}, y_{\text{ref}})$ is calculated by rotating the vector of length Δd pointing from the robot position to the desired position by an angle of γ. Practical aspects of calculating γ will be discussed latter, but it is essentially equal to the difference between the desired final orientation (θ_d) and the angle to move to the desired position (β):

$$\gamma = \beta - \theta_d \tag{8}$$

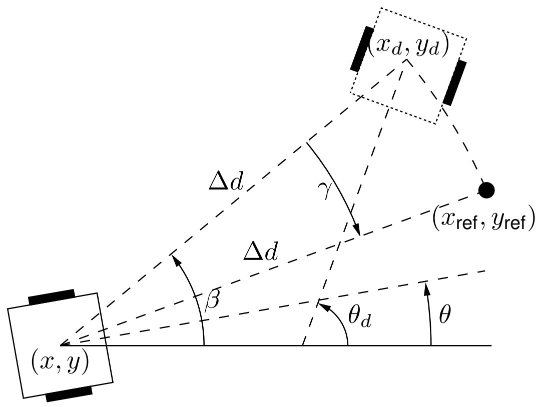

Figure 8: Robot orientating problem

If β and θ_d coincide, $\gamma = 0$ and the robot goes straight to the final position $(x_{\text{ref}}, y_{\text{ref}}) = (x_d, y_d)$. If not, as long as the robot tries to move to the internal reference $(x_{\text{ref}}, y_{\text{ref}})$, the difference between θ_d and β raises, γ decreases and $(x_{\text{ref}}, y_{\text{ref}})$ tends to (x_d, y_d).

The moving reference scheme is so driven by the following equations:

$$\begin{aligned} x_{\text{ref}} &= x + \Delta d \cdot \cos(\beta - \gamma) \\ y_{\text{ref}} &= y + \Delta d \cdot \sin(\beta - \gamma) \end{aligned} \quad (9)$$

where x and y are the robot Cartesian coordinates, θ_d is the desired orientation, and Δd and β are presented on figure 8:

$$\begin{aligned} \Delta d &= \sqrt{(x_d - x)^2 + (y_d - y)^2} \\ &= \sqrt{(\Delta x_d)^2 + (\Delta y_d)^2} \\ \beta &= \tan^{-1}\left(\frac{y_d - y}{x_d - x}\right) = \tan^{-1}\left(\frac{\Delta y_d}{\Delta x_d}\right) \end{aligned} \quad (10)$$

4.2.1 Practical aspects

The γ angle in equation 8 should be restrained to the interval $-\pi/2 \leq \gamma \leq \pi/2$. The specific way to do this restriction depends on the robot capabilities.

If the robot can move as well in one direction as in the order, it is irrelevant if the robot reaches the desired orientation with or without a rotation of π. In the example of figure 9, we want a final orientation $\theta_d = \pi/2$, but the robot finished the mouvement with an orientation of $\theta_d = -\pi/2$. This behavior (bidirectional orientation) is achieved by normalizing the γ angle, as illustrated by the graphic in figure 9.

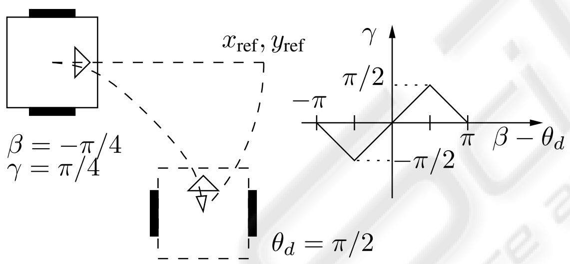

Figure 9: Bidirectional orientation

Sometimes a non-inversed final orientation is required. In this case (unidirectional orientation), the γ angle should be saturated (and not normalized), as illustrated by the graphic in figure 10. This orientating control strategy should be combined with a position control strategies that favors forward mouvements, as described in section 4.1.1. The limitation of this kind of position control (see 4.1.1) imposes that the unidirecional orientating control strategy can only be applied when the robot is far enough from the desired reference point.

4.3 The linear controller

The controller appearing on figures 2 and 4 can be designed using any of the classical control techniques

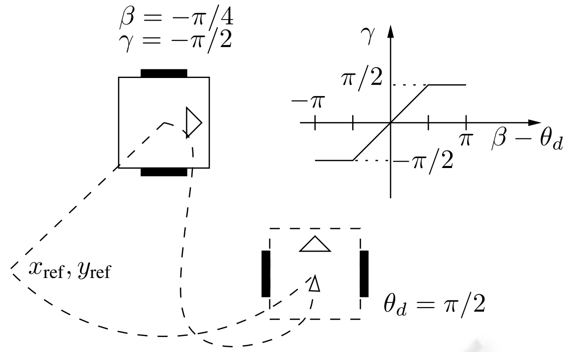

Figure 10: Unidirectional orientation

that can be used with a linear multi-variable system described by the model in equation 4. We will exemplify with a simple controller based on decoupled PIDs, but in no way it should be assumed that the control strategy presented on sections 4.1 and 4.2 must necessarily be used with a PID-based controller.

If the robot is symmetrical and driven by two identical DC motors, the $\mathbf{K}$, $\mathbf{M}$ and $\mathbf{B}$ matrices in equation 2 have the following properties (Yamamoto et al., 2003):

$$\mathbf{K} = \begin{bmatrix} \alpha & \alpha \\ \beta & -\beta \end{bmatrix} \qquad \mathbf{M}, \mathbf{B} \text{ are diagonals}$$

We can define two new input signals, v_s and v_θ and a new input vector $\mathbf{w} = [\begin{array}{cc} v_s & v_\theta \end{array}]^T$ such that:

$$v_s = \frac{u_l + u_r}{2} \qquad v_\theta = \frac{u_l - u_r}{2}$$

$$\mathbf{w} = \frac{1}{2}\left[\begin{array}{cc} 1 & 1 \\ 1 & -1 \end{array}\right]\mathbf{u} \Rightarrow \mathbf{u} = \left[\begin{array}{cc} 1 & 1 \\ 1 & -1 \end{array}\right]\mathbf{w} \quad (11)$$

If we introduce the new $\mathbf{w}$ input vector in equation 4 and calcute the transfer model equivalent to the space stade equation, we obtain a decoupled system:

$$\left[\begin{array}{c} S(s) \\ \Theta(s) \end{array}\right] = \left[\begin{array}{cc} G_s(s) & 0 \\ 0 & G_\theta(s) \end{array}\right] \cdot \left[\begin{array}{c} V_s(s) \\ V_\theta(s) \end{array}\right] \quad (12)$$

Using equation 12, it is very simple to design two independent PID controllers for s and θ, based on the desired linear and angular behavior. The output of these controllers are the virtual input signals v_s and v_θ: to calculate the real input signals u_l and u_r, we use equation 11.

5 RESULTS

Simulated results of the proposed strategy are presented on this section. A simple PD controller has been implemented as the positioning controller. The robot dynamic model has been derived via experimental identification of a real mobile robot (Guerra et al., 2004).

On figure 11 a simulation for the robot stabilisation control problem is shown, where the initial conditions are $x = 0$, $y = 0$ and $\theta = 0$, and the desired configuration is $x_d = 1$, $y_d = 1$ and $\theta_d = 0$. The moving reference scheme can also be observed on figure 11.

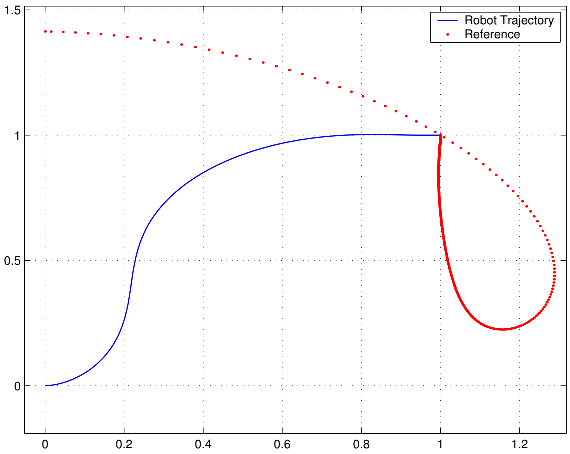

Figure 11: Mobile robot stabilisation.

Figure 12 shows the linear and angular errors convergence to zero, thus, assuring the achievement of the control objective. It must be noticed that the controller performance can be improved through the PD gains adjustment.

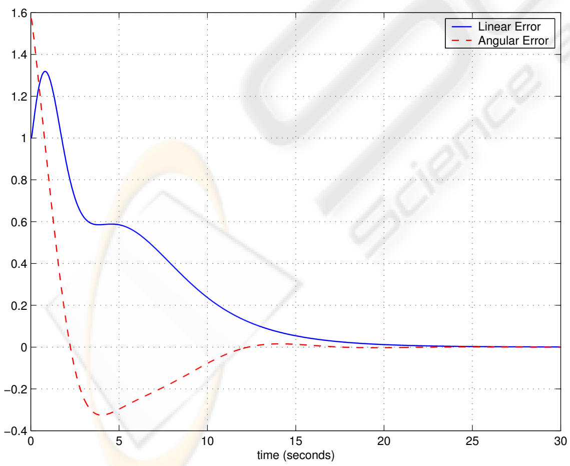

Figure 12: Linear and angular errors for the robot stabilisation simulation.

On figure 13 a set of simulations with the same final configuration ($x = 0$, $y = 0$ and $\theta = 0$) and different initial conditions is presented.

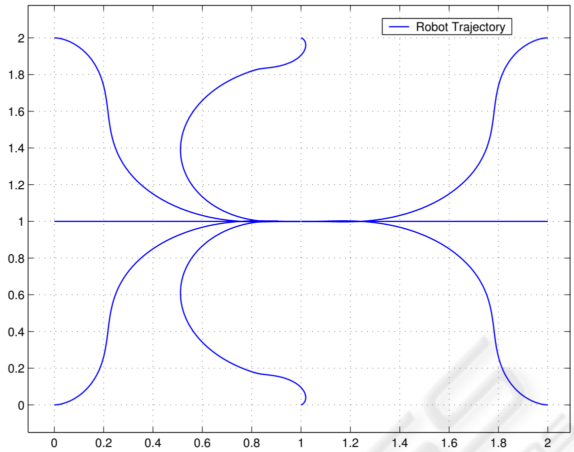

Figure 13: Simulation for different initial conditions.

6 CONCLUSIONS

This paper introduces a new approach to the stabilisation problem of two-wheeled nonholonomic mobile robots, considering the robot dynamic. The implementation of the proposed control strategy is very simple and the simulations have shown very satisfactory results.

Since the proposed control strategy can be implemented with linear controllers, the system performance adjustment is simpler and very meaningful (for example, the adjustment of PID controller gains). We can also use a classical technique to identify in real-time the model of the system (Guerra et al., 2004) and adopt an adaptive controller.

The proposed control strategy does not require any information about the robot body velocities. The only information needed is the robot Cartesian coordinates and its orientation. Such information can be obtained via any kind of absolute positioning system.

Future works will consider the proof for the proposed moving reference control scheme.

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