

CONSOLIDATED TREE CONSTRUCTION ALGORITHM: STRUCTURALLY STEADY TREES

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Abstract: This paper presents a new methodology for building decision trees or classification trees (Consolidated Trees Construction algorithm) that faces up the problem of unsteadiness appearing in the paradigm when small variations in the training set happen. As a consequence, the understanding of the made classification is not lost, making this technique different from techniques such as bagging and boosting where the explanatory feature of the classification disappears. The presented methodology consists on a new meta-algorithm for building structurally more steady and less complex trees (consolidated trees), so that they maintain the explaining capacity and they are faster, but, without losing the discriminating capacity. The meta-algorithm uses C4.5 as base classifier. Besides the meta-algorithm, we propose a measure of the structural diversity used to analyse the stability of the structural component. This measure gives an estimation of the heterogeneity in a set of trees from the structural point of view. The obtained results have been compared with the ones get with C4.5 in some UCI Repository databases and a real application of customer fidelisation from a company of electrical appliances.

1 INTRODUCTION

Several pattern recognition problems need an explanation of the made classification together with a good performance of the classifier related to its discriminating capacity. Diagnosis in medicine, fraud detection in different fields, customer fidelization, resource assignation, etc, are examples of applications where an explanation of the classification made becomes as important as the accuracy of the system. Among the set of classification techniques that are able to give an explanation for the classification made, we can find the decision trees [Quinlan 93]. However, classification trees have a problem; they are too sensitive to the sample used in the induction process. This weakness of decision trees is called unsteadiness or instability [Dietterich 00a, Chawla et al 02]. Due to this behaviour, several algorithms for building classification trees with greater discriminating capacity and steadiness have been developed.

Bagging [Breiman 96], boosting [Freund et al 96], and different variants of them [Chawla et al 02]; are

examples of techniques used to face the problem. All of them use resampling techniques. Bagging uses a set of trees induced from different bootstrap subsamples, extracted from the initial sample (training set). In the multiple classifier built, classification is made by simple voting or weighted voting [Duin et al 00, Bauer et al 99]. When the used technique is boosting, the different trees are built sequentially and the patterns are reweighted depending on the error made by the preceding classifier. This makes the next tree in the sequence concentrate in the hardest zones of the classification field, so, in the decision boundaries. The final decision is based on a weighted vote that takes into account the error estimated during the learning process of each one of the trees composing the system.

These techniques obtain improvements in the behaviour of the global system but they lose the explaining capacity of the classifier, because in every case the final classifier is a combination of different trees with different structures. Our approximation introduces a new methodology that brings together both characteristics: stability and

explaining capacity. We have developed a new algorithm for building trees. We denominate the built trees *consolidated trees*. This methodology uses different training subsamples and builds a single accorded tree among all of them. Every technique mentioned before to make more stable classification trees, loses its explanatory component, because a set of trees with very different structures is used to make the final decision. Our aim has been to design a new methodology for building trees that maintaining the performance of standard classification trees, reduces the complexity and adds structural stability to the induced tree. This makes the explanation related to classification much more robust and steady, so, less sensible to small changes in the sample used to build the classifier. In order to analyse the structural stability of classification trees, we have defined a diversity measure or distance that measures how similar or different a set of trees is. The paper proceeds with the description of our new methodology for building consolidate trees, Section 2. In Section 3, the structural diversity measure is introduced, followed in Section 4 by the description of the data sets and the experimental set-up. The results of our experimental study are discussed in section 5. Finally, Section 6 is devoted to summarise the conclusions and future research.

2 CTC LEARNING ALGORITHM

The tree building methodology we propose, Consolidated Trees Construction (CTC) algorithm, is based on resampling techniques. Several training subsamples are extracted from the original training set and a single tree is built based on consensus among the partial trees that are being built from each subsample. The main difference with bagging, which makes it radically different, is that the consensus is achieved in each step of the trees' building process. The decision about which variable will be used to make the split in a node is accorded among the different proposals coming from the trees (using the base classifier: C4.5 in our case) being built from the different subsamples. The decision is made by a voting process. Once the decision about the variable selected to split the trees is made, all the trees (each one associated to a different subsample) are forced to use that variable to make the split. The process is repeated iteratively until no more divisions are possible, due to some stop criterion. An schema of the algorithm can be found in (Figure 1) and it proceeds the following way:

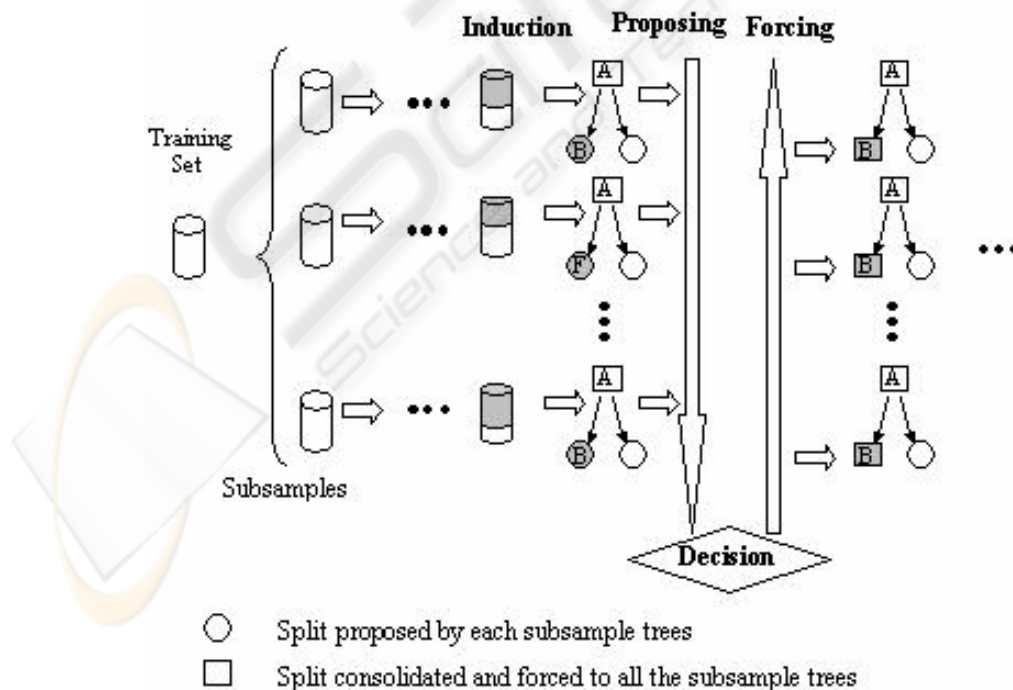


Figure 1: Consolidation of a node.

1. Extract a set of subsamples (*Number_Samples*) from the original training set using the desired resampling technique (*Resampling_Mode*): size of the subsamples (100%, 75%, 50%, etc; with regard to the original set's size), with replacement or without replacement, stratified or not, etc.
2. The final tree is built node by node in preorder. Each consolidated node is built the following way:
 - a. For each subsample, induce the variable that would be used to make the split at that level of the partial tree; in the example (B, F,...,B).
 - b. Analyse the number of partial trees that propose to make an split and decide depending on the established criteria *Crit_Split* (ex: simple majority, absolute majority, etc.) whether to split or not. If the decision is not to split (a leaf node is created), jump to the next node to consolidate and go to step 2a.
 - c. Analyse the number of votes that has the most voted variable (votes of variable B in Figure 1). If based on *Crit_Variable* the variable has not enough votes, consolidate the node as a leaf node and go to step 2a. When the number of votes is enough this variable will be the one used to split the consolidated node.
 - d. Decide the branches the node to split will have, depending on *Crit_Branches* criteria. If the variable to split is continuous, determine the cutting point (ex: using the mean or the median of the values proposed for that variable). If the variable to split is discrete, decide the set of categories for each branch (ex: a branch for each category, using heuristics such as C4.5's subset option, etc.).
 - e. Force the accorded split (variable and stratification) in every tree associated to each subsample (every partial tree in the example is forced to make the split with the consolidated variable B). Jump to next node and go to step 2a.

The different decisions can be made by voting, weighted voting, etc. Once the consolidated tree has been built, its behaviour is similar to the one of the used base classifier. Section 5 will show that the trees built using this methodology, have similar discriminating capability (the differences are not statistically significant) but they are structurally more steady and less complex. In order to analyse this second aspect, we have defined the structural diversity measure that we present in next section.

3 STRUCTURAL DIVERSITY MEASURE

We will use this section to define the diversity measure or structural distance, that will allow us to analyse the stability of the consolidated trees and compare them to the standard ones. The aim is to be able to measure the heterogeneity existing in sets of trees built using each of the methodologies to be compared. The estimation of the degree of structural diversity in a group, is made analysing the structural differences among each possible pair of trees in the group, and, calculating average values of the differences obtained.

The defined metric or distance (*Structural_Distance*, *SD*) is based on a vector (M_0, M_1, M_2) with three values used to compare two trees (T_i, T_j). Both trees are looked through in preorder, node by node, and the corresponding split variables are compared to know whether they match or not. The three components are calculated the following way:

- M_0 : Number of common nodes in T_i and T_j . We understand as common nodes the ones that being in the same position in both trees, make the split based on the same variable.
- M_1 : measures the number of times that looking through a common branch, a tree makes a split in a node and the other does not. Each increment is weighted depending on the complexity of the subtree beginning in this node.
- M_2 : measures the number of times that looking through a common branch and arriving to a common node, the variables chosen to make the split in both trees are different. Each increment is weighted depending on the complexity of both subtrees.

The pseudo-code of the algorithm used for calculating each one of the components of the proposed measure is in Appendix.

The definitions show that to increase the value of M_0 , the same variable has to appear in the same node, so that the variable used to make the split has been chosen at the same level in both trees. Once a different split appears, the remaining subtrees of the compared trees have an effect on the value of M_1 or M_2 . This is important because in the tree construction process (top-down), the variables used to split a node are selected depending on their statistical importance (entropy, p-value, ...). As a consequence, this measure of similarity/diversity takes also into account when making the

comparison, the statistical importance each tree has given to each of the independent variables.

We have defined two measures, M_1 and M_2 , because M_2 indicates higher diversity level among the trees than M_1 does, and we think that it is important to differentiate both cases.

Evidently, when greater is the first component and smaller the two others, more homogeneous is the set of trees used to calculate the values.

In order to calculate the diversity in the set of trees we estimate the mean of the vector:

$$SD^{M_x}_{mean}(T_{set}) = \frac{2}{m(m-1)} \sum_{\substack{k,l=0 \\ k < l}}^{m-1} SD^{M_x}(T_k, T_l) \quad (1)$$

with $x = 0, 1, 2$

Once the vector has been obtained, the comparison between groups can be done at vector level or at scalar level making any combination of the different components depending on the importance we want to give to each of the measures get.

In this case we define a scalar measure ($\%Common_{mean}$) based in three percentages calculated from the three components normalised in respect to the sum of the number of nodes of the two trees compared. The scalar is computed with a lineal combination of the three percentages with the vector of weights (1,-1,-1). In this experimentation we don't take into account the different level of importance of the two last components. If the scalar measure is positive, the compared trees are more similar than different among them, and, when the value is negative the common part of the trees is smaller than the different one. The range of values the scalar can take is between -100 and +100. This measure allows the comparison of groups of trees with different complexities (at the structural level), because the components are normalised in respect to the number of nodes of the comparison.

4 EXPERIMENTATION

Six databases of real applications have been used for the experimentation. Most of them belong to the well known UCI Repository benchmark [Blake et al 98], widely used in the scientific community. Table 1 shows the characteristics of the databases used in the comparison. The last database is a real data applications from our environment, and does not belong to UCI. The data set called Faithful is centred in the electrical appliance's sector. In this case, we try to analyse the profile of the customers during the time, so that a classification related to their fidelity to the brand can be done. This will allow the

company to follow different strategies to try to increment the number of customers that are faithful to the brand, so that the sales increase. In this kind of applications is very important to use a system that provides information about the factors taking part in the classification (the explanation), because it is nearly more important to analyse and explain why a customer is or is not faithful, than the own categorisation of the customer. This is the information that will help the corresponding department to make good decisions.

Table 1: Description of experimental domains

Domain	N. of patterns	N. of features	N. of classes
Breast cancer W	699	10	2
Heart disease C	303	13	2
Hypothyroid	3163	25	2
Lymphography	148	18	4
Segment	210	19	7
Faithful	24507	49	2

The CTC methodology has been compared to the C4.5 tree building algorithm Release 8 of Quinlan [Quinlan 93], using the default parameters' settings. The methodology used for the experimentation [Hastie et al 01] is a 10-fold stratified cross validation. Cross validation is repeated five times using a different random reordering of the examples in the data set. This methodology has given us the option to compare 5 times groups of 10 trees CTC and C4.5 in both senses the structural point of view and the discriminating capacity (50 executions for each instance of the analysed parameters). In each run we have calculated the average error and its standard deviation, together with the $SD_{mean}(T_{set})$ of each of the groups of trees and the values $Comp_{mean}$ (estimated as the number of internal nodes of the tree) and $\%Common_{mean}$ (explained in Section 3).

In order to evaluate the structural improvement achieved in a given domain by using the algorithm CTC compared to the algorithm C4.5, we have calculated, $(\%Comm_{CTC} - \%Comm_{C4.5}) / \%Comm_{C4.5}$, the relative improvement.

For every result we have tested the statistical significance [Dietterich 98, Dietterich 00] of the differences of the results obtained with the two algorithms using the paired t-test (with significance level of 95%).

5 RESULTS

We have analysed several parameters of the CTC meta-algorithm. The ranges of the different parameters analysed are:

1. *Number_Samples*: 3, 5, 10, 20, 30, 40, 50, 75, 100, 125, 150, 200. The obtained results show that 10 is the minimum number of samples to achieve satisfactory results with CTC. So, the average results presented in this section do not take into account the values 3 and 5, except for *Faithful* where due to the size of the database the study has been done from 3 to 40 subsamples.
2. *Resampling_Mode*: due to the importance of the diversity of the subsamples used to build a classifier [Skurichina et al 00], different options have been proved (samples of size 50% and 75% of the original training set drawn without replacement, and bootstrap samples). The best results have been obtained with the 75% (results shown in the paper), probably because this is the case where each subsample has larger amount of information of the original sample.
3. *Crit_Split* and *Crit_Variable*: simple majority among the *Number_Samples* for both. The possibility of introducing thresholds to these

criteria has not been considered in this paper because the comparison of our consolidated trees (CTC) with the simple classification trees C4.5 has been done pruning the trees with the pruning algorithm of the C4.5 Release 8 software. The pruning has been done in order to obtain two systems with similar complexity level (the same zone in the learning curve) and also to be able to make the structural comparison with similar development conditions for all trees. We can not forget that developing too much a classification tree leads to a greater probability of overtraining and on the other hand, the fact that a variable appears in the tree is less significant because the system takes into account very specific details of the training set.

4. *Crit_Branches*: the selection of the new branches to be created when the variables are continuous has been made based on the median. Experimentation with the mean has also been done, but the results were worse, probably because this measure has smaller stability. When the variables were discrete we have not used the subset option of C4.5 and as many branches as categories has the variable selected to make the split have been proposed.

Table 2: Error and complexity comparison among C4.5 and CTC. The best values related to *Number_Samples* in each data set are shown

	C4.5		CTC (ERR _{MIN})				CTC (COMP _{MIN})					
	Err	Comp	Err	R.Dif	Comp	R.Dif	N_S	Err	R.Dif	Comp	R.Dif	N_S
<i>Breast-W</i>	5.63	3.16	5.40	-4.12	3.11	-1.41	125	5.49	-2.49	3.04	-3.52	30
<i>Heart-C</i>	23.96	15.11	22.85	-4.61	13.51	-10.59	20	22.92	-4.35	13.20	-12.65	30
<i>Hypo</i>	0.71	5.38	0.72	0.56	4.51	-16.12	20	0.72	0.84	4.40	-18.18	30
<i>Lymph</i>	20.44	8.84	19.65	-3.88	9.24	4.52	30	19.93	-2.51	9.09	2.76	40
<i>Segment</i>	13.61	11.71	11.52	-15.31	13.82	18.03	50	13.51	-0.75	12.64	7.97	10
<i>Faithful</i>	1.48	39.47	1.48	-0.13	32.44	-17.79	20	1.50	0.94	28.56	-27.65	05
Average	10.97	13.94	10.27	-4.58	12.77	-3.89		10.68	-1.38	11.82	-8.54	

Table 2 shows the results related to error (Err) and complexity (Comp) for the different data sets. The comparison among C4.5 and CTC we present in the table has been calculated using the best value of the parameter *Number_Samples* (N_S) for CTC in order to minimise the error or the complexity of the classifier. The relative differences among the two algorithms (R.Dif) are also presented, for both parameters, the error and the complexity. The last row shows the mean of the results obtained with the two algorithms, for the different domains. It can be observed that in average, the CTC algorithm obtains a relative improvement in the error of 4.58%. In this case, when talking about the complexity of the generated trees, the relative improvement is 3.89%. Even when the best *Number_Samples* for

minimising the complexity has been selected, the CTC has smaller errors than the C4.5, being the improvement 1.38%. The relative complexity reduction obtained in this case is 8.54%. In the larger database we have used (*Faithful*), where the reduction of the complexity becomes more important, the obtained improvement is 27.65%. This reduction is statistically significant for every value of the parameter *Number_Samples*.

To confirm the robustness of the algorithm when the parameter *Number_Samples* is changed, Table 3 (left side) shows the average values of the error and the complexity obtained with all the analysed values for that parameter.

Table 3: Average results of the error and the complexity in CTC taking into account the whole range of N_S (left side) and for N_S=30 (right side)

	CTC (Average N_S=10,20,30,40,50,75,100,125,150,200)				CTC N_S=30			
	Err	R.Dif	Comp	R.Dif	Err	R.Dif	Comp	Dif
	<i>Breast-W</i>	5.56	-1.21	3.09	-2.11	5.49	-2.49	3.04
<i>Heart-C</i>	23.43	-2.21	13.84	-8.41	22.92	-4.35	13.20	-12.65
<i>Hypo</i>	0.73	2.30	4.65	-13.60	0.72	0.84	4.40	-18.18
<i>Lymph</i>	20.03	-2.01	9.18	3.74	19.65	-3.88	9.24	4.52
<i>Segment</i>	12.42	-8.70	14.08	20.23	13.13	-3.48	13.47	14.99
<i>Faithful</i>	1.49	0.84	30.03	-23.92	1.49	0.67	29.53	-25.17
Average	10.61	-1.83	12.48	-4.01	10.57	-2.11	12.15	-6.67

It can be observed that a relative improvement of 1.83% in the error is maintained and the complexity improves in a 4.01%. If we would like to tune the parameter in order to optimise the error/complexity trade off in the analysed domains, the results address us to N_S=30. The results for N_S=30 appear in Table 3 (right side). They do not differ substantially from the results obtained finding in each domain the best value for *Number_Samples*. The CTC maintains its improvement if compared to C4.5 in both cases. If we analyse the statistical significance of the differences among the two algorithms, we won't find significant differences in the error parameter for none of the values of *Number_Samples*. However, when analysing the complexity significant

differences in favour of CTC are found in three of the six databases (*Heart-C*, *Hypo*, *Faithful*). Regarding to the structural stability of the trees obtained with each algorithm, and taking into account the metric presented in Section 3, Table 4 shows that except in one of the databases (*Lymph*), the trees built using CTC algorithm, are more similar among them than the ones generated with C4.5. This means that the induction mechanism is able to extract more information about the explanation of the classification, and besides, in a more steady way. The improvement is in average of 16.82%, and, the differences are statistically significant in every case where better results are obtained.

Table 4: Comparison of C4.5 and CTC related to the structural diversity measure, using the best value of *Number_Samples* in each domain

	C4.5		CTC (%COMM _{MIN})				
	%Comm	Err	%Comm	R.Dif	Err	R.Dif	N_S
<i>Breast-W</i>	56.99	5.63	65.89	15.62	5.49	-2.49	30
<i>Heart-C</i>	-59.01	23.96	-57.06	3.30	23.16	-3.32	150
<i>Hypo</i>	27.99	0.71	41.11	46.90	0.73	2.53	75
<i>Lymph</i>	-27.72	20.44	-35.50	-28.08	20.18	-1.25	75
<i>Segment</i>	-67.09	13.61	-36.19	46.06	12.76	-6.26	20
<i>Faithful</i>	-57.95	1.48	-48.01	17.15	1.50	0.94	05
Average	-21.13	10.97	-11.63	16.82	10.64	-1.64	

Table 5: Average results of the structural metric in the CTC algorithm. The whole range of N_S is taken into account in left side and N_S=30 in right side

	CTC (Average N_S=10..200)		CTC N_S=30	
	%Comm	R.Dif	%Comm	R.Dif
<i>Breast-W</i>	60.51	6.19	65.89	15.62
<i>Heart-C</i>	-61.10	-3.53	-58.57	0.75
<i>Hypo</i>	35.15	25.60	37.14	32.69
<i>Lymph</i>	-36.71	-32.43	-36.76	-32.62
<i>Segment</i>	-47.16	29.70	-43.33	35.41
<i>Faithful</i>	-50.71	12.49	-50.94	12.09
Average	-16.67	6.33	-14.43	10.66

Table 5 shows in the left side the mean of the differences in the structural metric of both

algorithms, for all the experimented values with parameter *Number_Samples*. The average value favours CTC, and, we should not forget that the error also favours it. This proves again the stability of the meta-algorithm in respect to the tuning of the parameter *Number_Samples*.

The same table, right side, presents results of the structural diversity among both algorithms, when N_S=30. The results in this case are near the optimal results we found and better than the average results obtained (left side). As a consequence we can ensure that the influence of the parameter *Number_Samples* in the final result is not critical in any of the analysed criteria.

6 CONCLUSIONS AND FURTHER WORK

A new algorithm to build classification trees that are structurally more steady and with smaller complexity level has been presented (Consolidated Trees Construction, CTC). This algorithm achieves and even improves the discriminating capacity of C4.5. The proposed algorithm maintains the explanatory feature of the classification and this is very important in many real life's domains. The algorithm builds trees that reduce the error rate and the complexity of the classifier if compared to the C4.5.

Besides, and to prove the goodness and the stability of the explanation related to the classification, a measure of the structural diversity of two trees is proposed. This measure analyses the stability of the variables and their statistical importance (level where they appear in the tree). The measure allows the analysis of the heterogeneity of a set of trees from the structural or explanatory point of view.

In this paper we have proven that for the analysed domains, the CTC is able to extract more information about the explanation of the classification, and, in a more steady way. The differences are statistically significant in most of the analysed domains.

On the other hand, the stability of the meta-algorithm when varying the parameter *Number_Samples* has been proved; $N_S=30$ is an adequate value for all the databases used in the experimentation.

The first work to do in the future is to enlarge the set of domains analysed. We are also thinking on experimenting with other possibilities for the parameter *Resampling_Mode* (different amount of information or variability of the subsamples) as further work.

Other interesting possibility is to generate new subsamples dynamically, during the building process of the CTC, where the probability of selecting each of the cases is modified based on the error (similar to *boosting*).

We are also analysing a possibility where the own meta-algorithm builds trees that do not need to be pruned. With this aim, we would make a tuning of the parameters *Crit_Split* and *Crit_Variable*, so that the generated trees are situated in a better point of the learning curve and the computation load of the training is minimised. Heuristic techniques for the stratification of the discrete variables can also be

studied in order to build trees with greater explaining capacity.

This methodology can be very useful when resampling is compulsory (large databases, class imbalance, ...).

APENDIX

```
int CalculateSD (Tree Ti, Tree Tj,
                int VMetric[])
{
    if ((Ti->NodeType != LEAF) &&
        (Tj->NodeType != LEAF))
        if (Ti->Variable == Tj->Variable)
        {VMetric[0]++;
         if (Ti->Forks != Tj->Forks)
             return (-1);
         ForEach(k,1, Ti->Forks)
             CalculateSD (Ti->Branch[k],
                          Tj->Branch[k],VMetric);
        }
    else //different division variables
    {Vmetric[2]+=
     CalcNumberDescendents(Ti);
     Vmetric[2]+=
     CalcNumberDescendents(Tj);
     return (0);
    }
    else
        if ((Ti->NodeType == LEAF) &&
            (Tj->NodeType == LEAF))
            return (0);
        else
            if (Ti->NodeType = LEAF)
            {Vmetric[1] +=
             CalcNumberDescendents(Tj);
             return (0);
            }
            else
            {Vmetric[1] +=
             CalcNumberDescendents(Ti);
             return (0);
            }
    return (-1);
}
```

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REFERENCES

- Quinlan J. R., 1993. *C4.5: Programs for Machine Learning*. Morgan Kaufmann Publishers Inc.(eds), San Mateo, California.
- Dietterich T.G., 2000. Ensemble Methods in Machine Learning. *Lecture Notes in Computer Science, Vol. 1857. Multiple Classifier Systems: Proc. 1st. Inter. Workshop, MCS, Cagliari, Italy*, 1-15
- Chawla N.V., Hall L.O., Bowyer K.W., Moore Jr., Kegelmeyer W.P., 2002. Distributed Pasting of Small Votes. *Lecture Notes in Computer Science Vol. 2364. Multiple Classifier Systems: Proc. 3th. Inter. Workshop, MCS, Cagliari, Italy*, 52-61
- Breiman L., 1996. Bagging Predictors. *Machine Learning*, 24, 123-140
- Freund, Y., Schapire, R. E., 1996. Experiments with a New Boosting Algorithm. *Proceedings of the 13th International Conference on Machine Learning*, 148-156
- Duin R.P.W, Tax D.M.J., 2000. Experiments with Classifier Combining Rules. *Lecture Notes in Computer Science 1857. Multiple Classifier Systems: Proc. 1st. Inter. Workshop, MCS, Cagliari, Italy*, 16-29
- Bauer E., Kohavi R., 1999. An Empirical Comparison of Voting Classification Algorithms: Bagging, Boosting, and Variants. *Machine Learning*, 36, 105-139
- Blake, C.L., Merz, C.J., 1998. UCI Repository of Machine Learning Databases. *University of California, Irvine, Dept. of Information and Computer Sciences* <http://www.ics.uci.edu/~mlearn/MLRepository.html>
- Hastie T., Tibshirani R. Friedman J., 2001. *The Elements of Statistical Learning*. Springer-Verlang (es). ISBN: 0-387-95284-5
- Dietterich T.G., 1998. Approximate Statistical Tests for Comparing Supervised Classification Learning Algorithms. *Neural Computation*, 10, 7, 1895-1924
- Dietterich T.G., 2000. An Experimental Comparison of Three Methods for Constructing Ensembles of Decision Trees: Bagging, Boosting, and Randomization. *Machine Learning*, 40, 139-157
- Skurichina M., Kuncheva L.I., Duin R.P.W., 2002. Bagging and Boosting for the Nearest Mean Classifier: Effects of Sample Size on Diversity and Accuracy. *Lecture Notes in Computer Science 2364. Multiple Classifier Systems: Proc. 3th. Inter. Workshop, MCS, Cagliari, Italy*, 62-71