AN LMI-BASED GENETIC ALGORITHM FOR GUARANTEED COST CONTROL

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Keywords: Genetic Algorithms, Linear Matrix Inequalities, Robust Control, Guaranteed Cost Control

Abstract: In this paper a new approach for the Guaranteed Cost Control Problem (GCCP) is presented, using two efficient tools, Linear Matrix Inequalities (LMIs) and Genetic Algorithms (GAs). A linear system with parametric uncertainty is considered for which a control law is to be found, minimizing a performance index. In a previous paper, an efficient method has been presented by using an LMI optimisation technique. A combined use of LMIs and GAs is proposed in the present approach that allows further improvement of the design procedure.

1 INTRODUCTION

The guaranteed cost control problem (GCCP) has drawn considerable attention in the last few years (Kosmidou, 1991), since the inclusion of uncertainties in the system model, with the appearance of robust control theory, is a standard practice. Most of the approaches to this problem make use of linear optimization techniques to find best solutions that satisfy some constraints of a specified problem. For convex problems, LMI techniques are now very efficient (Fischman, 1996).

However, searching methods have the major drawback due to the fact that they may converge to a local minimum or maximum.

Genetic Algorithms (GAs) come to negotiate this drawback, providing an alternative stochastic searching process, in which the natural evolutionary theory is adapted. Successive application of GAs in control theory has led to very promising results (Kundu, 1996).

In this paper, a new genetic algorithm is presented, which makes use of the LMI tool to compute the fitness value, of its candidate solution of the current population.

The paper is organized as follows: In Section 2 the guaranteed cost control problem is formulated. A general presentation of Genetic Algorithms and the basic operators they used is given in Section 3. The main idea of the proposed technique is described in details in Section 4, while an experimental verification of this method is performed through a numerical example, in Section 5. Finally, Section 6 presents conclusions and perspectives.

2 PROBLEM FORMULATION

Consider the linear uncertain system in state-space representation

$$\dot{x}(t) = (A_0 + \Delta A(t))x(t) + (B + \Delta B(t))u(t) \qquad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control vector, A_0 and B_0 are the state and control matrices, respectively, having appropriate dimensions.

The system uncertainties are described by

$$\begin{aligned} \Delta A(t) &= \sum_{i=1}^{p} a_i(t) A_i \ , \quad a_i^2(t) \le 1 \\ \Delta B(t) &= \sum_{i=1}^{q} b_i(t) B_i \ , \quad b_i^2(t) \le 1 \end{aligned} \qquad (2)$$

where the scalars $a_i(t)$, $b_i(t)$ are uncertain parameters, possibly time-varying, belonging to specified ranges, and A_i, B_i are given constant matrices determining the uncertainty structure. Without loss of generality, one can always assume that A_i, B_i have unity rank

A. Papakostas G., I. Kosmidou O. and E. Antonakis I. (2004).
AN LMI-BASED GENETIC ALGORITHM FOR GUARANTEED COST CONTROL.
In *Proceedings of the First International Conference on Informatics in Control, Automation and Robotics*, pages 327-333
DOI: 10.5220/0001145303270333

and thus they may be decomposed in form of products of vectors of appropriate dimensions, as follows:

$$A_i = d_i e_i^T \ ; \ B_i = f_i g_i^T \tag{3}$$

This is called the *rank-1 decomposition*. By using these vectors, define the matrices

$$\begin{aligned} D &:= \lfloor d_1 \ldots d_p \rfloor & F &:= \lfloor f_1 \ldots f_q \rfloor \\ E &:= \left[e_1 \ldots e_p\right]^T & G &:= \left[g_1 \ldots g_q\right]^T \\ S &:= diag\left(s_1 \ldots s_p\right) & T &:= diag\left(t_1 \ldots t_q\right) \end{aligned} \tag{4}$$

where s_i, $i=1,\ldots,p$ and t_i, $i=1,\ldots,q$ are positive scalars. Since decomposition (3) is not unique, these scalars may be chosen to determine a suitable rank-1 decomposition in order to satisfy different design requirements. In other words the elements S, T will be treated as free parameters, in the design procedure.

Consider also the quadratic performance index of the form:

$$J\left(x_0, u(t)\right) = \int_0^\infty [x^T(t) Q_0 x(t) + u^T(t) R_0 u(t)] dt \tag{5}$$

with $Q_0>0$, $R_0>0$.

As shown in (Kosmidou, 1996, Fischman, 1996) the *guaranteed cost control* law of the form

$$u^*(t) = -\delta R^{-1} B^T P x(t) \tag{6}$$

ensures an upper bound of the quadratic performance index (5) for all parameter variations consistent with (2), called *guaranteed cost*,

$$J\left(u^*(t)\right) \leq J^* = x(0)^T P x(0) \tag{7}$$

The nxn matrix P is the positive definite solution of the modified Riccati equation associated with the GCC problem,

$$\begin{aligned} &A^T P + PA - P[\delta\,(BR^{-1}B^T - \\ &\delta\, BR^{-1}G^T T^{-1} G R^{-1} B^T) \\ &- FTF^T - DSD^T]P + E^T S^{-1} E + Q = 0 \end{aligned} \tag{8}$$

The scalar δ and the scaling matrices S, T are chosen by the designer.

Since the GCC problem is often related with conservatism, i.e. the resulting upper bounds are too large with respect to the minimal J obtained from the LQR optimal design for the system without uncertainty, it is desirable to make J^* as small as possible. This leads to an auxiliary optimization problem, which is often analytically not tractable. An efficient solution has been proposed in (Fischman, 1996) by solving an LMI objective minimization problem.

That approach is being improved using the proposed method, by involving genetic algorithms, for the searching of the free parameters (S, T, δ) of the system.

3 GENETIC ALGORITHMS

Genetic Algorithms (GAs) have played a major role in many applications of the Engineering Science. As mentioned above, GAs constitute a powerful tool to optimization tasks. In other words, a simple GA is a stochastic method that performs searching in wide search spaces and depends on some probability values. For these reasons as well as its parallel nature, it has the ability to converge to the global minimum or maximum, depending on the specific application, and to skip possible local minima or maxima, respectively.

The main idea in which GAs are based, was first inspired by J. Holland (Holland, 2001). He tried to find a method to imitate the evolutionary process that characterizes the evolution of living organisms. This theory is based on the mechanism qualified by the survival of the fittest individuals over a population. In fact, there are some specific procedures taking place until the predominance of the fittest individual.

In the sequel, terminology in the field of genetic methods for optimization and searching purposes is given:

Individual is a solution of a problem satisfying the constraints and demands of the system in which it belongs.

Population is a set of candidate solutions of the problem, which contains the final solution.

Fitness is a real number value, which characterizes any solution and indicates how proper is this solution for the problem under consideration.

Selection is an operator applied to the current population, in a manner similar to the one of natural selection found in biological systems. The fitter individuals are promoted to the next population and poorer individuals are discarded.

Crossover is the second operator that follows the previous one. This operator allows solutions to exchange information, in such a way that the living organisms use in order to reproduce themselves. More specifically, two solutions are selected to exchange their sub-strings from a single point and after (single point crossover), according to a predefined probability P_c. The resulting offsprings carry some information from their parents. In this way new individuals are produced and new candidate solutions are tested in order to find the one that satisfies the appropriate objective.

Mutation is the third operator that can be applied to an individual. According to this operation each single bit of an individual binary string, can be flipped with respect to a predefined probability P_m.

There is a different procedure that can be considered for a single iteration of a genetic algorithm, called *Elitism*. During this operation the probability of discarding the fittest individual is minimized, since at each generated population, the fittest individual is checked whether it has a lower fitness than the elite member of the previous generation. If so, a randomly selected individual is replaced by the old elite member. Thus, it is guaranteed that the fittest individual will be promoted to the next population, and also an acceleration of the overall speed of the algorithm is derived in this way.

In the present algorithm, an elitism based reinsertion method is used. According to this process, a predefined number of individuals (the least suitable) are replaced.

Since these operators have been applied to the current population, a new population will be formed and the generational counter will be increased by one. This process will continue until a predefined number of generations is attained or some form of convergence criterion is met.

A simple Genetic Algorithm, which uses some of the operations discussed above, is presented in Fig.1.

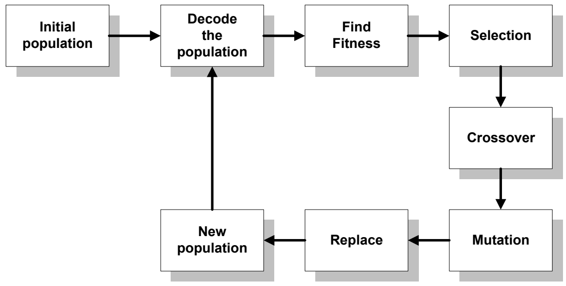


Figure 1: Block diagram of a simple GA

In the next section, an LMI-based genetic algorithm is proposed and applied to the guaranteed cost control problem described in Section 2.

4 LMI-GENETIC METHOD

As previously mentioned, a first attempt of using LMI optimizations in Guaranteed Cost Control (Fischman, 1996) appeared very promising. Besides, genetic algorithms seem to be very efficient in solving various optimization problems in which the searching space is complex.

In the proposed method, a combination of two optimization tools is used. More precisely, the genetic algorithm is used to find a suitable variable set (S, T, δ), while the LMI optimization, used in (Fischman, 1996) is applied to find an optimum matrix P that satisfies some prespecified constraints. By using the obtained matrix P, a fitness value is assigned to the corresponding candidates and process goes on.

This method can be viewed as a simple genetic algorithm that uses the LMI tool to find the fitness of the resulting candidate individuals of the problem, under consideration.

To be more specific, consider the modified genetic algorithm depicted in Fig. 2. This algorithm is similar to the simple genetic algorithm of Fig.1, with the difference that an LMI optimization mechanism is used, to compute the objective value of the current candidate solutions.

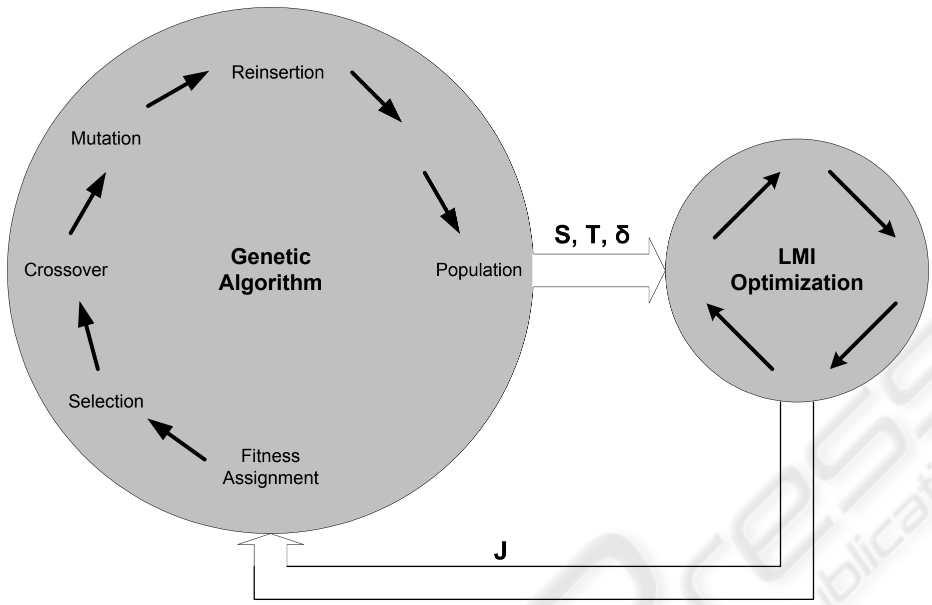


Figure 2: The proposed LMI-based Genetic Algorithm

The LMI optimization procedure described in Fig.2, consists of the following *Theorem* which is a modified version of *Theorem 3* in (Fischman, 1996) and stated as,

Theorem

The minimization of the value of the guaranteed cost (5) for the uncertain system (1) is ensured by a control law of the form (6), if the optimization problem described by equation (9) has a non empty set of feasible solutions (M, W) with M, W being symmetric positive definite matrices, and $P = W^{-1}$.

The difference of the above *Theorem* from *Theorem 3* of (Fischman, 1996) is that the matrices S, T and the scalar δ, are not variables of the LMI optimization process, but constant values that come from the genetic algorithm.

In other words, the genetic procedure manages to find candidate sets of the (S, T, δ) variables, and passes these sets into the LMI optimization for finding optimal value of the matrix P.

The resulted matrix P is used in equation (7) to derive the corresponding guaranteed cost, with a known initial condition x_0.

Before the beginning of the proposed algorithm, one has to take some decisions about the parameters that must be defined, in order to initialize the procedure. Some of these parameters are (Coley, 2001)

- Type of individual representation (real, binary, etc.)
- Population size (typical values are 20-100)
- Length of individuals (L, depends on the range of the parameters)
- Crossover probability (P_c: typical values are 0.4-0.9)
- Mutation probability (P_m=1/L, 0.01)
- The selection operator
- Number of Generations

The proposed algorithm can be summarized in the following steps:

Algorithm

Step 1: Generation of the initial population, consisting of 100 individuals. Each individual contains, three variables (S, T, δ), in binary representation, of length 20 each one.

Step 2: The candidate sets of the variables (S, T, δ) are appeared as solutions of the problem. Each one of this set, is passed as constant matrices into equation (9), and the LMI procedure is started.

$$\min_{M,W} Tr(M)$$

$$\begin{cases} (i)\ \begin{bmatrix} M & I \\ I & W \end{bmatrix} > 0 \\ (ii) \begin{bmatrix} -WA^T - AW + \delta BR_0^{-1}B^T - DSD^T - FTF^T & WE^T & \delta BR_0^{-1}G^T & W \\ EW & S & 0 & 0 \\ GR_0^{-1}B^T\delta & 0 & T & 0 \\ W & 0 & 0 & Q_0^{-1} \end{bmatrix} > 0 \end{cases} \quad (9)$$

Step 3: The resulted matrix P is used to compute the guaranteed cost that corresponds to the set of (S, T, δ), by using (7).

Step 4: The computed cost J, consists the objective value of the respective set (S, T, δ), and is used for the fitness assignment operator of the genetic algorithm.

Step 5: The algorithm continues, by applying the genetic operators, presented in Fig.2, with the appropriate settings.

Step 6: A new population is obtained and the algorithm goes to *Step 2* until a fixed number of generations is achieved.

When the above algorithm has been terminated, the individual with the higher fitness value is the solution of the respective problem. Thus the variables that consist the fitter individual, is the optimum set of problem variables (S, T, δ), which obtains a minimum guaranteed cost control J^*.

The above algorithm is now illustrated by means of a numerical example.

5 NUMERICAL EXAMPLE

A common problem (Kosmidou, 1996, Fischman, 1996) is considered in this section in order to investigate the performance that can be achieved by using the proposed method.

This problem represents a 4th order model of a helicopter in a vertical plane for an airspeed range of 60 knots to 170 knots. For this range of operating conditions significant changes occur in elements a_{32}, a_{34} and b_{21} of the system matrices, where the nominal matrices are the following:

$$A_0 = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.2855 & -0.7070 & 1.3229 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.0447 & -7.5922 \\ -5.5200 & 4.9900 \\ 0 & 0 \end{bmatrix}$$

with the uncertainty matrices:

$$\Delta A(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.2192a_1(t) & 0 & 1.2031a_1(t) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

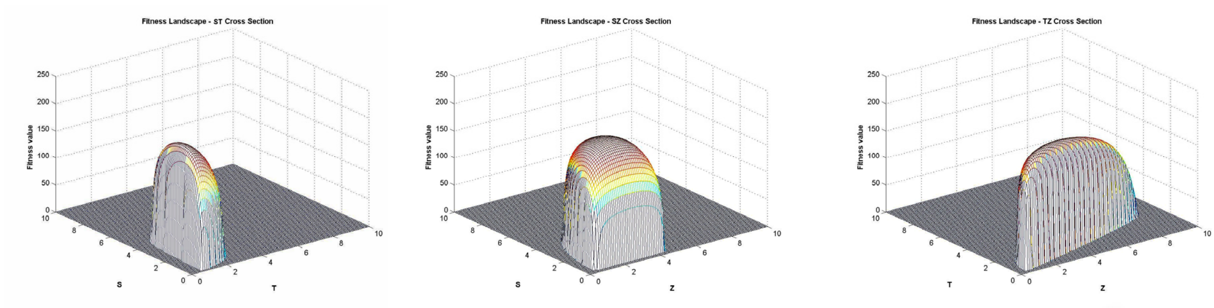
$$\Delta B(t) = \begin{bmatrix} 0 & 0 \\ 1.0673b_1(t) & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Let the uncertainty decomposition matrices be as follows:

$$D = \begin{bmatrix} 0 \\ 0 \\ 1.2031 \\ 0 \end{bmatrix}, \ E = \begin{bmatrix} 0 & 0.1822 & 0 & 1 \end{bmatrix},$$

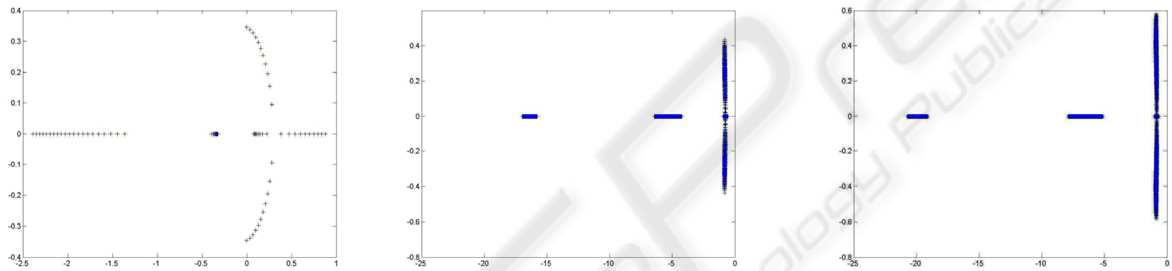
$$F = \begin{bmatrix} 0 \\ 2.1346 \\ 0 \\ 0 \end{bmatrix}, \ G = \begin{bmatrix} 0.5 & 0 \end{bmatrix}$$

while the $R_0 = I_2$ and $Q_0 = I_4$.



(a) (b) (c)

Figure 3: Fitness landscape in three cross sections, of the proposed genetic algorithm, (a) ST, (b) SZ and (c) TZ cross sections



(a) (b) (c)

Figure 4: The complex plane with the poles of (a) open system, (b) closed loop system with the control law of (Fischman, 1996), and (c) closed loop system with the control law obtained by the proposed algorithm

By applying the proposed LMI-based Genetic algorithm the following values of the three degrees of freedom are obtained

$S = 2.1663 \qquad T = 1.2036 \qquad d = 2.4998$

The corresponding guaranteed cost control law is:

$$\hat{u}(t) = \begin{bmatrix} -2.5665 & 0.6334 & 2.4866 & 3.8876 \\ 1.2727 & 1.5956 & -0.3127 & -1.8963 \end{bmatrix} x(t)$$

The guaranteed cost obtained by the proposed method is lower than those of the previous methods (Kosmidou, 1996, Fischman, 1996), and it is more close to the optimal one, as illustrated in the following Table 1. For the computation of the guaranteed cost, equation (7) is used with initial condition

$$x_0 = [1 \quad 1 \quad 1 \quad 1]'$$

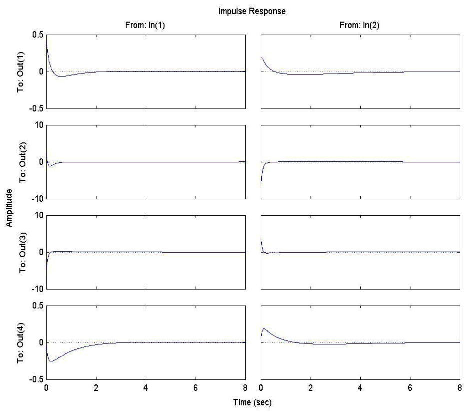
Table 1: Guaranteed cost and gain norms for the case of the three methods

Method	Guaranteed Cost	Norm of the gain
Proposed	4.8978	5.7141
Fischman's	5.8000	3.0400
Kosmidou's	5.2591	4.4038
Optimal Control	3.4890	2.0545

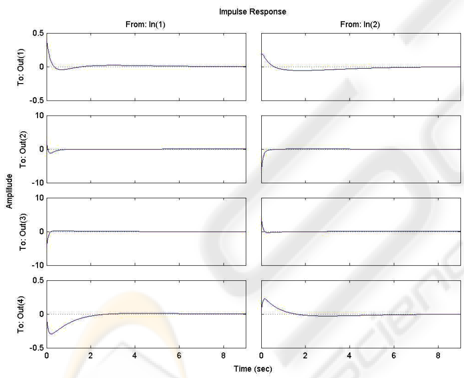
In Figure 3, the landscape of the proposed genetic algorithm is presented in three cross sections. The combination of these three landscapes consists the entire search space of the algorithm.

Also, in Figure 4, the poles of open system and the closed loop system, for a range of uncertainties ($|a_i(t)| \le 1$, $|b_i(t)| \le 1$) applying the control law derived by using the proposed method and the method of (Fischman, 1996) are depicted. As can be seen, by this figure, the open system is unstable, while in the case of the proposed law, is more stable than the one obtained by (Fischman, 1996).

Finally, the state responses of the closed systems, in the case of the proposed law and the (Fischman, 1996) one, are presented in Figure 5. This figure points, that the proposed gain feedback, behaves quite more efficiently than the other, since the responses are smoother.



(a)



(b)

Figure 5: State responses (a) proposed method and (b) Fischman's method

6 CONCLUSIONS

A novel method of finding a robust control law, that guarantees an upper bound of a quadratic performance index, has been presented in this paper.

The proposed method is based on the combination of the Genetic Algorithms and the LMI optimization tool (Gahinet, 1995). It makes use of an LMI approach to the guaranteed cost, presented in (Fischman, 1996), of the form of an objective computation method.

The LMI proposed in (Fischman, 1996), is modified to find only an optimal matrix *P*, while the rest of free parameters are derived through a genetically processed algorithm.

The results are very promising, since the resulted guaranteed cost is lower than previous ones (Kosmidou, 1996, Fischman, 1996), with an additional quite better system behavior, in the sense of stability.

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