APPLICATION OF UNCERTAIN VARIABLES TO STABILITY ANALYSIS AND STABILIZATION FOR ATM ABR CONGESTION CONTROL SYSTEMS

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Abstract: The paper presents the application of uncertain variables to stabilization for ATM ABR congestion control systems. The time-varying system with unknown parameters is considered. The unknown parameter is assumed to be a value of uncertain variable described by the certainty distribution given by an expert. The estimation of the certainty index that the system is stable is presented. The estimation consists in the determination of the lower and upper bounds for the certainty index via the determination of sufficient and necessary stability conditions. A specific stabilization problem is considered.

1 INTRODUCTION

A theory of uncertain variables and their applications (Bubnicki, 2001a, 2001b, 2002b, 2004) developed in the recent years has been successfully applied to stability analysis for uncertain systems (Bubnicki, 2000; 2002a). The uncertain variable $\bar{x}$ is defined by a set of values X and the certainty distribution $h_x(x)=v(\bar{x}\cong x)$ given by an expert, where v denotes the certainty index that $\bar{x}$ is approximately equal to x. For the given set $D\subset X$ the certainty index that $\bar{x}$ approximately belongs to D is defined as follows

$$v(\bar{x}\,\tilde{\in}\,D)=\max_{x\in D} h_x(x)\,.$$

Computer networks are often treated as control systems, for which well-known methods of the control theory and artificial intelligence are applied. Taking into account a specific character of a network as a system, the control may concern various functions, for example – the prevention of congestion (Altman, et al., 1999; Benhomed, et al., 1993; Kolarov, et al., 1999). The purpose of this paper is to demonstrate how uncertain variables can be used for stability analysis and stabilization of the selected congestion control system in the ATM network, under assumption that in the description of uncertainty concerning the time-varying system, the parameters that are the values of uncertain variables occur.

2 THE MODEL OF THE CONTROL SYSTEM

One of the significant functions of the computer network is prevention of congestions. In this paper, the congestion control system proposed in (Kolarov, et al., 1999) is considered, where the problem of determination of the explicit rate for the ABR source has been resolved as a classical control problem in a closed-loop system (Figure 1). The control system is a computer network. The model of the system has the following form

$$\begin{aligned} y_{n+1} &= y_n + l_n^{(0)}u_{n+1} + l_n^{(1)}u_n + l_n^{(2)}u_{n-1} + \\ &\quad ... + l_n^{(k)}u_{n+1-k} + w_n, \end{aligned} \tag{1}$$

where the input of the system u_n is the common rate of the ABR sources, the output y_n is the number of ABR cells in a buffer of a switch port, w_n - the available bandwidth to ABR sources, is treated as an unmodelled disturbance. In the equation of the

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model, there appears the vector of the parameters,

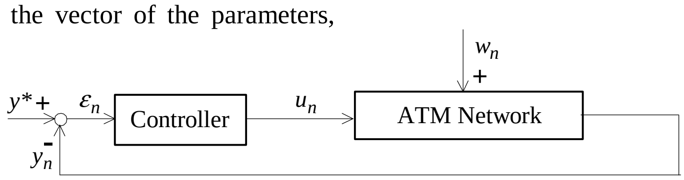

Figure 1: Block diagram of the closed-loop system

$l_n = [l_n^{(0)} \; l_n^{(1)} \ldots \; l_n^{(k)}]^T$, where $l_n^{(i)} \geq 0$ is the number of connections with a round trip delay equal to i $i \in \overline{0, k}$ time slots and k means the largest, measured round-trip delay. In this paper, it is assumed that l_n is a vector of unknown time-varying parameters. For determination of the u_n quantity, it is proposed to use the control algorithm described by the equation

$$u_{n+1} = u_n + a^{(0)}\varepsilon_n + a^{(1)}\varepsilon_{n-1} - b^{(0)}u_n - b^{(1)}u_{n-1} - b^{(2)}u_{n-2} - \ldots - b^{(k)}u_{n-k}, \quad (2)$$

where the control error $\varepsilon_n = y^* - y_n$ is the input of a controller, y^* is a buffer set point, $a = [a^{(0)} \, a^{(1)}]^T$ and $b = [b^{(0)} \, b^{(1)} \ldots b^{(k)}]^T$ are the vectors of controller parameters. The description of the above-mentioned system in a state space, assuming the state vector $x_n = [\varepsilon_{n-k-1} \; \varepsilon_{n-k} \; \ldots \; \varepsilon_n]^T$, has a form

$$x_{n+1} = C(a, b, l_n)x_n = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ \alpha_1 & \alpha_2 & \alpha_3 & \ldots & \alpha_{k+2} \end{bmatrix} x_n, \quad (3)$$

$\alpha_1 = a^{(1)} l_n^{(k)} - b^{(k)}$,

$\alpha_2 = a^{(0)} l_n^{(k)} + a^{(1)} l_n^{(k-1)} + b^{(k)} - b^{(k-1)}$,

$\alpha_3 = a^{(0)} l_n^{(k-1)} + a^{(1)} l_n^{(k-2)} + b^{(k-1)} - b^{(k-2)}$,

$\ldots$

$\alpha_{k+2} = a^{(0)} l_n^{0} + b^{(0)} - 2$.

3 STABILITY ANALYSIS

Applying the methods proposed in (Bubnicki, 2000, 2002a), the stability analysis of the considered congestion control system can be carried out, under assumption of time-varying character of a control system. This analysis consists in evaluation of the certainty index that considered system is stable.

Let us assume that the uncertainty concerning unknown parameters of the considered control system, and thus the number of the connections with the specified round-trip delay, is described by a set of allowable values

$$D_l(e, z) = \{ l \in L : \bigwedge_{i \in \{0,1,\ldots,k\}} (e^{(i)} \leq l^{(i)} \leq z^{(i)}) \} \quad (4)$$

in a following way

$$\bigwedge_n l_n \in D_l(e, z), \quad (5)$$

where $e = [e^{(0)} \, e^{(1)} \ldots \, e^{(k)}]^T$ is a vector of lower bounds of parameters l_n, $e^{(i)}$ is the minimum possible number of connections with a round-trip delay equal to i time slots, $z^{(i)}$ is the upper limit, which is an unknown parameter, the value of the uncertain variable with the certainty distribution $h_{z,i}(z^{(i)})$, ($i \in \overline{0, k}$) given by an expert.

The problem of a stability assessment can be formulated in the following way: for given certainty distributions $h_{z,i}(z^{(i)})$, one should determine estimation of the certainty index v_s that considered system is stable

$$v_w \leq v_s \leq v_g. \quad (6)$$

This estimation can be achieved by determination of the necessary condition $G(z)$ and sufficient stability condition $W(z)$ and then by determination of certainty indexes that conditions are satisfied

$$v_w = \max_{z \in D_{zw}} h_z(z), \qquad v_g = \max_{z \in D_{zg}} h_z(z),$$

where $D_{zw} = \{ z \in Z : W(z) \}$, $D_{zg} = \{ z \in Z : G(z) \}$ and $h_z(z) = \min\{ h_{z_1}(z_1), h_{z_2}(z_2), \ldots, h_{z_{(k+2)}}(z_{k+2}) \}$ is a joint certainty distribution.

In general $D_{zw} \subseteq D_{zg}$ and $D_{zg} - D_{zw}$ is called "a grey zone" which is a result of an additional uncertainty caused by the fact that: $W(z) \neq G(z)$.

Consider a congestion control system, in which, in respect of delays, only one kind of connections exists, i.e. all connections have the same round-trip delays ($k = 0$), the number of these connections equals l_n. The uncertainty concerning an unknown parameter l_n is described by the following formula

$$\bigwedge_n e \le l_n \le z, \tag{7}$$

where e (the minimum allowable number of connections) is given, whereas z is an unknown parameter, about which it is known that this parameter is a value of an uncertain variable with a certainty distribution $h_z(z)$. The description of the system in the state space has the following form

$$x_{n+1} = \begin{bmatrix} 0 & 1 \\ -a^{(1)} l_n - 1 & -a^{(0)} l_n + 2 \end{bmatrix} x_n. \tag{8}$$

Applying stability conditions proposed in (Bubnicki, 2000, 2002a) from the sufficient condition we obtained

$$D_{zw} = \{ z \in Z : \frac{1}{2a^{(0)}} < z < \frac{-1}{2(a^{(0)} + 2a^{(1)})},$$
$$a^{(0)} + a^{(1)} > 0,\ \ 3a^{(0)} + 4a^{(1)} < 0,\ a^{(1)} < 0,\ \ a^{(0)} > \frac{1}{2e} \},$$

from the necessary condition

$$D_{zg} = \{ z \in Z : (z < \frac{4}{a^{(0)}},\ a^{(1)} > 0) \vee (z < \frac{4}{a^{(0)} - a^{(1)}},$$
$$a^{(1)} < 0),\ a^{(0)} + a^{(1)} > 0 \}.$$

For the given triangular certainty distribution (z^*, g) described by the formula

$$h_z(z) = \begin{cases} \frac{1}{g} z - \frac{z^*}{g} + 1 & \text{for } z^* - g \le z \le z^*, \\ -\frac{1}{g} z + \frac{z^*}{g} + 1 & \text{for } z^* \le z \le z^* + g, \\ 0 & \text{otherwise}, \end{cases}$$

the certainty index that the necessary stability condition is satisfied

$$v_g = \begin{cases} 1 & \text{for } (\frac{4}{a^{(0)}} \ge z^*,\ a^{(1)} > 0) \\ & \wedge (\frac{4}{a^{(0)} - a^{(1)}} \ge z^*,\ a^{(1)} < 0), \\ & a^{(0)} + a^{(1)} > 0, \\ \frac{4}{g\,a^{(0)}} - \frac{z^*}{g} + 1 & \text{for} \\ & z^* - g \le \frac{4}{a^{(0)}} \le z^*, \\ & a^{(1)} > 0,\ a^{(0)} + a^{(1)} > 0, \\ \frac{4}{g(a^{(0)} - a^{(1)})} - \frac{z^*}{g} + 1 & \text{for} \\ & z^* - g \le \frac{4}{a^{(0)} - a^{(1)}} \le z^*, \\ & a^{(1)} < 0,\ a^{(0)} + a^{(1)} > 0, \\ 0 & \text{otherwise}, \end{cases}$$

and for $a^{(0)} + a^{(1)} > 0$, $3a^{(0)} + 4a^{(1)} < 0$, $a^{(0)} > \frac{1}{2e}$, $a^{(1)} < 0$ the certainty index that the necessary condition is satisfied

$$v_w = \begin{cases} 1 & \text{for } \frac{1}{2a^{(0)}} \le z^*,\ \frac{-1}{2(a^{(0)} + 2a^{(1)})} \ge z^*, \\ \frac{-1}{2g(a^{(0)} + 2a^{(1)})} - \frac{z^*}{g} + 1 & \text{for} \\ & z^* - g < \frac{-1}{2(a^{(0)} + 2a^{(1)})} < z^*, \\ \frac{-1}{g\,a^{(0)}} + \frac{z^*}{g} + 1 & \text{for} \\ & z^* \le \frac{1}{2a^{(0)}} \le z^* + g, \\ 0 & \text{otherwise}. \end{cases}$$

For example, for $z^* = 35$, $g = 15$, $a^{(0)} = 0.02$, $a^{(1)} = -0.018$, we obtained: $0.75 \le v_s \le 1$.

4 SYSTEM STABILIZATION

Having determined the stability conditions and corresponding certainty indexes, a task consisting in adequate design of the examined control system that aims at making the system stable, i.e. stabilization of the system, can be considered. The task of designing is a parametric problem, i.e. parameters in the set form of a control algorithm of an uncertain system should be determined. Stabilization of an uncertain system consists in maximization of the certainty index that considered system is stable. However, bearing in mind the fact that usually, as a result of a stability analysis, the value of the certainty index is not obtained directly but only its estimation (6) is obtained, the task of stabilization can be formulated as a selection of controller parameters that maximize the certainty index:

- v_w determined from the sufficient condition,
- v_w subject to constraint $v_g \le \alpha$, for a given $0<\alpha<1$ or $v_g - v_w \le \beta$, for a given $0<\beta<1$,
- v_g subject to constraint $v_w \ge \gamma$, for a given $0<\gamma<1$ or $v_g - v_w \le \lambda$, for a given $0<\lambda<1$.

The task of stabilization for the first of above-mentioned approaches consists in determination of a set of controller parameters D_c that are maximizing v_w:

$$D_c = \{ \begin{bmatrix} a^* \\ b^* \end{bmatrix} : \begin{bmatrix} a^* \\ b^* \end{bmatrix} = \arg\max_{a,b} v_w \}.$$

For the considered system, the result of this task is the set

$$D_c = \{ \begin{bmatrix} a^* \\ b^* \end{bmatrix} : z^* = [z^{*(0)} \quad z^{*(1)} \quad \dots \quad z^{*(k)}]^T \in D_{zw} \},$$

where $z^{*(i)} = \arg\max_{z^{(i)}} h_{z,i}(z^{(i)})$, $(i \in \overline{0,\, k})$.

The value of the certainty index v_w for controller parameters selected from the D_c, is equal 1.

For example, for $z^* = 50$, $g = 15$ and $e = 25$:

$$D_c = \{ \begin{bmatrix} a^{(0)} \\ a^{(1)} \end{bmatrix} : \frac{1}{2a^{(0)}} \le 50,\ \frac{-1}{2(a^{(0)} + 2a^{(1)})} \ge 50,$$

$$a^{(0)} + a^{(1)} > 0,\ 3a^{(0)} + 4a^{(1)} < 0,\ a^{(0)} > 0.02,\ a^{(1)} < 0 \}.$$

By selection of controller parameters from D_c, for example $a^{(0)} = 0.03$, $a^{(1)} = -0.018$ the certainty index that the system is stable $v_s = 1$ is obtained.

5 FINAL REMARKS

In this paper, the stability analysis and stabilization of the congestion control system in the ATM network with application of uncertain variables has been presented. The specific results in a form of estimation of the certainty indexes that considered control system is stable has been obtained. The selection of controller parameters, which assures stabilization of the examined system, has been considered. Uncertain variables can be utilized also for the quality analysis of control in the considered control system.

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