

ANALYSIS OF THE ITERATED PROBABILISTIC WEIGHTED K NEAREST NEIGHBOR METHOD, A NEW DISTANCE-BASED ALGORITHM

J. M. Martínez-Otzeta and B. Sierra
Dept. of Computer Science and Artificial Intelligence
University of the Basque Country
P.O. Box 649
20080 San Sebastián, Spain

Keywords: Supervised Classification, Nearest Neighbor, k-NN, Machine Learning, Pattern Recognition.

Abstract: The k-Nearest Neighbor (k-NN) classification method assigns to an unclassified point the class of the nearest of a set of previously classified points. A problem that arises when applying this technique is that each labeled sample is given equal importance in deciding the class membership of the pattern to be classified, regardless of the typicalness of each neighbor.

We report on the application of a new hybrid version named Iterated Probabilistic Weighted k Nearest Neighbor algorithm (IPW-k-NN) which classifies new cases based on the probability distribution each case has to belong to each class. These probabilities are computed for each case in the training database according to the k Nearest Neighbors it has in this database; this is a new way to measure the typicalness of a given case with regard to every class.

Experiments have been carried out using *UCI Machine Learning Repository* well-known databases and performing 10-fold cross-validation to validate the results obtained in each of them. Three different distances (Euclidean, Canberra and Chebychev) are used in the comparison done.

1 INTRODUCTION

The nearest neighbor classification method assigns to an unclassified point the class of the nearest among a set of previously classified points. This rule is independent of the underlying joint distribution on the sample points and their classifications. An extension to this approach is the k-Nearest Neighbor (k-NN) method, in which classification is made taken into account the k nearest points and classifying the unclassified point by a voting criteria among this k points.

We present a new voting method which takes into account the fact that not all the cases in the database are typical representatives of the class they belong to (i.e., they could have some degree of exceptional-ity). This new method gives to each point in the training database a measure of its typicality regarding its neighbors and their typicality as well.

The undergone experimentation suggests that this new approach improves k-NN results in most of the databases tested.

The structure of this paper is as follows. The new proposed method is introduced in section 2, as well as a brief description of k-NN and Probabilistic Weighted

k Nearest Neighbor (PW-k-NN) methods. In section 3 we show the experimental results obtained and in final section 4 concluding remarks are given.

2 THE ITERATED PROBABILISTIC WEIGHTED K NEAREST NEIGHBOR METHOD (IPW-k-NN)

In this section the new proposed approach is presented as a new member of the distance based classification algorithms family. In order to introduce it, we present first the well known k-NN paradigm, then an extension of it that weights the neighbors with their probability of belonging to its class, and finally the new proposed approach IPW-k-NN is introduced.

2.1 The K-NN Method

A set of pairs $(x_1, \theta_1), (x_2, \theta_2), \dots, (x_n, \theta_n)$ is given, where the x_i 's take values in a metric space X upon which is defined a metric d and the θ_i 's take values

in the set $\{1, 2, \dots, M\}$ of possible classes. Each θ_i is considered to be the index of the category to which the i th individual belongs, and each x_i is the outcome of the set of measurements made upon that individual. We use to say that " x_i belongs to θ_i " when we mean precisely that the i th individual, upon which measurements x_i have been observed, belongs to category θ_i .

A new pair (x, θ) is given, where only the measurement x is observable, and it is desired to estimate θ by using the information contained in the set of correctly classified points. We shall call

$$x'_n \in \{x_1, x_2, \dots, x_n\}$$

the nearest neighbor of x if

$$\min d(x_i, x) = d(x'_n, x) \quad i = 1, 2, \dots, n$$

The Nearest Neighbor (NN) classification decision method gives to x the category θ'_n of its nearest neighbor x'_n . In case of tie for the nearest neighbor, the decision rule has to be modified in order to break it. A mistake is made if $\theta'_n \neq \theta$. An straightforward extension to this decision rule is the so called k-NN approach (Cover and Hart, 1967), which assigns to the candidate x the class which is most frequently represented in the k nearest neighbors to x .

Much research has been devoted to the k-NN rule (Dasarathy, 1991). We could give different weights to the variables in the distance computation, or different weights to each neighbor in the voting process. This last approach has been developed in the Probabilistic Weighted k Neighbor Method.

2.2 The Probabilistic Weighted K Nearest Neighbor Method (PW-k-NN)

In the k-NN algorithm each of the labeled samples is given equal importance in order to decide the class membership of the pattern to be classified, regardless of their "typicalness". Taking into account this fact, another approach might estimate each case's probability of belonging to its real class.

Sierra and Lazkano (Sierra and Lazkano, 2002) described a k-NN variation, the so called Probabilistic Weighted k Nearest Neighbor Method (PW-k-NN). In their work they use Bayesian Networks (Cowell et al., 1999) to estimate the probability distribution each case has of belonging to each class.

They use a Bayesian Network learned for classification task (Sierra and Larrañaga, 1998) among the predictor variables selected by a forward simple selection technique (Inza et al., 2000), and present a new voting method which takes into account the fact that not all the cases of the database are typical in the class they belong to. This new method gives to each point in the training database a measure of its typicality by using the learned Bayesian Network.

2.3 The Iterated Probabilistic Weighted K Nearest Neighbor Method (IPW-k-NN)

Our method is a new version of the Probabilistic Weighted k Nearest Neighbor Method, described in the previous section. Instead of using Bayesian Networks to estimate the probabilities, we take into account the nearest neighbors.

In a first step we estimate how typical is a case in its own class. This estimation is performed taken into account a number K_p of neighbors. Such estimation is stored in a bidimensional array P_{xy} , $x = 1 \dots n$, $y = 1 \dots M$, where P_{ij} stands for the probability that the case i belongs to class j . In each iteration (the number of iterations is given by a parameter called β), we compute the new value of each P_{ij} as a combination of the old value and an estimated P'_{ij} . A parameter called α determines the relative weight of these two values in the new P_{ij} . If, for example, provided $K_p = 10$, and five neighbors of case i belong to class θ_1 , four to class θ_2 and the last one to class θ_3 , we obtain, in a first step, that $P_{i1} = 0.5$, $P_{i2} = 0.4$ and $P_{i3} = 0.1$. In further iterations, the new value of P_{ij} is computed as $P_{ij} \leftarrow \alpha P'_{ij} + (1 - \alpha)P_{ij}$, where the weight of the former value of P_{ij} as well as P'_{ij} are weighted. When the first iteration has finished, each case in the training database has been associated to a probability array, which is, in its turn, used to compute P'_{ij} in following iterations.

In the second and last step a test case is classified according to their K_c neighbors. K_p^1 and K_c^2 may be different, because K_p is used to compute class probabilities in the training set and K_c to classify a test case.

The class which outputs IPW-k-NN for the case i is the class for which $\sum_{z=1}^{K_c} P_{zi}$ is maximum.

The new proposed method, IPW-k-NN is shown in its

¹ K_p stands for K neighbors in probability estimation task

² K_c stands for K neighbors in classification task

```

begin IPW-k-NN
As input we have the samples file TR, containing
n cases  $(x_i, \theta_i), i = 1, \dots, n$ ,
the value of  $K_p, K_c, \alpha$  and  $\beta$ 
and a new case  $(y, \theta)$  to be classified
 $\theta$  ranges over  $m$  classes
FOR each case  $(x_i, \theta_i)$  in TR DO
  BEGIN
  Search the k Nearest Neighbors of  $(x_i, \theta_i)$ 
  in TR -  $(x_i, \theta_i)$ 
  and store their TR index in  $Neigh_{ij}, j = 1, \dots, K_p$ 
  Initialize the probability array  $P_{ij}$  associated
  to the case  $i$  as follows:
    if  $\theta_i = j$  then  $P_{ij} \leftarrow 1$ 
    otherwise  $P_{ij} \leftarrow 0$ 
  END
FOR  $\beta$  iterations DO
  BEGIN
  FOR each case  $(x_i, \theta_i)$  in TR DO
    BEGIN
    Modify the associated probability array  $P_{ij}$ 
    as follows:
       $P'_{ij} \leftarrow (\sum_{z=1}^{K_p} P_{Neigh_{iz}j}) / K_p$ 
       $P_{ij} \leftarrow \alpha P'_{ij} + (1 - \alpha) P_{ij}$ 
    END
  END
  Search the  $K_c$  Nearest Neighbors of  $(y, \theta)$  in TR
  Reset the weights of all existing classes  $WC_i = 0$ 
  FOR each of the k-NN  $(x_k, \theta_k)$  DO
    BEGIN
    FOR each class  $i$  actualize its weight  $WC_i$ 
    as follows:
       $WC_i \leftarrow WC_i + P_{ki}$ 
    END
  Output the class  $\theta_i$  with greatest weight  $WC_i$ 
end IPW-k-NN

```

Figure 1: The pseudo-code of the Iterated Probabilistic Weighted k Nearest Neighbor Algorithm.

algorithmic form in Figure 1.

The algorithm works as follows: given

- a classification problem: to associate a case to a class among M different ones. Without loss of generality, we suppose classes are numbered from 1 to M .
- a set of n correctly classified cases, $(x_1, \theta_1), (x_2, \theta_2), \dots, (x_n, \theta_n)$, with θ_k the coded class number corresponding to case x_k
- a new case (y, θ) where the value of class θ is unknown
- four parameters, K_p, K_c, α and β ,

the following classification process is done:

1. For each case x_i in the training set TR (the set of cases whose class is known), compute the probability of belonging to each class. This is done taken into account the class of its neighbors. The more represented is a class among its neighbors the more weight will have this class in the probability distribution associated to x_i . As a first approach,

$$P_{ij} = (\text{number of neighbors in class } j) / K_p$$

But, given that we want to iterate this computation, because we would like to get a more accurate estimation of this probability (maybe the neighbor that says the case is not typical, it is a so called outlier in its turn!). Then, that equation becomes $P_{ij} = (\sum_{z=1}^k P_{Neigh_{iz}j}) / K_p$. And, given that we want to carry, in some way, the past value of P_{ij} , we use this expression to compute the definite value of P_{ij} in this iteration:

$$P_{ij} = \alpha \left(\sum_{z=1}^{K_p} P_{Neigh_{iz}j} \right) / K_p + (1 - \alpha) P_{ij}$$

where α controls the weight of new and old P_{ij} values and β is the number of iterations.

We would like to remark that this first step is a kind of preprocessing in the training set. Thus, it may be computed independently of the classification procedure and its result stored for further runnings.

2. Then, given the new case to be classified, search for its K_c nearest neighbors, and compute the sumatory of their probabilities for each class.

3. Associate to the new case the class θ with the highest value.

As it has been explained, this algorithm seeks in an iterative manner the neighbors of the neighbors of a case, in an attempt to associate to each case a more accurate probability distribution. This probability distribution guides further classification of new cases. Though this iterative search may be computationally expensive, it can be performed as a pre-process of the data, just computed once for each training set and used for every test case.

3 EXPERIMENTAL RESULTS

In this section the experimental work is presented. Several databases are used, and the new proposed

approach is executed over all of them, as well as the standard k-NN algorithm we are comparing with, in order to show the differences in the obtained results.

3.1 Databases

Seven databases are used to test our hypothesis. All of them have been obtained from the *UCI Machine Learning Repository* (Blake and Mertz, 1998). The characteristics of the databases are given in Table 1.

Parameter β (number of iterations) ranges from 1 to

Table 1: Details of experimental domains

Domain	Number of cases	Number of classes	Number of attributes
Diabetes	768	2	8
Heart	270	2	13
Ionosphere	351	2	34
Monk2	432	2	6
Pima	768	2	8
Wine	178	3	13
Zoo	101	7	16

10 and parameter α (relative weights of the two terms in expression 1) from 0 to 1, in steps of width 0.01.

3.2 Machine Learning Standard Classifiers Performance

We will describe briefly the paradigms we use in the undergone experiments, in order to compare the results with those obtained by the new approach. These paradigms come from the world of the Artificial Intelligence and are grouped in the family of *machine learning* (ML) paradigms (Mitchell, 1997).

3.2.1 Decision Trees

A *decision tree* consists of nodes and branches to partition a set of samples into a set of covering decision rules. In each node, a single test or decision is made to obtain a partition. The starting node is usually referred as the root node. In the terminal nodes or leaves a decision is made on the class assignment.

In each node, the main task is to select an attribute that makes the best partition between the classes of the samples in the training set. There are many different measures to select the best attribute in a node of the decision trees. In our experiments, we will use the well known decision tree induction algorithm C4.5 (Quinlan, 1993). Figure 2 shows the normal use of this kind of paradigms.

3.2.2 Rule Induction

One of the most expressive and human readable representations for learned hypothesis are the sets of *IF-THEN rules*, where in the *IF* part, there are conjunctions and disjunctions of conditions composed of the predictive attributes of the learning task, and in the *THEN* part, the class predicted for the samples that carry out the *IF* part appears.

We can interpret a decision tree like the set of rules generated by a rule induction classifier: the tests that appear in the way from the root of a decision tree to a leaf, can be translated to a rule's *IF* part, the predicted class of the leaf also in the *THEN* part appears. Some problems that may be overcome by the rule induction paradigm are: generation of simple rules when noise is present to avoid the overfitting and efficient rule generation when using large databases. In our experiments, we will use Clark and Niblet's (Clark and Niblet, 1989) *cn2* rule induction program. *Cn2* has been designed with the aim of inducing short, simple, comprehensible rules in domains where problems of poor description language and/or noise may be present. The rules are searched in a general-to-specific way, generating rules that satisfy large number of examples of any single class, and few or none of other classes. To use the rule set to classify unseen examples, *cn2* applies a "strict match" interpretation by which each rule is tried in order until one is found whose conditions are satisfied by the attributes of the example to classify.

3.2.3 Naive Bayes And Naive Bayes Tree Classifiers

Theoretically, Bayes' rule minimizes error by selecting the class y_j with the largest posterior probability for a given example \mathbf{X} of the form $\mathbf{X} = \langle X_1, X_2, \dots, X_n \rangle$, as indicated below:

$$P(Y = y_j | \mathbf{X}) = \frac{P(Y = y_j)P(\mathbf{X} | Y = y_j)}{P(\mathbf{X})}$$

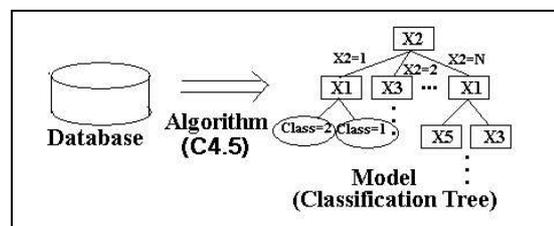


Figure 2: Typical example of a Machine Learning training algorithm.

Table 2: Details of accuracy level percentages for the databases.

Inducer	C4.5	NB	NBTree	CN2	Best
Diabetes	72.78	75.66	74.74	73.54	75.66
	±05.59	±04.43	±03.88	±05.14	
Heart	80.37	83.70	79.26	79.28	83.70
	±07.42	±05.84	±09.27	±07.23	
Ionosph.	89.75	84.90	89.20	92.32	92.32
	±04.67	±05.57	±04.51	±03.03	
Monk2	67.14	66.22	64.61	57.67	67.14
	±09.37	±08.86	±08.19	±09.16	
Pima	73.81	75.90	75.14	73.92	75.90
	±05.88	±05.96	±05.80	±26.08	
Wine	92.75	97.19	94.94	91.07	97.19
	±04.56	±03.96	±04.96	±08.93	
Zoo	93.10	87.09	96.09	93.09	96.09
	±06.71	±15.69	±05.05	±06.91	

Since \mathbf{X} is a composition of n discrete values, one can expand this expression to:

$$P(Y = y_j | X_1 = x_1, \dots, X_n = x_n) = \frac{P(Y = y_j)P(X_1 = x_1, \dots, X_n = x_n | Y = y_j)}{P(X_1 = x_1, \dots, X_n = x_n)}$$

where $P(X_1 = x_1, \dots, X_n = x_n | Y = y_j)$ is the conditional probability of the instance \mathbf{X} given the class y_j . $P(Y = y_j)$ is the a priori probability that one will observe class y_j . $P(\mathbf{X})$ is the prior probability of observing the instance \mathbf{X} . All these parameters are estimated from the training set. However, a direct application of these rules is difficult due to the lack of sufficient data in the training set to reliably obtain all the conditional probabilities needed by the model. One simple form of the previous diagnose model has been studied that assumes independence of the observations of feature variables X_1, X_2, \dots, X_n given the class variable Y , which allows us to use the next equality

$$P(X_1 = x_1, \dots, X_n = x_n | Y = y_j) = \prod_{i=1}^n P(X_i = x_i | Y = y_j)$$

where $P(X_i = x_i | Y = y_j)$ is the probability of an instance of class y_j having the observed attribute value x_i . In the core of this paradigm there is an assumption of independence between the occurrence of features values, that is not true in many tasks; however, it is empirically demonstrated that this paradigm gives good results in several tasks, typically in medical domains.

In our experiments, we use this Naive Bayes (NB) classifier. Furthermore, we use a Naive Bayes Tree (NBTree) classifier (Kohavi, 1996), which builds a decision tree applying the Naive Bayes classifier at the leaves of the tree.

Table 2 shows the accuracy obtained by these classifiers when applied over the chosen databases.

3.3 Experimental Method

To estimate the accuracy of the classifiers produced by our algorithm, we performed 10-fold cross-validation (Stone, 1974). In this validation method, the data set is partitioned into ten disjoint subsets. In each fold, one subset is held out as an independent test set and the remaining instances are used as the training set. A classification algorithm is then learned on the training set and tested on the test set.

In the new paradigm presented, a pre-processing is made on the data. All attribute values are scaled to the interval $[0,1]$. Extreme values are squashed by giving a scaled value of 0 (resp. 1) to any raw value that is less (resp. greater) than three standard deviations from the mean of that feature computed across all instances. We use this approach in order to limit the effect of outlying values.

3.4 k-NN VS. IPW-k-NN

We have tested our new method against the classical k-NN. Experiments have been carried out using three distances: Euclidean, Camberra and Chebychev (Michalsky et al., 1981). Table 3 shows the mathematical description of each of the used distances.

Table 3: Definition of Euclidean, Camberra and Chebychev distances.

Euclidean	$D(x,y) = \sqrt{\sum_{i=1}^m (x_i - y_i)^2}$
Camberra	$D(x,y) = \sum_{i=1}^m \frac{ x_i - y_i }{ x_i + y_i }$
Chebychev	$D(x,y) = \max_{i=1}^m x_i - y_i $

Tables 5, 6 and 7 show performances for each of this three distances obtained by both k-NN and IPW-k-NN over the seven databases.

In each table the best performance of k-NN and IPW-k-NN over the whole range of parameters along with the value of the parameters for which such performance is achieved (K for k-NN, K_p , K_c , α and β for IPW-k-NN).

The last five rows in the table show the percentage of

cases (among all the posible combinations of the four parameters) in which IPW-k-NN performance is significantly different from k-NN. We compare k-NN for a given K with IPW-k-NN for the same value of K_c . A Wilcoxon signed rank test (Wilcoxon, 1945) is carried out to check the significance level of differences between k-NN and IPW-k-NN. We have chosen a minimum significance level of 95%. The row labeled **equal** shows the percentage of cases where differences are no significatives under this level (95%). The row labeled **better 99**, shows the percentage of cases where IPW-k-NN outperforms k-NN with a significance of 99%. The row labeled **better 95** shows the percentage of cases where IPW-k-NN outperforms k-NN with a significance between 95% and 99%. The rows labeled **worse 99** and **worse 95** shows the percentage of cases where k-NN outperforms IPW-k-NN under those significance levels.

As it can be seen, in five out of seven databases, there are combination of value parameters for which improvement holds. But, in a blind search through the range of these parameters, we would have little chance to find these values, as they are fewer (in some cases they are very outnumbered) than values for which k-NN outperforms IPW-k-NN.

In the next section we show a first approach to try to characterize the parameter values for which IPW-k-NN outperforms k-NN.

3.5 Characterization Of Parameter Values

So far we have shown that IPW-k-NN outperforms k-NN for several values of K_p , K_c , α and β . But this would be almost useless if we are not able to characterize those values for which such improvement in performance holds.

To check if such characterization is possible we have constructed the set consisting of every combination of values of the four parameters along with the significance of the performance of the algorithm with those values over each database. So, the set consists of 700,000 instances, corresponding to 10 (range of K_p) x 10 (range of K_c) x 100 (range of α) x 10 (range of β) x 7 (number of databases). There is a different set for each distance (Euclidean, Camberra and Chebychev), so they are three different sets of 700,000 instances each. An instance is composed by four features (K_p , K_c , α , β) and the variable corresponding to the class it is associated to. There are three differents classes: IPW-k-NN outperforms k-NN with a significance of 95% (class 1), k-NN outperforms IPW-k-NN with a significance of 95% (class 2), and there is no significative difference between k-NN and IPW-k-NN (class 3).

ID3, C4.5 and Naive-Bayes classification algorithms

in MLC++ environment have been tested, in order to assess the success of all three in the task of characterizing the values of K_p , K_c , α and β , for which a significative improvement is achieved. The table shows the number of cases that each algorithm classifies as belonging to class 1, along with the real classification of that case. For example, in the first row we have that for the set of 700,000 instances corresponding to Euclidean distance, when using ID3 (Quinlan, 1986), a total amount of 8,564 cases are classified as belonging to class 1. But just 2,096 of them are really belonging to that class, because the rest of cases were misclassified, corresponding 2,130 cases to class 2, and 4,338 to class 3.

Table 4: Characterization of K_p , K_c , α and β using ID3, C4.5 and Naive-Bayes.

Distance	Classif.	class 1	class 2	class 3	Total
Euclidean	ID3	2096	2130	4338	8564
		24.47%	24.87%	50.65%	100%
	C4.5	2322	1502	2265	6089
		38.13%	24.67%	37.20%	100%
Naive-Bayes		33	10	35	78
		42.31%	12.82%	44.87%	100%
Camberra	ID3	535	1842	3067	5444
		9.83%	33.83%	56.34%	100%
	C4.5	301	295	284	880
		34.21%	33.52%	32.27%	100%
Naive-Bayes		0	0	0	0
		—	—	—	—
Chebychev	ID3	977	1135	4191	6303
		15.50%	18.01%	66.49%	100%
	C4.5	0	0	0	0
		—	—	—	—
Naive-Bayes		0	0	0	0
		—	—	—	—

Analyzing these results we observe that C4.5 algorithm over the set corresponding to Euclidean distance behaves rather well, with a 38% of cases well classified, against a 25% whose selection would make IPW-k-NN perform worse. We consider the rest of cases (37%) to be neutral, due to the no significance of the difference of performance over those parameters. From these tables we conclude that Camberra y Chebychev do not seem to be a good election, with bad classifications exceeding the number of good ones.

4 CONCLUSION AND FURTHER RESEARCH

In this work we have developed and tested a new distance based algorithm: Iterated Probabilistic

Weighted k Nearest Neighbor (IPW-k-NN). Three distance functions have been used in our experiments: Euclidean, Canberra and Chebychev.

We have shown that improvements over classic k-NN are achieved for some values of K_p , K_c , α and β , as well that a characterization of such values is possible using C4.5 and Euclidean distance.

Further research involves a more straightforward characterization of the values of parameters for which improvement holds.

An extension of the presented approach is to select among the feature subset that better performance presents regarding to classification. A Feature Subset Selection (Inza et al., 2000; Sierra et al., 2001) technique could be applied in order to select which of the predictor variables should be used. This could take advantage in the classifier execution process, as well as in the accuracy. A combination with another paradigms to improve the accuracy of each of them (Dietterich, 1997; Lazkano and Sierra, 2003) will also be experimented.

Experiments with different values of α , corresponding to different levels of neighbourhood β , might also be another line of research.

ACKNOWLEDGMENTS

This work is supported by the University of the Basque Country.

REFERENCES

- Blake, B. and Mertz, C. (1998). Uci repository of machine learning databases.
- Clark, P. and Niblett, T. (1989). The cn2 induction algorithm. *Machine Learning*, 3(4):261–283.
- Cover, T. and Hart, P. (1967). Nearest neighbour pattern classification. *IEEE Transactions on Information Theory*, 13(1):21–27.
- Cowell, R. G., Dawid, A. P., Lauritzen, S., and Spiegelhalter, D. J. (1999). *Probabilistic Networks and Expert Systems*. Springer.
- Dasarathy, B. (1991). *Nearest Neighbor (NN) Norms: NN Pattern Recognition Classification Techniques*. IEEE Computer Society Press.
- Dietterich, T. G. (1997). Machine learning research: four current directions. *AI Magazine*, 18(4):97–136.
- Inza, I., Larrañaga, P., Etxeberria, R., and Sierra, B. (2000). Feature Subset Selection by Bayesian network-based optimization. *Artificial Intelligence*, 123(1-2):157–184.
- Kohavi, R. (1996). Scaling up the accuracy of naive-bayes classifiers: a decision-tree hybrid. In *Proceedings of the Second International Conference on Knowledge Discovery and Data Mining*, pages 147–149.
- Lazkano, E. and Sierra, B. (2003). Bayes-nearest: a new hybrid classifier combining bayesian network and distance based algorithms. In *Proceedings of the EPIA 2003 Conference. Lecture Notes on Computer Science*. Springer-Verlag.
- Michalsky, R., Stepp, R., and Diday, E. (1981). *A Recent Advance in Data Analysis: Clustering Objects into Classes Characterized by Conjunctive Concepts*, pages 33–56. North-Holland.
- Mitchell, T. (1997). *Machine Learning*. McGraw-Hill.
- Quinlan, J. (1986). Induction of decision trees. *Machine Learning*, 1(1):81–106.
- Quinlan, J. (1993). *C4.5: Programs for Machine Learning*. Morgan Kaufmann Publishers, Inc.
- Sierra, B. and Larrañaga, P. (1998). Predicting survival in malignant skin melanoma using bayesian networks automatically induced by genetic algorithms. an empirical comparison between different approaches. *Artificial Intelligence in Medicine*, 14:215–230.
- Sierra, B. and Lazkano, E. (2002). Probabilistic-weighted k nearest neighbor algorithm: a new approach for gene expression based classification. In *KES'2002, Sixth International Conference on Knowledge-Based Intelligent Information Engineering Systems*, pages 932–939. IOS Press.
- Sierra, B., Lazkano, E., Inza, I., Merino, M., Larrañaga, P., and Quiroga, J. (2001). Prototype Selection and Feature Subset Selection by Estimation of Distribution Algorithms. A case Study in the survival of cirrhotic patients treated with TIPS. In *Proceedings of the Eighth Artificial Intelligence in Medicine in Europe. Lecture Notes on Artificial Intelligence*, pages 20–29. Springer-Verlag.
- Stone, M. (1974). Cross-validation choice and assesment of statistical predictions. *Journal of the Royal Statistic Society*, 36:111–147.
- Wilcoxon, F. (1945). Individual comparisons by ranking methods. *Biometrics*, 1:80–83.

Table 5: IPW-k-NN vs. k-NN (Euclidean distance)

Databases	Diabetes	Heart	Ionosphere	Monk2	Pima	Wine	Zoo
<i>k-NN Best</i>	74.16	83.33	85.71	76.51	74.68	95.00	94.00
	± 04.80	± 07.46	± 04.86	± 05.31	± 05.31	± 04.10	± 10.75
<i>K</i>	9	10	1	5	5	7	1
<i>IPW-k-NN Best</i>	76.75	84.82	86.00	76.74	76.88	95.00	95.00
	± 04.04	± 07.08	± 04.75	± 05.59	± 05.04	± 04.10	± 09.72
K_p	3	2	1	1	5	1	2
K_c	2	10	1	5	2	7	1
α	0.42	0.01	0.01	0.02	0.72	0.01	0.01
β	2	2	1	10	1	1	1
<i>equal</i>	64.46	85.56	85.25	20.17	64.45	76.37	56.07
<i>better 99%</i>	03.91	08.87	00.88	00.00	06.19	00.00	00.00
<i>better 95%</i>	04.59	04.37	02.25	00.82	05.23	00.00	00.00
<i>worse 99%</i>	19.04	00.03	07.56	62.06	19.42	09.98	33.26
<i>worse 95%</i>	08.00	01.17	04.06	16.95	04.71	13.65	10.67

Table 6: IPW-k-NN vs. k-NN (Camberra distance)

Databases	Diabetes	Heart	Ionosphere	Monk2	Pima	Wine	Zoo
<i>k-NN Best</i>	74.03	84.07	90.57	99.53	72.99	97.22	94.00
	± 03.35	± 07.21	± 04.27	± 00.98	± 03.76	± 02.93	± 09.66
<i>K</i>	5	10	1	1	5	1	1
<i>IPW-k-NN Best</i>	75.97	84.82	90.57	99.53	75.72	97.22	94.00
	± 04.91	± 08.45	± 04.27	± 00.98	± 04.86	± 02.93	± 09.66
K_p	3	3	1	1	1	1	1
K_c	6	10	1	1	8	1	1
α	0.33	0.13	0.01	0.01	0.07	0.01	0.01
β	4	9	1	1	10	1	1
<i>equal</i>	63.90	88.13	36.06	16.39	71.20	54.44	69.93
<i>better 99%</i>	05.79	01.36	00.00	00.00	07.67	00.00	00.00
<i>better 95%</i>	12.45	06.60	00.00	00.02	10.37	00.00	00.00
<i>worse 99%</i>	11.75	01.00	50.06	76.94	03.86	36.04	21.17
<i>worse 95%</i>	06.11	02.91	13.88	06.65	06.90	09.52	08.90

Table 7: IPW-k-NN vs. k-NN (Chebychev distance)

Databases	Diabetes	Heart	Ionosphere	Monk2	Pima	Wine	Zoo
<i>k-NN Best</i>	72.99	80.00	85.43	73.49	73.90	92.78	87.00
	± 04.81	± 11.48	± 05.94	± 06.86	± 04.56	± 06.44	± 08.23
<i>K</i>	10	3	5	2	9	8	1
<i>IPW-k-NN Best</i>	75.58	81.85	88.86	73.49	75.98	92.78	88.00
	± 05.26	± 09.63	± 05.12	± 06.86	± 05.59	± 07.88	± 07.89
K_p	9	1	5	1	1	1	2
K_c	7	3	1	2	9	3	1
α	0.72	0.51	0.08	0.01	0.37	0.01	0.01
β	1	3	10	1	4	1	1
<i>equal</i>	64.54	76.75	58.18	47.25	61.00	89.29	58.06
<i>better 99%</i>	02.73	00.00	15.48	00.00	06.12	00.00	00.00
<i>better 95%</i>	11.25	00.24	15.34	02.62	08.26	00.01	00.00
<i>worse 99%</i>	14.35	06.09	09.65	36.56	18.79	07.10	37.40
<i>worse 95%</i>	07.13	16.92	01.35	13.57	05.83	04.60	05.54