STABILIZING CONTROL FOR HIGHER ORDER SYSTEMS VIA REDUCED ORDER MODEL- A PASSIVITY BASED APPROACH

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Abstract: In this paper a methodology for design of stabilizing control for high order system via reduced order model is presented. In the first part a method is proposed for the reduction of original higher order passive system to a lower order stable model, using this reduced order model, a strictly passive controller of order equal to that of reduced order model is designed. It is shown that this lower order controller designed from reduced order model when applied to original higher order system results in to close loop stability.

1 INTRODUCTION

The reduced model makes the synthesis and analysis of controller simpler so the reduction of high order systems to a reduced order system has been a topic of interest of many researchers. However, the controller designed from reduced order model do not guarantee stability of resulting closed loop when it is applied to original higher order system. This problem of guaranteed stabilization of original system has been addressed by very few researchers such as (Bandyopadhyay et al, 1998), (Lamba and Rao, 1974), (Chidambara and Schanker, 1974). In this paper a methodology for lower order controller design is proposed, theory is developed to show that the lower order stabilizing passive controller designed for the reduced model by proposed method stabilizes original passive system.

The rest of the paper is organized as follows: Section 2 reviews theory of passivity and passivity based control. In Section 3 the conditions are derived under which the given lower order controllers are strictly-passive. In Section 4 new methods for passive system reduction preserving stability is presented with two numerical examples. In Section 5, new methodology for low-order controller design is described and based on this methodology a numerical example of low order controller design for higher order system is illustrated followed by conclusion.

2 THEORY OF PASSIVITY

In this section we will review the theory of passivity (Guillemin, 1957)-(Yengst, 1964),(Braess, 2003), (Van Der Schaft, 1999), (Lozano-leal and Joshi, 1988), (Wen, 1988), (Lozano-leal and Joshi, 1990) and (Tao and Ioannou, 1990). For a given transfer function it is possible to synthesize the network using the passive circuit components only if the given transfer function satisfy certain conditions. These conditions are known as the realizability conditions for the given transfer function. Any transfer function is realizable iff

- Numerator and denominator polynomials are Hurwitz.
- The given transfer function is positive real.

2.1 Positive real function

In this sub-section we will state various definitions, theorem and corollary related to the positive real function (Lozano-leal and Joshi, 1990; Tao and Ioannou, 1990).

Definition 1 : Let $H(s)$ be a rational function □

1. If $H(s)$ is positive real, then it has no poles and zeros in $\mathbf{C}^+$
2. Any pole and zero on the imaginary axis is simple and have positive real residue.

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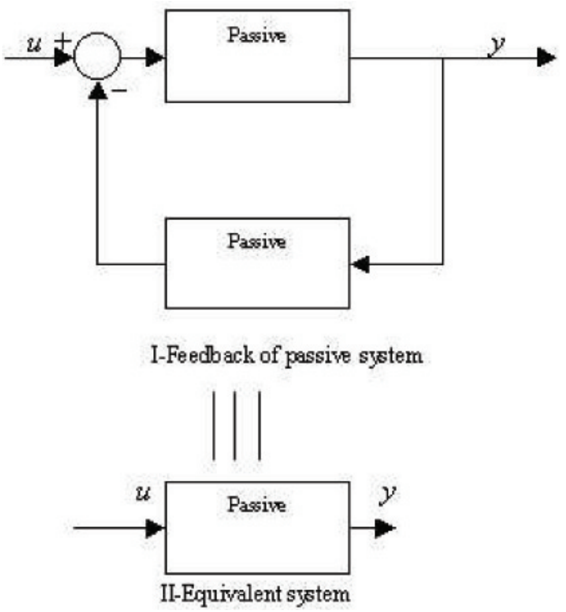

Figure 1: Feedback interconnection of passive systems and their Equivalence

3. The function $H(s)$ is positive real iff it has no poles in $\mathbf{C}^+$ and $\mathrm{Re}(H(j\omega)) \geq 0$ for all $\omega \in R$.

Definition 2 : A rational transfer function $H(s)$ is strictly positive real function iff □

1. All elements of $H(s)$ are analytic in $\mathrm{Re}(H(j\omega))$
2. $H(j\omega) + H^*(j\omega) \gneqq 0$ for all $\omega \in R$.
3. Strong definition imposes condition as $\lim_{\omega\to\infty} \omega^2(H(j\omega) + H^*(j\omega)) \gneqq 0$

Theorem 1 : A positive real function $Z(s)$ cannot have any poles or zeros in the r. h. s. plane. j axis poles of $Z(s)$ and $\frac{1}{Z(s)}$ must be simple with real positive residues. □

Theorem 2 : If $Z(s)$ is prf, the degree of the numerator cannot differ from that of the denominator by more than unity. □

2.2 Passivity based stabilizing control

Any system $H(s)$ satisfying Definition 1 or Definition 2 is passive system. Passivity based control is a methodology which consist in controlling a system with the aim at making the closed loop system, passive(M. Vidyasagar, 1983).

Theorem 3 : Consider two passive systems interconnected as shown in Figure 1. If one of the system is strictly passive and another strong strictly passive then the resulting close loop system will always be stable. □

This theorem allows a passivity based stability analysis(M. Vidyasagar, 1983). Alternatively it can be stated that a negative feedback loop consisting of two passive systems is passive(Sepulchre et al, 1997).

3 LOWER ORDER PASSIVE CONTROLLERS

In this section we will derive the conditions under which the lower order controller is strictly passive. We will restrict this discussion to third order controller. These conditions are extremely important in the design of lower order controller using the proposed method of controller synthesis. For the derivation of these condition we are referring the spr condition given in definition 2.

3.1 First order system

Conditions under which a first order system is spr are simple as it has only three parameters. Let the system be of the form

$$C_1(s) = \frac{x_0}{y_0 + y_1 s} \tag{1}$$

for this system to be spr the necessary and sufficient condition are met by $x_0, y_0$and y_1 being greater than zero.

3.2 Second order system

Let the second order strictly proper system be

$$C_2(s) = \frac{x_0 + x_1 s}{y_0 + y_1 s + y_2 s^2} \tag{2}$$

The $C_2(s)$ will be spr if $(C_2(j\omega)C_2^*(j\omega)) > 0$ for all $\omega \in R$ i. e

$$\frac{x_0 + x_1 s}{y_0 + y_1 s + y_2 s^2} + \frac{x_0 - x_1 s}{y_0 - y_1 s + y_2 s^2} > 0$$

$$\frac{2x_0 y_0 + 2(x_0 y_2 - x_1 y_1)s^2}{y_0^2 + 2y_2 y_0 s^2 + y_2^2 s^4 - y_1^2 s^2} > 0$$

$$\frac{2x_0 y_0 + 2(x_1 y_1 - x_0 y_2)\omega^2}{y_0^2 + (y_1^2 - 2y_2 y_0)\omega^2 + y_2^2 \omega^4} > 0$$

This condition will be true when both the numerator and denominator are of the same sign. We are restricting ourselves to the case when all the coefficients of the controller are greater than zero. Thus, conditions satisfying above inequality are $x_0, x_1, y_o, y_1$and $y_2$be positive with

$$x_1 y_1 \geq x_0 y_2 \text{ and } y_1^2 \geq 2y_2 y_0 \tag{3}$$

3.3 Third order controller

Let the system be of the form

$$C_3(s) = \frac{x_0 + x_1 s + x_2 s^2}{y_0 + y_1 s + y_2 s^2 + y_3 s^3} \quad (4)$$

Now again by spr definition, the conditions under which the $C_3(s)$ is spr are

$$\begin{aligned} 0 &< y_0, y_1, y_2, y_3, x_0, x_1 \text{ and } x_2 \quad (5) \\ x_1 y_1 &> y_0 x_2 + y_2 x_0, \\ x_2 y_2 &> x_1 y_3 \\ y_1^2 &\geq 2 y_2 y_0 \\ y_2^2 &\geq 2 y_1 y_3 \end{aligned}$$

4 A NEW MODEL ORDER REDUCTION TECHNIQUE

An approximation that is frequently used is the Pade technique. The approximated model by Pade technique matches first 2r time moments with the original higher order system, where r is the order of the reduced model. However, the Pade approximation does not guarantee the stability of the reduced model. This problem is addressed in (Shamash, 1975) and overcome by Routh-Pade approximation technique. In this method reduced model matches only initial r time moments with the original system thus compromising with the accuracy of the fit. In (Lepschy and Viaro, 1982) an improvement to this method is suggested to improve the accuracy of the fit, but method is cumbersome and in few cases it's possible to perfectly match only one additional time moment and approximately matching another. In this section a new method is proposed to reduce the order of a linear time invariant higher order stable system, using the Hermite-Biehler stability theorem and Pade approximation. The proposed method not only tackle the problem of the guaranteed stability but it can match additional time moments over the conventional Routh-Pade method.

4.1 The order reduction problem

Let the transfer function of a higher order linear time invariant stable system is given by

$$G(s) = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \cdots + b_n s^n} \quad (6)$$

The order of the original higher order system is n. We want a reduced order model of order r. Thus, the problem is to find the approximated reduced order model of order r such that it matches two additional time moments while preserving the stability.

4.2 Matching additional time moments

Let $G(s)$ be the transfer function of a higher order linear time invariant stable system. Let $D(s)$ be the denominator polynomial of order n and $N(s)$ is the numerator polynomial of order $(n-1)$. Then denominator and numerator of equation 6 can be expressed as

$$\begin{aligned} N(s) &= a_0 + a_1 s + a_2 s^2 + \cdots + a_{n-1} s^{n-1} \\ D(s) &= b_0 + b_1 s + b_2 s^2 + \cdots + b_n s^n \end{aligned}$$

These polynomials can be separated into even and odd parts as follows (Bhattacharya et al, 1995), For n odd

$$\begin{aligned} D^{even}(s) &= b_0 + b_2 s^2 + b_4 s^4 + \cdots + b_{n-1} s^{n-1} \\ D^{odd}(s)/s &= b_1 + b_3 s^2 + b_5 s^4 + \cdots + b_n s^{n-1} \end{aligned}$$

For n even

$$\begin{aligned} D^{even}(s) &= b_0 + b_2 s^2 + b_4 s^4 + \cdots + b_n s^n \\ D^{odd}(s)/s &= b_1 + b_3 s^2 + b_5 s^4 + \cdots + b_{n-1} s^{n-2} \end{aligned}$$

Let $(0 \pm \omega_{e,i}^d)$ and $(0 \pm \omega_{o,i}^d)$ denotes the roots of the $D^{even}(s)$ and $D^{odd}(s)/s$ respectively. Then for the stable plant, by interlacing property, the following condition must be satisfied(Bhattacharya et al, 1995),

$$0 < \omega_{e,1}^d < \omega_{o,1}^d < \omega_{e,2}^d < \omega_{o,2}^d < \omega_{e,3}^d \cdots \quad (7)$$

This concept of interlacing of roots of even and odd polynomials is used to construct a reduced degree stable denominator polynomial as follows:

Write $D^{even}(s)$ and $D^{odd}(s)$ in term of their roots $\omega_{e,1}^d, \omega_{e,2}^d \cdots$ and $\omega_{o,1}^d, \omega_{o,2}^d, \cdots$ For n even

$$\begin{aligned} D_n^{even}(s) &= \prod_{i=1}^{n/2} (s^2 + \omega_{de,i}^2) \\ D_n^{odd}(s)/s &= \prod_{i=1}^{n/2-1} (s^2 + \omega_{de,i}^2) \end{aligned}$$

Now if we want to obtain a reduced r^{th}order model, then the even and odd polynomials for the reduced order denominator polynomial can be written as, for r even

$$\begin{aligned} D_r^{even}(s) &= \prod_{i=1}^{r/2} (s^2 + \omega_{de,i}^2) \quad (8) \\ D_r^{odd}(s)/s &= \prod_{i=1}^{r/2-1} (s^2 + \omega_{de,i}^2) \end{aligned}$$

Using (8) a modified reduced denominator can be constructed as

$$D_{rm}(s) = K_1 D_r^{even}(s) + K_2 D_r^{odd}(s) \quad (9)$$

Where K_1 and K_2 are real numbers and should have same sign so that the denominator polynomial is interlacing and hence stable. Then the higher order system given by equation (6) can be approximated by r^{th} order system as

$$G_{hb}(s) = \frac{x_0 + x_1 s + x_2 s^2 + \cdots + x_{r-1}s^{r-1}}{y_0 + y_1 s + y_2 s^2 + \cdots + y_r s^r} \quad (10)$$

Now, we know that

$$\begin{aligned} D_{rm}(s) &= K_1 \prod_{i=1}^{r/2}(s^2 + \omega_{de,i}^2) \\ &\quad + K_2 s \prod_{i=1}^{r/2-1}(s^2 + \omega_{de,i}^2) \\ &= y_0 + y_1 s + y_2 s^2 + \cdots + y_r s^r \end{aligned}$$

Now, the problem becomes finding $(r+2)$ unknown coefficients of reduced model given in (10), this problem is addressed here with the help of Pade approximation. Let the original higher order system, given by equation (6), be represented as

$$G(s) = c_0 + c_1 s + \cdots + c_r s^r + c_{r+1}s^{r+1} + \cdots \quad (11)$$

Now, taking the power series expansion of the reduced model given by equation (10), around $s = 0$ and equating equal powers of s we get

$$\begin{aligned} x_0 &= y_0 c_0 \quad (12) \\ x_1 &= y_0 c_1 + y_1 c_0 \\ x_2 &= y_0 c_2 + y_1 c_1 + y_2 c_0 \\ x_{r-1} &= y_0 c_{r-1} + y_1 c_{r-2} + \cdots + y_{r-1}c_0 \\ 0 &= y_0 c_r + y_1 c_{r-1} + y_2 c_{r-2} + \cdots + y_r c_0 \\ &\vdots \\ 0 &= y_0 c_{r+2} + y_1 c_{r+1} + \cdots + y_{r-1}c_{r-1} \end{aligned}$$

Where, $C_i's$ are the coefficient of the equation (11) above. The above set of equations can be written in matrix form as

$$\begin{bmatrix} c_r & c_{r-1} & .. & c_1 \\ c_{r+1} & c_r & .. & c_2 \\ c_{r+2} & c_{r+1} & .. & c_{r-1} \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ : \\ y_{r-1} \end{bmatrix} = \begin{bmatrix} -c_0 \\ -c_1 \\ : \\ -c_{r-1} \end{bmatrix} \quad (13)$$

And

$$\begin{bmatrix} c_0 & 0 & .. & 0 \\ c_1 & c_0 & .. & 0 \\ c_{r-1} & c_{r-2} & .. & c_0 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ : \\ y_{r-1} \end{bmatrix} = \begin{bmatrix} -x_0 \\ -x_1 \\ : \\ -x_{r-1} \end{bmatrix} \quad (14)$$

It must be noted that in the above transformation of equation (12) to (14) that $y_r = 1$. The solutions to the equations (13) and (14) gives the coefficients of the reduced r^{th} order model for the given n^{th} order system.

Now, suppose that given higher order system is to be reduced to a 3^{rd} order system. Then even and odd parts of denominator can be written as

$$\begin{aligned} D_r^{even}(s) &= (s^2 + \omega_{de,1}^2) \\ D_r^{odd}(s)/s &= (s^2 + \omega_{do,1}^2) \end{aligned}$$

Hence, from equation (9)

$$\begin{aligned} D_{rm}(s) &= K_1(s^2 + \omega_{de,1}^2) + K_2 s(s^2 + \omega_{do,1}^2) \\ D_{rm}(s) &= K_1\omega_{de,1}^2 + K_2\omega_{do,1}^2 s + K_1 s^2 + K_2 s^3 \end{aligned}$$

This gives guaranteed stability for any value of K_1 and K_2 such that ratio K_1/K_2 is positive. Then this parameterized equation can be used to match two additional time moments exactly and approximately match the third.

$$D_{rm}(s) = b_0 + b_1 s + b_2 s^2 + b_3 s^3$$

So we have, $b_0 = K_1\omega_{de,1}^2$, $b_1 = K_2\omega_{do,1}^2, b_2 = K_1$ and $b_3 = K_2$. Observe that b_0 and b_2 are linear combination of K_1, where as b_1 and b_3 are linear combination of K_2. Lets assume $\omega_{de,1}^2$ to be unknown. Then using constraint optimization equations (13) and (14) can be solved for K_1, K_2 and $\omega_{de,1}^2$ such that

$$F = -b_0 c_5 + b_1 c_4 + b_2 c_3 - b_3 c_2$$

is minimum and the solution set satisfy the constraints for stability, that is, K_1 and K_2 have same sign and $0 < \omega_{e,1}^d < \omega_{o,1}^d$, which ensure the Hermite-Beihler stability of denominator polynomial as interlacing is preserved. Then $(r+2)$ moments of the approximated system will exactly match with the original higher order system while $(r+3)^{rd}$ will match approximately. Under this condition the reduced third order model will exactly match initial five time moments where as matching 6^{th} time moment will be matched approximately . The reduced 3^{rd} order model is given by

$$G_{hbr}^3(s) = \frac{x_0 + x_1 s + x_2 s^2}{y_0 + y_1 s + y_2 s^2 + y_3 s^3}$$

4.3 Numerical Examples

In this section we will consider two most critical examples taken in from literature.

4.3.1 Numerical Example 1

Let us consider an example where ordinary Pade approximation technique results in to an unstable model, while the method described gives directly a stable reduced model. The original fourth order system is given by

$$G(s) = \frac{100 + 395s + 527s^2 + 267s^3}{1 + 4s + 6s^2 + 4s^3 + s^4} \quad (15)$$

we will reduce this system to third order model. Now $G(s)$ can be expressed as

$$H(s) = 100 - 5s - 53s^2 + 109s^3 - 198s^4 + \cdots$$

The denominator of the original higher order system is

$$D(s) = 1 + 4s + 6s^2 + 4s^3 + s^4$$

We can write the polynomial in to even and odd parts as following

$$\begin{aligned} D(s) &= (1 + 6s^2 + s^4) + (4s + 4s^3) \\ &= (s^2 + 5.828)(s^2 + 0.1715) + 4s(s^2 + 1) \end{aligned}$$

separating this in to even and odd parts

$$\begin{aligned} D_{even}(s) &= (s^2 + 5.828)(s^2 + 0.1715) \\ D_{rodd}(s)/s &= (s^2 + 1) \end{aligned}$$

It can be easily observed that the system is stable as even and odd roots of this polynomial interlace ie $0 < (0.4141 = \omega_{de,1}) < (1 = \omega_{do,1}) < (2.4141 = \omega_{de,2})$. Now the stable denominator of the reduced order approximation can be obtained by using equation (9) as,

$$\begin{aligned} D_{rm}(s) &= K_1(s^2 + \omega_{de,1}^2) + K_2 s(s^2 + \omega_{do,1}^2) \\ D_{rm}(s) &= K_1\omega_{de,1}^2 + K_2\omega_{do,1}^2 s + K_1 s^2 + K_2 s^3 \end{aligned}$$

Putting $\omega_{de,1}^2 = 0.1715$ and solving equations (13) and (14) under the required stability constraints on we get $K_1 = 3.418, K_2 = 1$ and $\omega_{do,1}^2 = 2.7686$. We have 3rd order reduced model as

$$G_{PR}(s) = \frac{58.53 + 273.93s + 298.037s^2}{0.5853 + 2.7686s + 3.429s^2 + s^3} \tag{16}$$

This model matches initial 5-time moments exactly where as 6^{th} time moment is matched approximately with the original system. Where as, the approximated model by Routh-Pade method for the same system is obtained and is given by

$$G_{RPR}(s) = \frac{100 + 176.25s + 62.937s^2}{1 + 1.8125s + 1.25s^2 + 0.3125s^3}$$

The step responses of original higher order system, approximated model by proposed method, approximated model by Routh-Pade method are plotted in Figure 2. From the response it is clear that proposed method performs good than the conventional Routh-Pade method and it matches 2 more time moments exactly and one approximately over the Routh-Pade method, which matches only initial 3 time moments with the original system.

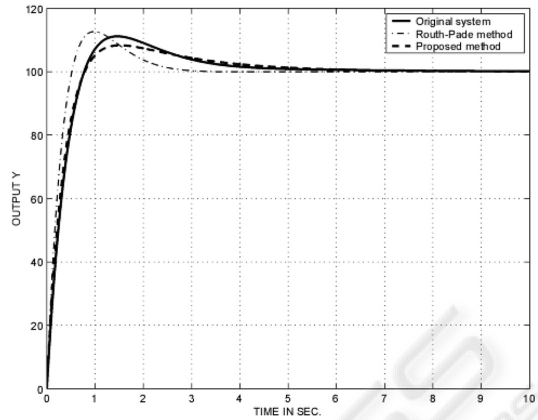

Figure 2: Step responses of original and approximated systems

4.3.2 Numerical Example 2

Now, Consider one more critical example suggested by A. Lipschy and U. Viaro(Lepschy and Viaro, 1982), where classical Pade approximation results into an unstable model and proposed method in to more appropriate reduced stable model. The original higher order system is given by

$$G(s) = \frac{2 + 8s + 8s^2 + 12s^3}{1 + 2s + 12^2 + 4s^3 + 2s^4}$$

Where

$$H(s) = 100 - 5s - 53s^2 + 109s^3 - 198s^4 + \cdots$$

The denominator of the original higher order system is

$$D(s) = 1 + 2s + 12s^2 + 4s^3 + 2s^4$$

We can write the polynomial in to even and odd parts as following

$$\begin{aligned} D(s) &= (1 + 12s^2 + 2s^4) + (2s + 4s^3) \\ &= (s^2 + 5.915)(s^2 + 0.085) + 2s(2s^2 + 1) \end{aligned}$$

splitting this in to even and odd parts

$$\begin{aligned} D_{even}(s) &= (s^2 + 5.915)(s^2 + 0.085) \\ D_{rodd}(s)/s &= (2s^2 + 1) \end{aligned}$$

It can be easily observed that the system is stable as even and odd roots of this polynomial interlace i.e. $0 < (0.085 = \omega_{de,1}^2) < (0.5 = \omega_{do,1}^2) < (5.915 =^2 \omega_{de,2})$. Now the stable denominator of the reduced order approximation can be obtained by using equation (9) as

$$\begin{aligned} D_{rm}(s) &= K_1(s^2 + \omega_{de,1}^2) + K_2 s(s^2 + \omega_{do,1}^2) \\ D_{rm}(s) &= K_1\omega_{de,1}^2 + K_2\omega_{do,1}^2 s + K_1 s^2 + K_2 s^3 \end{aligned}$$

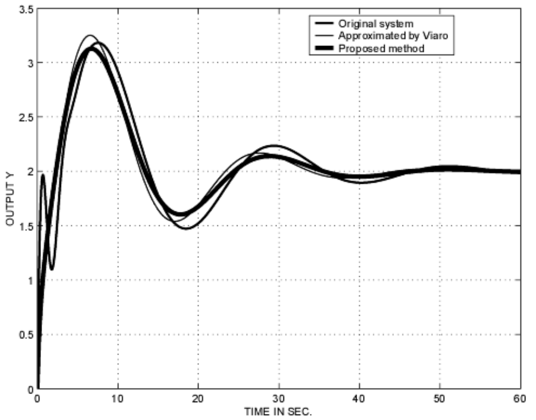

Figure 3: Step responses of original and approximated systems

Putting the values of $K_2 = 1$, $\omega^2_{de,1} = 0.085$, using Pade equations(14) and (13)We have 3rd order reduced model as

$$G_{hb}(s) = \frac{0.626 + 2.7504s + 2.8502s^2}{0.313 + 0.7492s + 3.6872s^2 + s^3}$$

This model matches initial 5-time moments exactly where as 6^{th} time moment is matched approximately with the original system. The approximated model by improved Routh-Pade method for the same system is obtained(Lepschy and Viaro, 1982), given by

$$G_{RPR}(s) = \frac{2 + 8.7309s + 7.4618s^2}{1 + 2.3654s + 11s^2 + 4.3853s^3}$$

The step responses original higher order system, approximated model by proposed method, approximated model by Routh-Pade method are plotted in Figure 3. From these plots it is clear that proposed method performs good even than the improved Routh-Pade method proposed by U. Viaro, it matches 1 more time moment exactly and another time moment approximately, over the improved Routh-Pade method, which matches only initial 4 time moments with the original system.

5 DESIGN OF LOW ORDER CONTROLLER

In this section an approach to design low order controller from reduced order model of the original plant is described. This method gives r^{th} order controller for n^{th} order plant, that is, the order of the controller is equal to the reduced order of the plant.

Let the controller be of the form

$$C_r(s) = \frac{N_c(s)}{D_c(s)} = \frac{x_0 + x_1 s + x_2 s^2 + .. + x_{r-1}s^{r-1}}{y_0 + y_1 s + y_2 s^2 + .. + y_r s^r} \quad (17)$$

and let the reduced order model be

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{c_0 + c_1 s + c_2 s^2 + .. + c_{r-1}s^{r-1}}{d_0 + d_1 s + d_2 s^2 + .. + d_r s^r} \quad (18)$$

Then, the characteristics equation of the closed loop is as

$$Q(s) = N_c(s)N_r(s) + D_c(s)D_r(s) \quad (19)$$

let it be in the following form

$$(s^2 + \lambda_1 s + \lambda_2)(s^r + \alpha_1 s^{r-1} + \alpha_2 s^{r-2} + \cdots + \alpha_r) \quad (20)$$

where λ_1 and λ_2 are free variables greater than zero while α_1, $\alpha_2 ....\alpha_r$ can be fixed. Equating the coefficients of s we obtain the coefficients of $C_r(s)$ in terms of λ_1 and λ_2 as

$$[A] \;\; [B] = [C] \begin{bmatrix} 1 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} \quad (21)$$

where A is $(2_{r+1}) \times (2_{r+1})$ non singular matrix whose inverse exist, B is $(2_{r+1}) \times 1$ matrix and C is $(2_{r+1}) \times 3$ matrix and are given by

$$A = \begin{bmatrix} d_r & 0 & 0 & 0 & . & 0 \\ d_{r-1} & d_r & 0 & 0 & . & 0 \\ d_{r-2} & d_{r-1} & c_{r-1} & 0 & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & c_0 & c_1 \\ 0 & 0 & 0 & . & 0 & c0 \end{bmatrix}$$

$$B = \begin{bmatrix} y_r \\ y_{r-1} \\ y_{r-2} \\ . \\ . \\ x_1 \\ x_0 \end{bmatrix}$$

and

$$C = \begin{bmatrix} 1 & 0 & 0 \\ \alpha_1 & 1 & 0 \\ \alpha_2 & \alpha_1 & 1 \\ . & . & . \\ . & . & . \\ 0 & \alpha_r & \alpha_{r-1} \\ 0 & 0 & \alpha_r \end{bmatrix}$$

In this way the coefficients of controller are obtained in terms of λ_1 and λ_2,the free variables. Now we can choose these two parameters in such a way that the resulting controller is spr system. Thus, when it is applied to the original higher order passive system it will stabilize it.

5.1 Numerical example

Here will consider a numerical example for the design of lower order controller using the proposed method, where a higher order system is reduced using the proposed methods. We will use proposed method of order reduction. Design a third order controller for a PR system given by

$$G(s) = \frac{100 + 395s + 527s^2 + 267s^3}{1 + 4s + 6s^2 + 4s^3 + s^4}$$

Here we have to design a third order controller. The order of controller designed by proposed method is equal to the order of the model, so we will reduce the given system to third order model. Using this model a third order spr controller can be designed.

In previous section, we have reduced this system given by equation (15) to a third order model given by equation (16), the reduced model is

$$G_r(s) = \frac{58.53 + 273.93s + 298.03s^2}{0.5883 + 2.7686s + 3.429s^2 + s^3}$$

Here, the reduced model is stable. Now, we will design a stabilizing strictly passive controller for this model. Let the controller be of the form

$$C_3(s) = \frac{N_c(s)}{D_c(s)} = \frac{x_0 + x_1 s + x_2 s^2}{y_0 + y_1 s + y_2 s^2 + y_3 s^3}$$

Then the characteristics equation of the closed loop becomes

$$Q(s) = N_c(s)N_r(s) + D_c(s)D_r(s)$$

Let it be equal to

$$(s^2 + \lambda_1 s + \lambda_2)(s^4 + +\alpha_1 s^3 + \alpha_2 s^2 + \alpha_3 s + \alpha_4)$$

lets assume that the four fixed closed loop poles to be at -1, -2, -3, -4. This gives $\alpha_1 = 10, \alpha_2 = 35, \alpha_3 = 50, \alpha_4 = 24$. Thus from equation(21) we have

$$\begin{aligned} A &= \begin{bmatrix} A_1 & A_2 \end{bmatrix} \\ A_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3.429 & 1 & 0 & 0 \\ 2.769 & 3.429 & 1 & 298 \\ 0.585 & 2.769 & 3.429 & 1 \\ 0 & 0.588 & 2.769 & 3.429 \\ 0 & 0 & 0.588 & 2.769 \\ 0 & 0 & 0 & 0.588 \end{bmatrix} \\ A_2 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 58.53 & 298 & 0 \\ 58.53 & 273.9 & 298 \\ 0 & 58.03 & 273.9 \\ 0 & 0 & 58.03 \end{bmatrix} \end{aligned}$$

and

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 10 & 1 & 0 \\ 35 & 10 & 1 \\ 50 & 35 & 10 \\ 24 & 50 & 35 \\ 0 & 24 & 50 \\ 0 & 0 & 24 \end{bmatrix}$$

we get

$$\begin{aligned} y_3 &= 1 \\ y_2 &= 6.5710 + \lambda_1 \\ y_1 &= 26.77 - 65.75\lambda_1 + 301.66\lambda_2 \\ y_0 &= -0.05 + 0.24\lambda_1 - 1 \\ x_2 &= 0.3375 - 1.05\lambda_1 + 3.48\lambda_2 \\ x_1 &= -0.2691 + 1.07\lambda_1 - 4.11\lambda_2 \\ x_0 &= 0.0006 - 0.0025\lambda_1 + 0.4238\lambda_2 \end{aligned}$$

Here free parameters λ_1 and λ_2 can be chosen such that the resulting controller is passive. Thus by referring the passivity condition for third order system given by equation(5) and choosing these two free variables λ_1 and λ_2 (both positive) to be $\lambda_1 = 0.4$ and $\lambda_2 = 0.03$ we get

$$\begin{aligned} y_3 &= 1, y_2 = 6.97,\ y_1 = 9.519, y_0 = 0.0095 \\ x_2 &= 0.018, x_1 = 0.6359,\ x_0 = 0.0123 \end{aligned}$$

Thus the third order spr controller obtained is

$$C_3(s) = \frac{N_c(s)}{D_c(s)} = \frac{0.0123 + 0.6359s + 0.018s^2}{0.009 + 9.519s + 6.97s^2 + s^3} \tag{22}$$

This low order strictly passive controller designed from the reduced order model will stabilize the model and the original higher order passive system.

6 CONCLUSION

Most modern robust controller design methods normally result in a complex controller. The controller so designed generally has an order atleast equal to that of the original system. Thus, the reduction of high order system to a lower order model is necessary. However, the reduced order model must capture the essential properties of the original higher order system. In control system design stability of the system is most important where as least possible error estimate is preferred. Here in this paper, this issues is addressed by developing an improved method for system reduction. From the illustrated examples it is observed that reduced model by this method not only preserve the stability but also has same dynamic response.

If a controller is designed from the reduced model, it does not guarantee the close loop stability when it is applied to the original system. In this the higher order system is reduced to a lower order model. A stabilizing passive low order controller for the stable reduced order model is designed, which when applied to original higher order passive system results in a stable closed loop. Though, the proposed method is applied for the order reduction in this paper, any stability-preserving system reduction method can be applied for this purpose.

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