

# AUTOLOCALIZATION USING THE CONVOLUTION OF THE EXTENDED ROBOT

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Abstract: In order to construct autonomous robots which they move in a indoor environment, it is necessary to solve several problems such as the autolocalization. The problem of the autolocalization in a robot mobile consists of it must find its location within an apriori known map of its surroundings using the perceived distances by its sensors. The difficulties come from the fact that the signals of the sensors have noise, as well as the control signals and also the map could differ from the reality of the surroundings.

The method which we presented joins the measures of the sensors and the signals of control in the called map of the extended robot; through of the convolution of this map and the a priori map of the environment, we can find the best matching between them, after a search into this calculated values, the location is obtained as a configuration that corresponds to the global maximum convolution.

The method was implemented in an sonar-based robot, with kinematics differential. The results have validated widely our proposal.

## 1 INTRODUCTION

During the last years, mobile robots have been thought to perform task in an autonomous way or work at high risk environment. In this way, the autolocalization is considered as one of the most important ability to be implemented.

Ones developed works to solve the autolocation, are based in probabilistic filters; the most referenced author is Kalman(Bar-Shalom and Li, 1995). Although one of its main problems is that the signals caught by the sensors differ from a signal with Gaussian error.

In additional, the Monte Carlo algorithm (S. Turn and Dellaer, 2000) recalculate the new hypothetical states of the position of robot according to the action model (kinematics). Between the disadvantages of the probabilistics methods it is that could not converge to the right solution if the generated hypotheses were not near enough of it.

Many researchers use the icon-based location(G. Schaffer and Stentz, 1992) where the caught information of the surroundings is plotted on a map that is matched with the a priori map of the environ-

ment, based on the minimal distance. The position of the robot is obtained so that the error between the distance of the landmarks is the smallest. A similar approach is the matched based on grid maps (Schiele and Crowley, 1994), the first grid map, centered in the robot and modeling its local environment using the last readings of the sensors, and the second grid map is a global model of the environment. A correlation index is increased when the grids are in the same state and diminishes when they have different states. At the end, with the maximum in the correlation index is the transformation that generates the maximum index, so that we obtain the correspondence between the local and the global map. Until now the matched problem grid to grid has been the processing time due to the high computacional load. The method presented here, perform the matching between the first map of the robot including the readings of the sensors and the second a priori map of the environment through a convolution. We can find the best matching between them, after a search into this calculated values, the location is obtained as a configuration that corresponds to the global maximum convolution. Using the fast Fourier transformed diminishing the

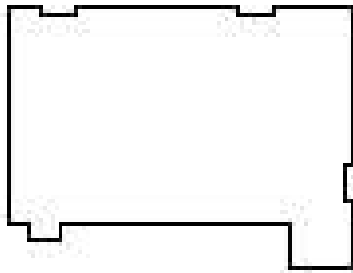


Figure 1: Map of the environment  $W$  in a matrix of 128X128 pixels.

computacional load significantly. In the section 2 we describe process of matching the environment map and the extended robot map, in section 3 we present some improvement to the method and in section 4 we present results and conclusions of our work.

## 2 MATCHING THE ENVIRONMENT AND THE EXTENDED ROBOT MAPS

### 2.1 The environment map

The a priori map is a matrix ( $W$ ), where the obstacles or the walls<sup>1</sup> are ones and the free place are zeros (fig 1).

$$w(x, y) = \begin{cases} 1 & (x, y) \in \text{Obstacle} \\ 0 & (x, y) \notin \text{Obstacle} \end{cases} \quad (1)$$

### 2.2 The extended robot

With the readings of the robot's sensors we can built a map of the position of the obstacles, which we will call the extended robot.

The initial representation of the robot  $P_0$  is determined by a set of points that in this case could be enough to consider only the locations of the sensors ( $s$  elements), because they are in contour of the mobile robot (fig. 2b). Therefore the vector of representative points of the robot  $P_0$  has  $s$  elements  $[p_{0,0}, \dots, p_{s-1,0}]^T$  referred to a local coordinates system<sup>2</sup>.

We define as extended robot to the composition of the robot's point defined by the representative points

<sup>1</sup>In general, word *obstacle* will include walls too.

<sup>2</sup>The center of the robot, the midpoint of the segment that links the two driving wheels, is placed in the coordinates (0,0) and the axis  $x$  oriented to the frontal part of the robot.

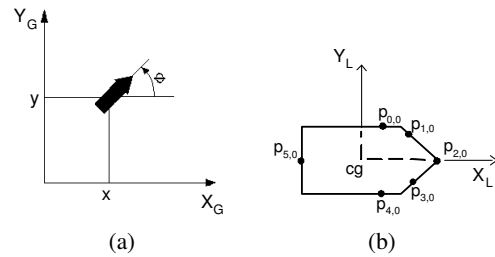


Figure 2: (a) The configuration  $q$  is given by three elements  $x, y, \theta$ . (b) Representative points of the robot  $P_0$  for a configuration  $q_0$ .

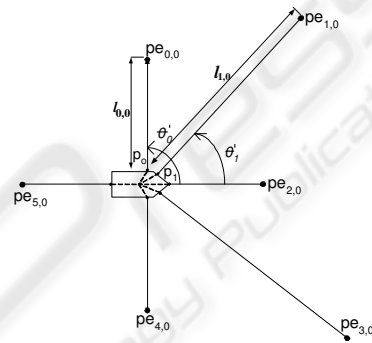


Figure 3: Distances Captured by the Robot

( $p_{i,0} \in P_0$ ) with the distances ( $l_{i,0} \in L_0$ ) captured by the sensors in its direction  $\theta'_i$  in the local coordinates system (figure 3).

The vector of Points of the extended Robot  $PE_0$  ( $[pe_{0,0}, \dots, pe_{s-1,0}]^T$ ) is calculated of the following way:

$$PE_0 = P_0 + L_0 \begin{bmatrix} \cos\theta'_0 & \sin\theta'_0 \\ \dots & \dots \\ \cos\theta'_i & \sin\theta'_i \\ \dots & \dots \\ \cos\theta'_{s-1} & \sin\theta'_{s-1} \end{bmatrix} \quad (2)$$

Finally we can build the Extended Robot Map  $A_0$  like a matrix which size is  $N \times N$  (fig. 4) and it is obtained using the  $PE_0$  vector, of the following way:

$$a_0(x, y) = \begin{cases} 1 & (x, y) \in PE_0 \\ 0 & (x, y) \notin PE_0 \end{cases} \quad (3)$$

As in the location problem we don't know the angle of the robot, we could find one extended robot for each possible orientation of the robot. The considered angles are  $\theta_k = k \cdot 2\pi/N$  where  $k : 0, 1, \dots, (N - 1)$  and  $N$  is the number of possible orientations in the discrete space. Then we must rotate the coordinates

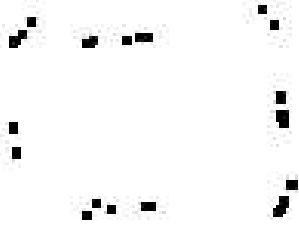


Figure 4: The extended Robot Map ( $A_0$ ) in a matrix of 128X128 pixels..

of  $PE_0$  to obtain a new vector of representative points of the extended robot, according to the following relation:

$$\mathbf{R}(\theta_k) = \begin{bmatrix} \cos\theta_k & \sin\theta_k \\ -\sin\theta_k & \cos\theta_k \end{bmatrix} \quad (4)$$

$$PE_k = PE_0 \cdot \mathbf{R}(\theta_k) \quad (5)$$

The matrix of the extended robot in a  $\theta_k$  orientation is calculate as the equation 3 in the following form:

$$a_k(x, y) = \begin{cases} 1 & (x, y) \in PE_k \\ 0 & (x, y) \notin PE_k \end{cases} \quad (6)$$

### 2.3 Matching Maps by Convolution

The matching between the environment and each extended robot maps is performed using a measurements of similarities based in the convolution of maps.

Each point of the extended robot vector has a bit of information about position of obstacles in the environment, so we can find point of similarities between the robot extended map ( $A_k$ ) and the map of the environment ( $W$ ). The grade of similarity can be estimate putting each map of the extended robot over the map of the environment, in this position, we can calculate the addition of products of each pixel of the extended robot with the under pixel of the environment.

$$c_k(0, 0) = \sum_{i=0}^{i=N-1} \sum_{j=0}^{j=N-1} w(i, j) \cdot a_k(i, j) \quad (7)$$

A new value can be calculate if the extended robot map is moved to a new  $(x, y)$  position over the environment map.

$$c_k(x, y) = \sum_{i=0}^{i=N-1} \sum_{j=0}^{j=N-1} w(i, j) \cdot a_k(i-x, j-y) \quad (8)$$

Using a change of variables we define a new matrix  $A'_k$  where :

$$a'_k(x, y) = a_k(-x, -y) \quad (9)$$

So that the equation 8 in matricial form result in:

$$C_k = A'_k \otimes W \quad (10)$$

Where  $\otimes$  is the convolution 2-dimensional<sup>3</sup>.

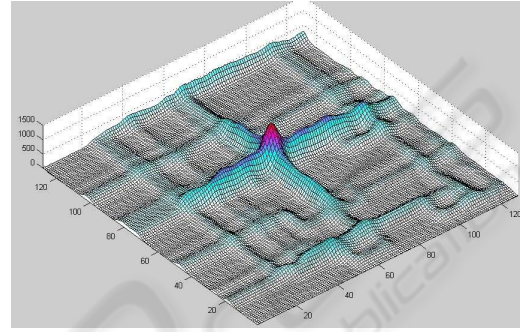


Figure 5: The Convolution Matrix  $C_0$ .

Figure 5 show the convolution matrix  $C_k$  for  $A_0$  and  $W$ . We can see a local maximum value the near to the center of the matrix.

The greater value of  $c_k(x, y)$ , the greater similarity between maps, so we think that the global maximum value in  $C_k$  ( $k : 0, \dots, N - 1$ ) given us the position  $(x, y, \theta_k)$  of the extended robot refered to the environment map.

As the  $C$  matrix is calculated by layers ( $C_k$ ), the process is repeated  $N$  times. In  $C$ , we will do a search for the configurations  $\hat{q}$  that satisfies the condition

$$C(\hat{q}) = \max(C) \quad (11)$$

This configuration will be the location of the robot<sup>4</sup>.

If we considered the maps  $W$  and  $A_k$  as square matrixs  $N$  dimensional, the time required for the calculation of  $C_k$  in 2D is  $O(N^2 \log N)$  and in addition, we must consider  $N$  possible directions, reason why, the operation must be repeated  $N$  times, thus, the time required to calculate  $C$  is  $O(N^3 \log N)$ .

## 3 IMPROVING THE METHOD

To implement the method exposed in subsection 2.3 was necessary to make variations that assure the

<sup>3</sup>The procedure for the fast calculation of the  $C$  matrix (Kavraki, 1995), using the discrete Fourier transformed, is  $C_k = DFT^{-1}(DFT(A'_k) \times DFT(W))$ .

<sup>4</sup>The search is performed varying  $x$  from 0 to  $(N - 1)$ ,  $y$  from 0 to  $(N - 1)$  and  $\theta$  from 0 to  $(2\pi * (N - 1)/N)$

unique solution, due to there could be more than one configuration  $\hat{q}$ . We found that the method is lacking in:

- Insufficient information to detect asymmetries and the limits of the environment.
- No consider no-Gaussian Error information of sensors.
- No consider Gaussian Error information of environment map and sensors

So we propose in 3.1, 3.2 and 3.3 some operations to overcome those problems.

### 3.1 Increasing the number of points in PE vectors

To increase the number of points in PE vectors, we chosed a specific exploration; the core is that the robot must complete  $N$  small turns, until  $2\pi$  radians. At each position<sup>5</sup>  $q_j$ , the robot takes readings from the sensors ( $L_j$  vector<sup>6</sup>) and with them, it can build the extended robot ( $PEX_j$ ) respect to a local coordinate system for the initial configuration  $q_0$  by performing a frame rotation.

$$PEX_j = \left( P_0 + L_j \cdot \begin{bmatrix} \dots & \dots \\ \cos\theta'_i & \sin\theta'_i \\ \dots & \dots \end{bmatrix} \right) \cdot R(\phi_j) \quad (12)$$

Since, the  $N$  vectors of representative points ( $PEX_0, PEX_1 \dots PEX_{N-1}$ ) are referred to the same coordinate system (refereed to  $q_0$ ) we can associate them in a single vector, the new Total Vector of Points of the Extended Robot ( $\vec{PE}_0$ ) with length  $s.N$ , which contains the coordinates of the end of points for all extended robots generated.

$$\vec{PE}_0 = \begin{bmatrix} \vec{pe}_{0,0} \\ \dots \\ \vec{pe}_{s_j+i,0} \\ \dots \\ \vec{pe}_{s.N-1,0} \end{bmatrix} = \begin{bmatrix} pex_{0,0} \\ \dots \\ pex_{i,j} \\ \dots \\ pex_{s-1,N-1} \end{bmatrix} \quad (13)$$

This exploration has the additional advantage that the final configuration  $q_N$  of the robot is approximately equal to the initial configuration  $q_0$ , so that if our calculations are based on the initial configuration, we will find the present configuration of the robot at the moment of the calculation.

<sup>5</sup>Applying the kinematics of the robot, the sequences of configurations  $q_0, q_1 \dots q_{N-1}$  is invariant in two first elements  $x_j$  and  $y_j$  and only the third element,  $\phi_j$ , changes, being  $\phi_j = j * 2\pi/N$ . In addition  $q_0$  is the configuration 0,0,0 and it is represented in the center of the A matrix

<sup>6</sup>the matrix of distances  $L$  where each element  $l_{i,j}$  corresponds to the reading of the sonar  $i$  in the configuration  $q_j$ .

### 3.2 Filtering the signals caught by the sonars (L matrix)

In a real case, using the total vector of points  $\vec{PE}_0$  to build the map of the extended robot, we obtained a figure where it is not possible to find any similarity between the environment map and the extended robot, so the first conclusion is the errors are positive generally or the measures captured by the sonars are greater than the measures calculated by the model of the environment.

We made a comparison of the measures captured by the sonars with the theoretical values, calculated from the model of the environment; for the theoretical values, we can observe 4 zones of local minima, also it is possible to see the similarity between the theoretical values and the caught ones by the sensors in the zones of minimums. These zones of local minimums correspond at the moments in which the sound waves fall perpendicularly to the surface of obstacle. Consequently we can conclude the readings closer to local minimums, will be more reliable and we will accept those values, discarding the other readings.

The filter algorithm for the length  $l_{i,j}$ , is performed by the following equation:

$$\vec{l}_{i,j} = \begin{cases} l_{i,j} & l_{i,j} < l_{i,j-1} \text{ AND } l_{i,j} < l_{i,j+1} \\ 0 & l_{i,j} > l_{i,j-1} \text{ OR } l_{i,j} > l_{i,j+1} \end{cases} \quad (14)$$

Now we can recalculate the  $PEX_j$  matrix using the new  $\vec{L}_j$  matrix in the equation 12.

### 3.3 Rebuilding of the environment map (W) and the extended robot (A) maps.

To consider the inaccuracies in the process of building the map, we should to calculate the convolution between the map and a mask, this mask matrix ( $e$ ) has the form like a probabilistic distribution. We use the cosene function to approximate this probabilistic distribution. The value of the central element is one, the others elements have a value less than one, in the bounds of the mask matrix there are zeros values(fig 6).

$$\vec{W} = W \otimes e \quad (15)$$

After the convolution, we obtain a new map of the environment ( $\vec{W}$ ), where there are greater values are in the same place of the bitmap original, the lower values are in the surroundings of the obstacles.

Similarly, we could consider the inaccuracies in the extended robot maps, using the mask matrix ( $e$ ).

$$\vec{A}_k = A_k \otimes e \quad (16)$$

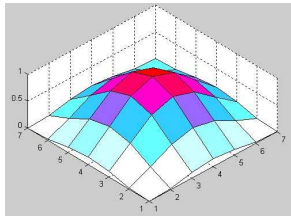


Figure 6: The mask matrix ( $e$ ) of the probabilistic distribution with 7X7 elements

## 4 RESULTS AND CONCLUSIONS

In order to prove the validity of this procedures, it was applied in an experimental robot AmigoBot developed by center SRI of Stanford. This robot, 8 sonars-based, performs the exploration in sweeping type (32 rotations). We has been chosen to prove the method in 3 locations different (real position) in a rectangle room, which approx dimensions are 6 x 9 meters and has asymmetries.

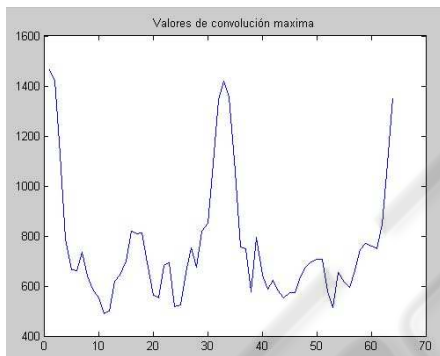


Figure 7: Local maxima values for each possible orientation  $\theta_k$

After calculating the Convolution  $C$ , we mark the maximum value in each possible orientation and we show it in the figure 7. The search of the  $q$  give us the configuration  $\hat{q}$  which is the estimated position of the robot, and it could be compared with the real position in the Table 1. The range of errors is in decimeters, and the resolution of the matrixes is one decimeter for a information pixel, the dimension of the matrixes was of 128 X 128, and the resulting  $C$  was of 128 X 128 X 64. If it is necessary greater resolution, the dimension of the matrixes will be greater therefore the time of calculation is increased.

### 4.1 Conclusions

The development of a tool for the automatic localization of mobile robots, which navigate in structured environment, is the main target of this work.

Table 1: Results for the points of test A, B and C

point of test	real position real $q_0$ $x, y, \phi$	estimate position $\hat{q}_0$ $\hat{x}, \hat{y}, \hat{\phi}$	error dm
A	47.10, 67.48, $0^0$	46.68, 67.81, $0^0$	0.53
B	62.11, 67.48, $0^0$	59.97, 66.22, $0^0$	2.48
C	77.05, 67.48, $0^0$	74.83, 65.07, $0^0$	3.27

The implementation only requires a bit map as a model of the environment, without having limitation for the shape or for the position between obstacles, being able to deduce that the time of processing does not depend on the complexity of the environment.

It is a global deterministic location, because, the method verifies all the possible configurations. The filter of minimums rejects noisy readings therefore the method is robust to great disturbances like in sonars. The calculation of the Convolution is a low load in the processor, thanks to the fast Fourier transformed applied.

Finally the algorithm of autolocalizacin in an experimental robot has been implemented and it has been possible to validate the results in real situations. Therefore, the robot has been equipped of the capacity to find its location, thus, it has greater autonomy.

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