# SOLVING THE LONGEST WORD-CHAIN PROBLEM

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- Keywords: The Longest Distance Problem, Word–Chain Game, Integer Programming, Heuristic Search, Linear Programming, Branch-and-Bound Method, Optimal Solution, Local-Maximum Solution
- Abstract: The SHIRITORI game is a traditional Japanese word-chain game. This paper describes the definition of the longest SHIRITORI problem (a kind of the longest distance problem) as a problem of graph and the solution based on the integer problem (IP). This formulation requires the exponential order variables from the problem size. Against this issue, we propose a solution based on the LP-based branch-and-bound method, which solves the relaxation problems repeatedly. This method is able to calculate the longest SHIRITORI sequences for 130 thousand words dictionary within a second. In this paper, we compare the performances for the heuristic-local search and investigate the results for several conditions to explore the longest SHIRITORI problem.

# **1 INTRODUCTION**

The SHIRITORI game (Word-Chain game) is a traditional Japanese game using words (usually basic nouns). There are many local rules for this game, but the basic rule is simple as below:

#### SHIRITORI rule

N-players sit in a circle. The first player presents a word. A player of his turn also shows a word such that its first character is same as the last character of a word presented by the previous player. While a game, any words are prohibited to be shown twice. And the last character "N" is also prohibited because any words cannot start with this in Japanese. A player is a loser when he cannot say any words.

An example of a SHIRITORI sequence is like below.

# $\frac{\text{RIN}\underline{\text{GO}} \text{ (apple)} \rightarrow \underline{\text{GO}} \text{RI}\underline{\text{RA}}(\text{gorilla}) \rightarrow}{\underline{\text{RA}}\underline{\text{PPA}} \text{ (trumpet)} \rightarrow \underline{\text{PA}}\underline{\text{IPU}}(\text{pipe)} \rightarrow \dots$

The SHIRITORI provides a way of child education. The instance research was appeared to develop the association ability (Kanasugi, 1996). In this research, any words could be accepted to present, if these words presented were related to the previous words. From a view of computer games, a research (Ito, 2002) showed a perfect solution under a model of the two-players, perfect information game. At that point, the model of N-players game was hard to solve under the limitation of time. This research, however, showed the small-SHIRITORI results (limitations of words and characters appeared in his experiment). The problem of handling several thousand words or an N-players game could not be computed under his proposal.

When we consider the SHIRITORI as an educational game, many works would be devoted to the long-continuation of a game. It is possible to draw many words from human-players if a computer makes a good plan for human to think of his words. As a result, a system can develop the human-ability of a way of thinking. A system (Kanasugu, 1996) fails to do this, because a computer proposes words randomly. We, as a basic research, propose a method of solving the longest SHIRITORI problem. We expect that our method provides useful information for child education systems and general N-players SHIRITORI game. The problem is defined as below:

#### The longest SHIRITORI problem

The longest SHIRITORI problem is a problem of finding a longest SHIRITORI sequence from headwords in a pre-defined dictionary (we use a term 'word' as a headword in this paper).

The longest SHIRITORI problem described in the next section is modelled under the directed graph in

214 Inui N., Shinano Y., Kounoike Y. and Kotani Y. (2004). SOLVING THE LONGEST WORD-CHAIN PROBLEM. In *Proceedings of the First International Conference on Informatics in Control, Automation and Robotics*, pages 214-221 DOI: 10.5220/0001138902140221 Copyright © SciTePress this paper. It is known that a longest distance algorithm for Directed Acyclic graphs (DAG) shows the O(N + M) performance for the number of vertices N and arcs M. Though, for Directed Cyclic graphs (DCG), it is known that Liao-Wong algorithm or Bellman-Ford algorithm show reasonable performances, these are only applied to DCG with negative cycles. We cannot apply these algorithms for the longest SHIRITORI problem, since word-chain sequences as a graph contain positive cycles.

Solving the longest SHIRITORI problem by a brute-force search algorithm requires dealing with O(N) average arcs and O(MN<sup>2</sup>) depths of tree. Here, "N" is the number of vertices and "M" is the maximum number of arcs between two vertices (Ito, 2002). This means that we cannot actually solve the longest SHIRITORI problem by brute-force methods. Instead of using brute-force methods, we divide the longest SHIRITORI problem into two problems: determining the number of arcs of the longest SHIRITORI sequences. The first problem is solved under a framework of the integer programming in this paper.

# 2 MODELING OF THE LONGEST SHIRITORI PROBLEM

The SHIRITORI described in the section 1 is modelled under the graph theory. A basic idea is coming from a model where vertices and arcs are corresponding to characters and words, respectively. "V" is a set of vertices corresponding to characters which are the first or the last characters of words. We denote V as  $V = \{1, 2, \dots, n\}$ , these numbers are corresponding to the characters. In this case, 'n' characters are used. "A" is a set of arcs defined as  $A = B_{11} \cup B_{12} \cup \dots \cup B_{nn}$ .  $B_{ij}$  means a set of words where the beginning character is *i* and the terminating character is *j*. So we define a graph *G* for expressing the transition graph for SHIRITORI as G = (V, A). Fig.1 shows an example of this graph. From the SHIRITORI rule, each arc is followed at once or none.

We introduce a super-source S and a super-sink t, for the beginning vertex and the terminating vertex, respectively. These vertices are added to the initial transition graph G like this way: The super-source has arcs for all other vertices and all vertices have arcs for the super-sink. By the two super vertices, we can find a longest SHIRITORI sequence without trying all combinations of vertices as the beginning and terminating vertices.

Though G is a basic idea, we do not need to consider words themselves. For example, we can

select an arbitrary word, 'AKAI' or 'ATAI', while moving from 'A' to 'I'. So we only consider the number of arcs between two vertices in the graph G. We define  $f_{ij} = |B_{ij}|$  as the number of arcs between two vertices such that the beginning character is *i* and the terminating character is *j*.



Figure 1: Transition Graph G for SHIRITORI.

From the graph G like Fig.1, we can find many SHIRITORI sequences like  $(A \rightarrow I \rightarrow U')$ ,  $(U \rightarrow SA \rightarrow SA'$  and so on. A SHIRITORI sequence can be viewed as a semi-Eulerian subgraph on the graph G. The longest SHIRITORI problem is equivalent to the largest semi-Eulerian subgraph problem. Generally, finding the largest semi-Eulerian subgraph is equivalent to finding the maximum Hamiltonian path. This means that the longest SHIRITORI problem is a NP-complete problem (Nakayama, 1995, Lai, 2001, Skina, 1990).

When the longest SHIRITORI problem is treated as a search problem, a brute-force search method enumerates SHIRITORI sequences. In this case, the depth of the enumeration tree is  $O(MN^2)$ , where N is the number of vertices and M is the maximum number of arcs between two vertices (Ito, 2002). This depth means that the brute-force search algorithm cannot find the optimal solution within reasonable time.

In this paper, we handle two sub-problems for the longest SHIRITORI problem. The first problem is to make a semi-Eulerian subgraph from a graph G. The second problem is to constitute an actual SHIRITORI sequence on the semi-Eulerian subgraph. We start to solve the first problem using the integer programming.

# **3** SOLVING THE LONGEST SHIRITORI PROBLEM

In many researches, parallel algorithms are focused on calculating the longest distance problem (Gu, 1996). There is the previous research on the longest distance problem using Integer Programming (Fischetti, 2002). We use the same way to describe the longest SHIRITORI problem. We denote the number of words  $x_{ij}$  on a SHIRITORI sequence which first character and last character is i and j, respectively, and  $f_{ij}$  means the number of words in a word-dictionary. Obviously, the upper bound of  $x_{ij}$  is given by  $f_{ij}$ . The next two conditions are for the semi-Eulerian graph of directed graphs.

- 1) A graph is a connected graph.
- 2) In-degree and out-degree at every vertex but two are same. A vertex with one more out-degree becomes a beginning one and another vertex with one more in-degree becomes a terminating one (in our model, these vertices are *s* and *t*).

(P) Maximize 
$$z_0 = \sum_{i \in V \cup \{s\}} \sum_{j \in V \cup \{t\}} x_{ij}$$
  
Subject to :  

$$\sum_{i \in V} x_{si} = 1$$

$$\sum_{i \in V} x_{ij} - \sum_{i \in V} x_{ji} = 0 \quad \forall j \in V$$

$$\sum_{i \in V} x_{ii} = 1$$

$$2^{|V \cup \{s,i\}|-2} \sum_{k=1} (1 - y_k) = 1$$

$$\sum_{k=1}^{i \in S_k} (1 - y_k) = 1$$

$$\sum_{k=1}^{i \in S_k} x_{ij} \ge 1, \forall S_k$$

$$g_k + \sum_{j \in S_k} x_{ij} \ge 1, \forall S_k$$

$$g_k \sum_{j \in S_k} f_{ij} + \sum_{j \in S_k} x_{ij} \le \sum_{j \in S_k} f_{ij}, \forall S_k$$

$$0 \le x_{ij} \le f_{ij}, \forall i \in V, \forall j \in V$$

$$0 \le x_{ij} \le 1, \forall j \in V$$

$$0 \le x_{ij} \le 1, \forall j \in V$$

$$x_{ij} \in \mathbf{Z}, \forall i \in V \cup \{s\}, \forall j \in V \cup \{t\}$$

$$y_k \in \{0,1\}, \forall k$$

We formulate these conditions as an integer programming problem (P), but the condition 1) requires exponential order variables and constraints in this problem formulation.

Instead of solving the integer programming problem (P), we use the LP-based branch-and-bound method. Our proposed method solves the problem like: the first, solve the longest SHIRITORI problem under the condition 2) that is formulated as the problem ( $RP_0$ ). If the condition 1) is not satisfied in the solution of ( $RP_0$ ), add conditions to satisfy the condition 1). The detail about LP-based branch-and-bound method is described later.

There are two states below after finding the solution of  $(RP_0)$ .

- a) The graph of the solution of (RP<sub>0</sub>) is a connected graph. In this case, the solution is also optimal for problem (P).
- b) If the graph is disconnected, every connected component of the solution of (RP<sub>0</sub>) is a semi-Eulerian graph or Eulerian graph, then two cases below is possible:
  - i) The Eulerian path from s to t in the connected component gives an optimal solution of problem (P).

(RP<sub>0</sub>) Maximize 
$$z_0 = \sum_{i \in V \cup \{s\}} \sum_{j \in V \cup \{t\}} x_{ij}$$
  
Subject to :  
 $\sum_{i \in V} x_{si} = 1$   
 $\sum_{i \in V} x_{ij} - \sum_{i \in V} x_{ji} = 0 \quad \forall j \in V$   
 $\sum_{i \in V} x_{it} = 1$   
 $0 \le x_{ij} \le f_{ij}, \forall i \in V, \forall j \in V$   
 $0 \le x_{sj} \le 1, \forall j \in V$   
 $0 \le x_{it} \le 1, \forall i \in V$ 

x<sub>ij</sub> ∈ Z, ∀i ∈ V ∪ {s}, ∀j ∈ V ∪ {t}
ii) The path does not give an optimal solution

of problem (P).

The connectivity of graph is easy to check within  $O(n^2)$ . We now denote the number of arcs in the Eulerian path in the graph of solution of  $(RP_0)$  as  $N_0$ . For the case ii), in order to find the alternative of the solution, a new problem  $(RP_1)$  is generated by adding a constraint such that there is at least an arc from the Eulerian path (including super vertices, *s* and *t*) to the outside. Let the objective value of  $(RP_1)$  be  $N_1$ . If  $N_1 < N_0$ , then the solution of  $(RP_0)$  is optimal for problem (P). The algorithm of finding the optimal solution is illustrated in Fig. 2. We call this procedure the LP-based branch-and-bound method. A state b) corresponds to a branch operation. The branch operation continues until the connected graph is obtained or no solution is found in LP solver.



Figure 2: LP-Based Branch-and-Bound Method.

 $(\operatorname{RP}_{k}) \operatorname{Maximize} z_{0} = \sum_{i \in V \cup \{s\}} \sum_{j \in V \cup \{t\}} x_{ij}$ Subject to :  $\sum_{i \in V} x_{si} = 1$  $\sum_{i \in V} x_{ij} - \sum_{i \in V} x_{ji} = 0 \quad \forall j \in V$  $\sum_{i \in V} x_{ii} = 1$  $\sum_{i \in V_{i}^{i}} x_{ij} \ge 1, l = 0, 1, \dots, k - 1$  $0 \le x_{ij} \le f_{ij}, \forall i \in V, \forall j \in V$  $0 \le x_{sj} \le 1, \forall j \in V$  $0 \le x_{ii} \le 1, \forall j \in V$  $x_{ij} \in \mathbf{Z}, \forall i \in V \cup \{s\}, \forall j \in V \cup \{t\}$ 

In the solution of RP<sub>k</sub>,  $V_k^*$  means a set of vertices on a Eulerian path from s to t.  $V \setminus V_k^*$  means the all vertices except  $V_k^*$ . So the conditions adding to past solutions mean that there is at least an arc from vertices on the semi-Eulerian graph to the outside vertices. Two cases, i) and ii), partition the problem into two. We can find the problem when IP solver stops whether the graph of the solution is connected or the solver cannot find the optimal solution because of the severe conditions added.

Note that the solution of  $(RP_0)$  is always integer solution, because the coefficient matrix of constraints of  $(RP_0)$  is a totally unimodular matrix. However,  $(RP_k)$  is not.

# 4 CONSTITUTION OF SHIRITORI SEQUENCE

After executing the method described in section 3, the number of words between vertices is available. The number of words of a longest SHIRITORI sequence is already founded until this point. In addition, we constitute the concrete word sequence from the transition graph. Generally, a transition graph includes numerous sequences. But we would like to line words up in a sequence. The algorithm for extracting an Eulerian path is below and Fig.3 shows an example of the process.



a) Example of Eulerian Transition Graph



Figure 3: Making SHIRITORI sequence.

T: a transition graph of the longest SHIRITORI

- L: loop sequences
- **R: SHIRITORI sequence**
- 1) All Loop are extracted from T, and put them to L.
- 2) A sequence, not including loops, is remained in T, and put it to R. In this time, T becomes 0.
- 3) Insert an arbitrary loop in the L which starts a vertex in R.
- 4) Repeat 3) while L is not 0.

As described in the experiment section, there are many largest semi-Eulerian subgraphs from a word dictionary and many SHIRITORI sequences for a semi-Eulerian subgraph. Since what kind of sequence is better depends on the application used, we do not deal with the quality of SHIRITORI sequences in this paper.

#### **5 LOCAL SEARCH METHOD**

In this paper, we compare the IP-based method with the local-search method. Usually, a top-down brute-force search method, where a SHIRITORI sequence is created from the first character to the last one, is not suitable for the longest SHIRITORI problem, because the depth of search tree is too deep to try alternatives. Instead of such a brute-force search method, we use the local search method. The local search method is previously applied to puzzles like N-queen (Sosic, 1990). Fundamentally, two component from different sources are exchanged like genetic cross-over operation. Of course, there is a risk to fall the solution into the local optimal solution. The local optimal solution is a solution that there are not any improved solutions around it. To avoid getting local optimal solutions, we use a TABU search technique which controls the next candidate of exchainging by the history. The TABU search avoid exchaining the same items repeatedly. An example of exchainging routes are shown in Fig.4 and we describe the algorithm below:



Figure 4: Example of Local Search.

T: a initial transition graph

U: a Eulerian transition graph

U(i,j): the value of the matrix from i to j

TA: a tabu list. In case that the

queue overflows, the most past item is throwed.

- 1) U=0, U(s,t)=1
- 2) Find a path P1 from the shortest path from i to j on U. In this case, (i,j) is not a member of TA. Find a longer path P2 than P1 from i to j on T. If P2 exists, exchange P1 on U and P2 on T and push (i,j) to TA and goto 2).
- Find a path P1 from the shortest path from i to j on U. In this case, (i,j) is a member of TA. Find

the longer path P2 than P1 from i to j on T. If P2 exists, exchange P1 on U and P2 on T and goto 2).

- 4) Find a path P1 from the shortest path from i to j on U. In this case, (i,j) is not a member of TA. Find a path P2 as same length as P1 from i to j on T. If P2 exists, exchange P1 on U and P2 on T and push (i,j) to TA and goto 2).
- 5) Output U.

# **6 EXPERIMENTS**

We experiment our method for the longest SHIRITORI problem using actual words in dictionary and virtual small words. All experiments are done under the environment as below:

PC: Xeon Dual Processor 2.8GHz, 2GB memory IP solver: GNU GLPK(ver. 4.2) OS: cygwin (ver. 2.218)under WINDOWS XP Compiler: GNU GCC(ver. 3.2)

#### 6.1 Experiment for Word Dictionary

First, we use actual words in KOUJIEN (Japanese Dictionary)(Niimura, 1992). Three kinds of datasets below are used in this experiment. The experimental results for these three datasets are shown in Table.1, 2 and 3. In all cases, the number of vertices is 70.

Dataset 1: All words except word with voice consonants as the beginning characters, like, DA,DI, ZO, and so on. The total number of words is 137,335.

- Dataset 2: All nouns are used. The total number of words is 192,687.
- Dataset 3: All nouns are used. But the same transcribed words are omitted. The total number of words is 154,150.

In Table.1, 2 and 3, fractions are omitted after divided by 2 for each arc. The result shows that the ratio of longest SHIRITORI sequences is not constant. It is because the number of arcs between vertices is not distributed uniformly. From our commonsense, the number of words deeply affects the length of SHIRITORI sequences. From our experimental results, when the number of words is large, the ratios between the number of words and the length of the longest SHIRITORI sequences appear to be approximately constant. However, when the number of words is small, the ratios are not constant. This is considered because of the connectivity of vertices. The difference between the numbers of arcs affects the length of the longest SHIRITORI sequences. In the previous research, the length of Eulerian sequences is estimated to be 2/3 (Li, 2004) for undirected graph. The length of the longest SHIRITORI sequences seems to be shorter than undirected graphs.

Table 1: Result (Dataset 1)

			/	
Num. of	Length of	Ratio	Num. of	Time
Word(N)	SHIRITORI(L)	(L/N)	times IP	(sec)
137,335	56,519	41%	1	0.53
68,417	27,718	41%	1	0.49
33,497	13,339	40%	1	0.47
16,079	6,183	38%	1	0.39
7,406	3,654	36%	1	0.30
3,219	1,016	32%	1	0.20
1,265	303	24%	1	0.16
444	70	16%	1	0.13
124	15	12%	1	0.11

Table 2: Result (Dataset 2)

Num. of	Length of	Ratio	Num. of	Time	
Word(N)	SHIRITORI(L)	(L/N)	times IP	(sec)	
192,687	86,788	45%	2	1.36	
95,225	42,236	44%	1	1.20	
46,596	20,077	43%	1	1.02	
22,351	9,115	41%	3	0.84	
10,361	3,792	37%	1	0.50	
4,535	1,372	30%	1	0.27	
1,853	401	22%	1	0.17	
697	91	13%	1	0.12	
231	18	8%	1	0.11	

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Num. of	Length of	Ratio	Num. of	Time	
Word(N)	SHIRITORI(L)	(L/N)	times IP	(sec)	
154,150	75,777	50%	1	1.30	
75,984	36,759	48%	1	1.16	
36,979	17,340	47%	1	0.97	
17,543	7,769	44%	1	0.70	
7,987	3,199	40%	1	0.47	
3,396	1,130	34%	1	0.25	
1,290	292	23%	1	0.14	
446	62	14%	1	0.12	
123	12	10%	1	0.09	

In our experiments, IP solver was activated once in almost cases. Even if the solver is called twice or more, the CPU execution time is short. This is mainly because of the performance of GNU GLPK. The reason why the number of times IP is called is almost 1 seems that the longest SHIRITORI consisting of loops includes vertices which are shared in other loops. In other word, it is difficult to make isolated loops from initial transition graphs made from an actual dictionary.



Figure 5: Performance of Local Search.

Fig.5 shows the performance of the local search described in the section 5. From this, we could find the optimal solution for the dataset 1 but not for the dataset 2. Our local search method cannot judge whether the final solution is optimal or not. This result shows that the performance of the local search is deeply dependent on datasets. However, since the optimal solution can be found in a dataset, the longest SHIRITORI problem seems not to be difficult.

Table 4: Result of Specific Beginning and Ending

Length	Kinds of	Kinds	Kinds	Example.
of	pairs of	of beg.	of end.	
SHIRIT	beg. and			
ORI	end char.			
56,519	51	3	17	A.GU, HA,GO
56,518	197	13	26	MO.BE.FU.GE
56,517	389	29	35	MU.ZA,MA.BI
56,516	424	37	53	RO.BI,RI.ZA
56,515	449	38	65	RO.DE,WA.KU
56,514	504	38	53	WA.RE,YO.NO
56,513	395	35	44	WA.TA,MA.O
56,512	194	25	35	RI.SE,MU.HI
56,511	52	9	17	RO.HO,I.HA
56,510	5	1	5	RY.A,RU.HE

Table 4 shows the lengths of the longest SHIRITORI sequences for the specific beginning and terminating characters about the dataset 1. The length of these longest SHIRITORI sequences becomes over 56,510 or 0. Here, when the beginning character is the same as the terminating character (loop), the longest SHIRITORI sequence become the closed longest Eulerian circuit and the length is 56,514 in this case. This result shows loops are mainly composed of the longest SHIRITORI sequences. The difference between the lengths of longest SHIRITORI sequences comes from the points entering or getting out of the loops.

# 6.2 Comprehensive trials for the small longest SHIRITORI problem

In the previous section, the experimental results show that the branch-and-bound procedure was not activated in almost cases. This means that it is enough to solve the longest SHIRITORI problem by using the relaxation problem which is not concerned of the connectivity of graph. In this section, we investigate the nature of the longest SHIRITORI problem by the small size of comprehensive transition graphs. Table 5 shows the result of this. The size is the number of vertices and the transition graphs satisfy the following conditions. These conditions exclude the redundancies of transition matrix.

$$f_{ij} = \{0,1\}$$

$$\sum_{i=1}^{n/2} \sum_{j=1}^{n} f_{ij} < \sum_{i=n/2+1}^{n} \sum_{j=1}^{n} f_{ij}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n/2} f_{ij} < \sum_{i=1}^{n} \sum_{j=n/2+1}^{n} f_{ij}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{i} f_{ij} < \sum_{i=1}^{n} \sum_{j=i+1}^{n} f_{ij}$$
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There is at least a path from the first character to other characters.

The table 5 shows the number of times IP solver is called, which means the depth of the branch-and-bound method. From this table, we can find the difficulty of the longest SHIRITORI problem. The relaxation problem which describes node conditions is enough to solve the longest SHIRITORI problem in almost transition graphs. As, however, the number of vertices increases, the number of times IP solver is called also tends to increase. By contrast, when the number of times is small, the improved results, compared with the early results, can be obtained. In addition to this, after IP is called many times, the IP cannot find the solutions because of the severe conditions of the connectivity. From this result, IP is not called many times against the size of transition matrix.

Size of	The Number of Times				
Problem	1	2	3	4	6
3	8	0	0	0	0
4	291	0	0	0	0
5	29,869	4	0	0	0
		0		4	
6	11,338,759	5903	3315	20	0
		0	0	558	4

Table 5: The number of Times IP solver is called

Upper line: the number of improved solution by branch-and-bound.

Lower line: the number of solution where conditions become severe.



Figure 6: Improved Solution by Branch-and-Bound.

Fig.6 shows an example of the transition graphs obtained as the longest SHIRITORI sequences. Two separated graphs are obtained at the first solution. By adding a constraint, the optimal solution is found. In this case, the length of the longest SHIRITORI sequence is improved from 10 to 12. Apparently, the number of words of the first solution gives the upper bound of the number of words of the longest SHIRITORI sequence. Though the number of words is the same between two transition graphs in this example, it is possible to decrease the number of words after the branch-and-bound method is applied.

At the problem  $(RP_0)$ , we can always get the integer solution from any transition matrix. But by adding constraints about connectivity in (RPk), it is not guaranteed to get integer solutions. In our system, GNUS GLPK is running on as the integer problem mode after the linear problem mode. We checked the number of transition graphs which output non-integer solutions after the linear programming mode. There are no transition graphs which output the non-integer solutions! This might mean another index showing the easiness of the longest SHIRITORI problem. When we consider the another problem like the longest SHIRITORI word sequence problem, calculating the longest character length of SHIRITORI sequence, the branch-and-bound method would work more to find solutions.

#### 7 CONCLUSIONS

This paper describes the definition of the longest SHIRITORI problem and the solution using the integer programming and the LP-based branch-and-bound method. The length of the longest SHIRITORI problem becomes over 40 percents for words in the Japanese dictionary. This result surprises us very much, because usual human SHIRITORI sequences end up to several tens of words.

As future works, we try to solve the more difficult problem like the longest SIRITORO word problem described in the previous section shortly and the applications of the longest SHIRITORI problem. For examples, by applying the longest SHIRITORI problem, the system makes the game time long. It is possible to keep up human interest against the game by this. For another application, our method is useful for the complete analysis of N-players SHIRITORI game. As, actually, we can find the longest or the shortest SHIRITORI word sequences in which n-th player is defeated by adding the several conditions. This would be useful for finding the solution of N-players SHIRITORI game.

### ACKNOWLEDGEMENT

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#### APPENDIX

An example of the longest SHIRITORI sequence of the dataset 1

1:ABEMARIA,ANMONIA,ANPEA,ANPURIFAIA 5:ANFEA,ANPAIA,ANTIOKIA,ANDARUSIA, 9:ANDAWEA,ANTAKIA

56511:YOUSUMONO,NOAZAMI,MIATUME, 56514:MEISIIRE,REIIKI, 56516:KIETUKU,KUIRA,RAIHARU,RUMONDO

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