# MODEL PREDICTIVE CONTROL FOR DISTRIBUTED PARAMETER SYSTEMS USING RBF NEURAL NETWORKS

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Abstract: A new approach for the identification and control of distributed parameter systems is presented in this paper. A radial basis neural network is used to model the distribution of the system output variables over space and time. The neural network model is then used for synthesizing a non linear model predictive control configuration. The resulting framework is particular useful for control problems that pose constraints on the controlled variables over space. The proposed scheme is demonstrated through a tubular reactor, where the concentration and the temperature distributions are controlled using the wall temperature as the manipulated variable. The results illustrate the efficiency of the proposed methodology.

## 1 INTRODUCTION

In distributed parameter systems (DPS) inputs, outputs as well as parameters may change temporally and spatially due to diffusion, convection and/or dispersion phenomena. Such systems are quite common in chemical industries (tubular reactors, fluidized beds and crystallizers) and are mathematical described by systems of partial differential equations (PDE), where time and spatial coordinates are the independent variables.

The conventional approach for the synthesis of implementable control schemes for DPSs is based on methodologies that reduce the infinite order model to a finite (low) order model, which can capture the dominant behavior of the system. A comprehensive analysis of the recent developments in this direction can be found in Christofides (2001a). The most common approach found in the literature for an accurate model reduction implements a linear or a non linear Galerkin method to derive ODE systems that capture the slow (dominant) modes of the original DPS. In Christofides (2001b) one can find the analytical description of the linear Galerkin procedure as well as the nonlinear model reduction method which implements the concept of approximate inertial manifold. The resulting models are then used for synthesizing low dimensional robust output feedback controllers for quasi linear and nonlinear parabolic systems (Christofides and Daoutidis, 1996; 1997; Christofides, 1998; Shvartsman and Kevrekidis, 1998; Christofides and Baker 1999; Chiu and Christofides, 1999; El-Farra *et al.*, 2003; El-Farra and Christofides, 2004).

However, the analytical solution of the eigenvalue problem of the spatial differential operator is not always possible and consequently the selection of the appropriate basis to expand the PDEs is not an easy task. A systematic data driven methodology to address this problem is the Karhunen-Loève expansion (KL), also called proper orthogonal decomposition (POD) or empirical eigenfunctions (EEF) or principal component analysis. The KL expansion uses data snapshots and constructs the empirical eigenfunctions as a linear combination of those snapshots (Newman, 1996a; 1996b; Chatterjee, 2000). The resulting EEFs have been used as basis functions in the Galerkin procedure in a number of publications for accurate modelling and control in one-dimensional or two-dimensional systems. (Park and Cho, 1996a; 1996b; Park and Kim, 2000; Baker and Christofides, 1999; Shvartsman and Kevrekidis, 1998; Armaou and Christofides, 2002;)

The Galerkin procedure, mentioned so far uses analytical or empirical eigenfunctions and requires the mathematical description of the process, namely the exact system of PDEs. In case the PDEs are unknown, Gay and Ray (1995) proposed an identification procedure based on input-output data. The methodology employs integral equation models



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to describe the DPS and the singular value decomposition (SVD) of the integral kernel to produce an input/output model, suitable for model predictive control (MPC) methodologies. A comparison of the efficiency of this data driven model with the methods mentioned earlier can be found in Hoo and Zheng (2001). More recently, an identification method that combines KL and SVD for low order modeling and control have been presented (Zheng and Hoo, 2002; Zheng *et al.*, 2002a; 2002b; Zheng and Hoo, 2004). The discrete form of the SVD-KL method has also been used in MPC configurations with improved performance, comparatively to linear feedback controllers.

A neural network approach for the identification of DPSs has been attempted by Gonzáles-García *et al.* (1998) and more recently a combination of POD and neural networks has been proposed by Shvartsman *et al.* (2000). Padhi *et al.* (2001) used two sets of neural networks to map a DPS and a discrete dynamic programming format for the synthesis of an optimal controller. The same concept, also exploiting the POD technique for a lower order model, is presented by Padhi and Balakrishnan (2003).

In the present work, a radial basis function (RBF) neural network is proposed for the identification of non linear parabolic DPSs. RBF neural networks are quite popular for lumped system modeling because of their comparatively simple structure and their fast learning algorithms (Sarimveis *et al*., 2002). In this paper the RBF neural network is formulated, so that it is able to predict the distribution of the output variables over space. This way, an estimation of the system outputs is available in any position. The RBF model is then implemented in a nonlinear MPC configuration to predict the controlled variables in a finite number of positions.

The rest of the article is formulated as follows: In section 2 the structure of the RBF neural network for DPSs is presented. In section 3 the non linear MPC configuration is described in more details. The proposed methodology is tested through the application described in subsection 4.1 The efficiency of the RBF neural network is examined in subsection 4.2 and the controller performance in 4.3. In section 5, the final conclusions are summarized.

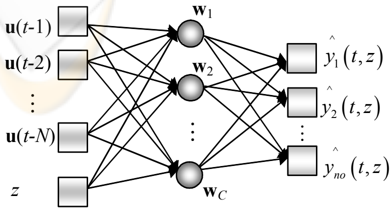


Figure 1: A radial basis function neural network of *C* hidden nodes for a distributed parameter system.

## 2 RBF NEURAL NETWORKS FOR MODELING DISTRIBUTED PARAMETER SYSTEMS

### 2.1 Quasi-linear parabolic DPS

In general, a quasi linear parabolic distributed parameter system is described by a set of partial differential equations and boundary conditions of the form of Eq. (1):

$$\begin{aligned}
&\frac{\partial \boldsymbol{\upsilon}(t,z)}{\partial t} = \mathbf{a}\frac{\partial^2 \boldsymbol{\upsilon}}{\partial z^2} - \mathbf{v}\frac{\partial \boldsymbol{\upsilon}}{\partial z} + \mathbf{u}(t,z) + \mathbf{G}(t,z), \quad \mathbf{a},\mathbf{v} > 0 \\
&\mathbf{y}(t,z) = \mathbf{C}(z)\cdot \boldsymbol{\upsilon}(t,z) \\
&\boldsymbol{\upsilon}(0,z) = \boldsymbol{\upsilon}_o, \quad 0 \le z \le L \\
&\frac{\partial \boldsymbol{\upsilon}}{\partial z}(t,0) = \mathbf{g}_o(t), \quad \frac{\partial \boldsymbol{\upsilon}}{\partial z}(t,L) = \mathbf{g}_l(t), \quad t > 0
\end{aligned} \tag{1}$$

where $\boldsymbol{\upsilon}(t,z)$ are the state variables, $\mathbf{u}(t,z)$ the manipulated variables and $\mathbf{y}(t,z)$ the controlled variables. $\mathbf{G}(t,z)$ is an additional non linear term of the model and $\mathbf{C}(z)$ is a function determined by the location of the sensors. Vectors $\boldsymbol{\upsilon}_o(z)$ and $\mathbf{g}_o(t)$, $\mathbf{g}_l(t)$ describe the initial and the Neumann boundary conditions of the system, respectively.

### 2.2 RBF neural network for DPS

Radial basis function networks are simple in structure neural networks that consist of three layers, namely the input layer, the hidden layer and the output layer. Development of an RBF network based on input-output data includes the computation of the number of nodes in the hidden layer and the respective centers and the calculation of the output weights, so that the deviation between the predicted and the real values of the output variables, over a set of training data, is minimized

An RBF neural network for modeling a DPS is constructed so that it can predict the values of the output variables at a specific spatial point (Figure 1). The input vector of such network at time point $t=kT_a$ (where $T_a$ is the sample time) contains past values of the input variables and the coordinates in space, where we wish to obtain a prediction:

$$\mathbf{x}(t,z)=\left[\mathbf{u}^T(t-1)\ \mathbf{u}^T(t-2)\ \ldots\ \mathbf{u}^T(t-N)\quad z\right]^T \tag{2}$$

For simplification we limit our analysis in only one dimension in space. Generalization to three dimensions is straightforward.

The neural network output is a vector containing the values of the process output variables at the location that is specified in the input vector:

$$\hat{\mathbf{y}}_{\mathrm{RBF}}(t,z)=\left[\hat{y}_1(t,z)\quad \hat{y}_2(t,z)\ \ldots\ \hat{y}_{no}(t,z)\right]^T \tag{3}$$

$$\hat{y}_j(t,z)=\sum_{c=1}^{C} w_{j,c}\cdot f\left(\left\|\mathbf{x}(t,z)-\mathbf{x}_c\right\|_2^2\right),\quad j=1,\ldots,no \tag{4}$$

In the previous equations $N$ is the number of past values for the input vector, $no$ is the number of the process output variables, $C$ is the number of hidden nodes, $\mathbf{w}_c$ is the weight vector corresponding to the output of the *cth* node, $f$ is the radial basis function and $\mathbf{x}_c$ is the center of the $c$ node. The method utilized to train neural networks in this work is based on a fuzzy partition of the input space and is described in details in Sarimveis *et al.* (2002).

# 3 NONLINEAR MPC FOR DPS

The nonlinear MPC configuration that is proposed in this work for controlling DPSs, uses the RBF model to predict the values of the controlled variables over a future finite horizon $ph$ at a number of locations $ns$, where measurements are available. Then, an optimization problem is solved, so that both the deviations of the controlled variables from their set points over the prediction horizon and the control moves over a control horizon $ch$, are minimized. The objective function is of the following form:

$$\min_{\substack{\mathbf{u}(t+k|t)\\ k=0,\ldots,ch-1}}\left(\sum_{j=1}^{ns}\sum_{k=1}^{ph}\left\|\mathbf{W}_{k,j}\left(\hat{\mathbf{y}}(t+k,z_j\,|\,t)-\mathbf{y}_j^{sp}\right)\right\|_2^2+\sum_{k=0}^{ch-1}\left\|\mathbf{R}_k\Delta\mathbf{u}(t+k\,|\,k)\right\|_2^2\right) \tag{6}$$

$$\hat{\mathbf{y}}(t+k,z_j\,|\,t)=\hat{\mathbf{y}}_{\mathrm{RBF}}(t+k,z_j)+\mathbf{d}(t,z_j\,|\,t),\quad j=1,\ldots,ns,\quad k=1,\ldots,ph \tag{7}$$

$$\mathbf{u}_{\min}\le\mathbf{u}(t+k\,|\,t)\le\mathbf{u}_{\max},\quad k=0,\ldots,ch-1 \tag{8}$$

where $\hat{\mathbf{y}}(t+k,z_j\,|\,t)$ is the prediction made at time point $t$ for the output vector at time $t+k$ and at location $z_j$, $ns$ is the number of sensors, $\mathbf{d}(t,z_j\,|\,t)$ is the estimated disturbance at time point $t$, considered constant over the prediction horizon and $\mathbf{y}_j^{sp}$ is the set point at the location of the $j$ sensor. For $k=ch,\ldots,ph$ the manipulated variables are considered to remain constant. $\mathbf{W}_k$ and $\mathbf{R}_k$ are weight matrices of appropriate dimensions.

# 4 APPLICATION

## 4.1 Description of the process

One typical distributed parameter system in chemical engineering is a tubular reactor, where variables depend on both time $t$ and reactor length $z$. The mass and energy balances, concerning a first order reaction, diffusion and convention phenomena, are described by two quasi-linear PDEs with Neumann boundary conditions (Eqs. (9-(12)).

$$\frac{\partial T}{\partial t}=\frac{1}{P_{eh}}\frac{\partial^2 T}{\partial z^2}-\frac{1}{L_e}\frac{\partial T}{\partial z}+\eta c\exp\left[\gamma\left(1-\frac{1}{T}\right)\right]+\mu\left(T_w(t,z)-T\right) \tag{9}$$

$$\frac{\partial c}{\partial t}=\frac{1}{P_{em}}\frac{\partial^2 T}{\partial z^2}-\frac{\partial c}{\partial z}-D_a c\exp\left[\gamma\left(1-\frac{1}{T}\right)\right] \tag{10}$$

$$z=0,\ -\frac{\partial T}{\partial z}(t,0)=P_{eh}\cdot\left(T_i(t)-T(t,0)\right),\ z=1,\ \frac{\partial T}{\partial z}(t,1)=0 \tag{11}$$

$$z=0,\ -\frac{\partial c}{\partial z}(t,0)=P_{em}\cdot\left(c_i(t)-c(t,0)\right),\ z=1,\ \frac{\partial c}{\partial z}(t,1)=0 \tag{12}$$

where $T(t,z)$, $c(t,z)$ are dimensionless temperature and concentration respectively inside the reactor, $T_i(t)$, $c_i(t)$ are dimensionless temperature and concentration at the entrance of the reactor and $T_w(t,z)$ is the wall temperature. The values of the parameters of Eqs. (9)-(12) can be found in previous publications (Hoo and Zheng, 2001; 2002).

## 4.2 RBF model efficiency

An input-output training set was created using the wall temperature $T_w$, at $z=$ [0 0.33 0.66] as the manipulated variable, while the output variables (temperature and concentration) were recorded at 21 spatial locations. The PDEs were solved using the PDE Matlab toolbox. More specifically, we simulated the system by changing randomly the input variables and recording the output responses using a sample period of $T_a$=0.5 time units. The training set consisting of 2000 data points was generated considering $N$=3 past values of each manipulated variable. Deviation variables were used by subtracting from all the input and output values the corresponding steady states. Several neural network structures were developed by changing the initial fuzzy partition in the fuzzy means training algorithm. The produced neural networks were tested using a new validation data set of 500 data that was developed in the same way with the training set, but was not involved in the training phase. The sum of squares errors (SSEs) for the different RBF structures are presented in Table 1. In Figure 2, the actual values and the predictions of the neural network consisting of 152 nodes are compared.

Table 1: Performance of RBF neural networks

| Hidden nodes *C* | SSE *T* | SSE *c* |
|---|---|---|
| 13 | 0.2012 | 0.8873 |
| 27 | 0.1251 | 0.7523 |
| 68 | 0.0535 | 0.3628 |
| 86 | 0.0404 | 0.2232 |
| 152 | 0.0332 | 0.1420 |
| 207 | 0.0295 | 0.1104 |

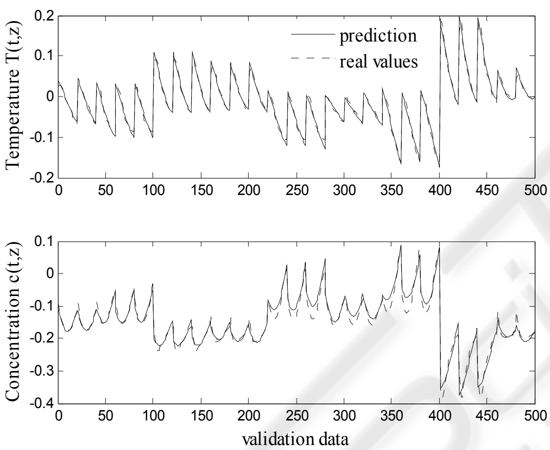

Figure 2: Actual values and predictions of the deviation variables for a neural network consisting of 152 hidden nodes

## 4.3 MPC performance

To test the proposed MPC configuration, we first simulated the example presented in Zheng and Hoo (2002). In that case, the temperature is the only controlled variable at $z$=[0:0.25:1] where we assume that sensors are available, while concentration is measured at $z$=1 but is not controlled. A disturbance is introduced to the system by decreasing the feed concentration $C_i$ by 5%. We tested the proposed MPC scheme using for prediction the RBF network that consists of 27 nodes and the following parameter values: $ch$=6, $ph$=10, $\mathbf{W}$=1, $\mathbf{R}=5\cdot\mathbf{I}_3$. The optimization problem that was formulated at each time instance was solved using the *fmincon* Matlab function. The performance of the controller is depicted in Figure 3, where the temperature distributions at the initial steady state and after 7 time units are compared. The responses at locations where sensors are available are also presented in the same figure. The proposed controller managed to reject the disturbance and produce zero steady state error. The obtained responses outperform the performances of a PI controller and an MPC configuration that utilizes the SVD-KL model. The responses of the two controllers are presented in Hoo and Zheng, (2002) and are not shown here due to space limitations. The temperature at the exit of the reactor returns to its initial value after 1.5 time units, while 6 time units are required by the system to produce zero steady state error along the length of the reactor.

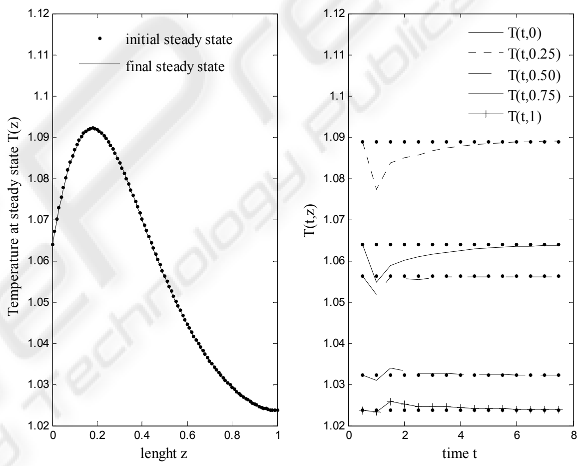

Figure 3: The final temperature distribution and the dynamic response to a 5% decrease in $C_i$ using the RBF model

A second performance test forces the system to reach a new steady state distribution. The actual steady state, where the temperature finally settles, is compared with the desired set point in Figure 4. The dynamic responses at locations where sensors are available are also presented in the same figure. The responses show that the system approaches the desired values quickly, avoiding overshoots. The behavior of the manipulated variables is depicted in Figure 5.

The last simulation presented in this work uses concentration at the reactor exit as an additional controlled variable. As far as the temperature profile is concerned, the target is to reach the same set point change as previously. Figures 6 and 7 present the responses of the temperature (at locations where sensors are available) and the concentration (at the reactor exit) respectively. They also present the final distribution of both variables, after 20 time units. Figure 8 depicts the control actions over time. It is obvious that due to the additional controlled variable the performance of the controller is slightly deteriorated as far as the dynamic behavior is concerned. However, the desired steady state is still approached satisfactorily.

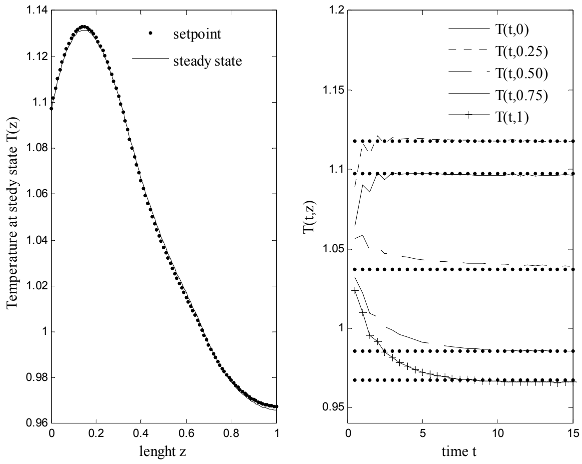


Figure 4: The temperature distribution after 15 time units and the dynamic response to a set point change

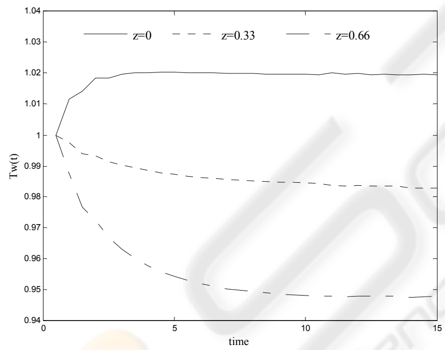


Figure 5: The manipulated variable $T_w(t)$ at $z$=0, 0.33 and 0.66

# 5 CONCLUSIONS

A nonlinear input/output identification method for distributed parameter systems is proposed is this paper. An RBF neural network capable to predict the output variables over space is developed. The accuracy of the neural network was established through a tubular reactor simulation. The model is then used for the synthesis of a MPC configuration that minimizes the deviation of the prediction of the controlled variables at a finite number of positions, where a sensor is assumed to exist. The proposed method produced satisfactory results in both disturbance rejection and set point change problems. The performance of the controller was found to be superior to PI controllers or linear MPC configurations presented in former publications.

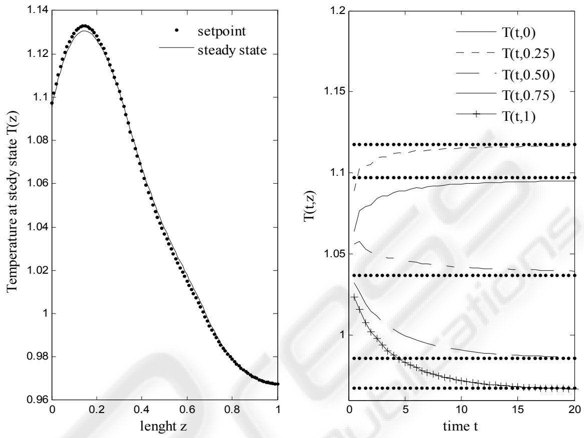


Figure 6: The temperature distribution after 20 time units and responses to a set point change when considering $c(t,1)$ as an additional controlled variable

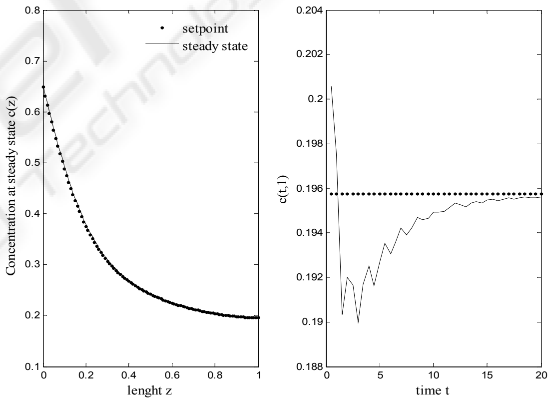


Figure 7: The concentration distribution after 20 time units and the response of $c(t,1)$

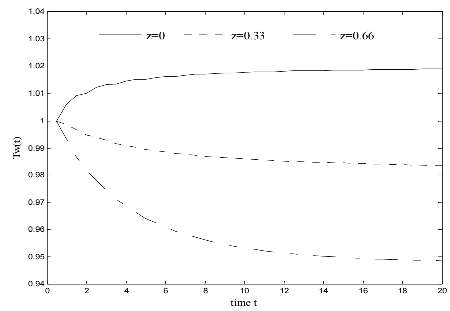


Figure 8: The manipulated variable $T_w(t)$ at $z$=0, 0.33 and 0.66 when considering $c(t,1)$ as an additional controlled variable

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