

JAVA BASED TOOLBOX FOR LINEAR REPETITIVE PROCESSES

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Abstract: In the paper a Java based toolbox has been presented. It is used in teaching of a special case of nD systems - Linear Repetitive Processes (LRP). Its predecessor has been developed in the Matlab environment so to use it a Matlab licence is necessary. This restriction has been removed after making it available in the Internet as a Java based program. Now a student, browsing a web page, may define a model together with initial / boundary conditions, then simulate a process as a continuous or discrete case, analyze the results in graphical or numerical form, modify visualization parameters of the plots and finally print the results. In the paper an overview of the tool has been given.

1 INTRODUCTION

The multidimensional (Roesser, 1975), (Fornasini and Marchesini, 1978) (nD) nature of dynamics of Linear Repetitive Processes (Rogers and Owens, 1992) is much more difficult to understand for students than dynamics of classical, e.g. 1-dimensional (1D) systems. Propagation of dynamics in more than one dimension, a built-in interactions of previous and current system variables (called passes), causes additional difficulties. In a repetitive process, on each pass, an output, termed the *pass profile*, is produced which acts as a forcing function on, and hence contributes to the dynamics of the next pass profile. The 2D systems structure of a repetitive process arises from information propagation in (i) the pass to pass direction, and (ii) along a given pass. Such a process may be presented graphically – see Figure 1 below.

We quote that the explicit interaction between successive pass profiles is the source of the novel control (and numerical) problems for these processes in that the output sequence of pass profiles can contain oscillations that increase in amplitude in the pass to pass direction.

Moreover, we define more than one stability no-

tion for repetitive processes (unlike as in classical 1D dynamic systems). A given process may be stable asymptotically, stable along the pass, stable horizontally and vertically. All of them have clear interpretation in physical processes. This fact may potentially decrease students understanding of the problem.

Even 'simple' processes (SISO case, 'smooth' control and initial conditions, 'rounded' matrices, etc.) often cause 'unpredictable' results of simulations. Another example is that we often observe a kind of wave oscillation that depends on the length of pass and dynamic properties of a given process. The number of such 'anomalies' seemed to be enough motivation to design and develop a proper educational tool.

Above we have mentioned only a few difficulties, that we often observe during students classes. The toolbox, in our intention, should help students in their in-deep understanding of a fascinating nature of repetitive processes.

1.1 Discrete case

The state space model of a discrete linear repetitive process has the following form (Rogers and Owens, 1992)

$$\begin{aligned}x_{k+1}(p+1) &= Ax_{k+1}(p) + Bu_{k+1}(p) + B_0 y_k(p) \\y_{k+1}(p) &= Cx_{k+1}(p) + Du_{k+1}(p) + D_0 y_k(p)\end{aligned}\tag{1}$$

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where indices $k = 0, 1, \dots$ and $p = 0, 1, \dots, \alpha - 1$ represent pass number(s) and points (position) on the given pass respectively and

- $x_k(p)$ is the state vector of dimension $n \times 1$,
- $y_k(p)$ is the output vector, called the pass profile vector, of dimension $m \times 1$,
- $u_k(p)$ is the bounded input vector of dimension $r \times 1$.

The real matrices are then as follows

$$A^{n \times n}, B^{n \times r}, B_0^{n \times m}, C^{m \times n}, D^{m \times r}, D_0^{m \times m}.$$

The constant α is a finite and fixed number called the *pass length*. This model, may be clearly recognized as a 2D state-space model which resembles the well known Roesser model (Roesser, 1975). It also posses some features of the second main 2D state-space model, i.e. the Fornasini-Marchesini one (Fornasini and Marchesini, 1978).

To complete the process description, it is necessary to specify the state and pass initial conditions, i.e. the initial state vector on each pass $x_{k+1}(0)$, $k = 0, 1, \dots$ and the initial pass profile $y_0(p)$, $p = 0, 1, \dots, \alpha - 1$. The simplest possible case is

$$\begin{aligned} x_{k+1}(0) &= d_{k+1}(0), k = 0, 1, \dots \\ y_0(p) &= f(p), p = 0, 1, \dots, \alpha - 1 \end{aligned} \quad (2)$$

where d_{k+1} is an $n \times 1$ vector with constant entries and $f(p)$ is an $m \times 1$ vector whose entries are known bounded functions of discrete time p . These conditions are frequently called, by analogy to the classical 1D systems, initial conditions. However, in fact they are clearly boundary conditions in their nature.

A more general form of (2) called dynamic initial conditions (extended state initial conditions) for (1) is defined as

$$x_{k+1}(0) = d_{k+1}(0) + \sum_{j=0}^{\alpha-1} K_j y_k(j). \quad (3)$$

Note. The process state space model (1) has the so-called unit memory property, i.e. it is only the pass profile on the previous pass which (explicitly) contributes to the current one. Non-unit memory linear repetitive processes are the natural generalization of (1) where a finite number, say $M > 1$, of previous pass profiles (explicitly) contribute to the current one. Such processes are not considered here since the results given for the unit memory special case generalize in a natural manner.

1.2 Continuous case

The state space model of a differential linear repetitive process has the following form over $0 \leq t \leq \alpha$, $k =$

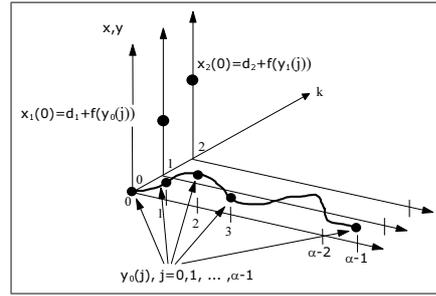


Figure 1: Schematic illustration of the dynamics of a repetitive process (1) with extended version of state initial conditions (3)

$0, 1, \dots$ (Rogers and Owens, 1992)

$$\begin{aligned} \dot{x}_{k+1}(t) &= \hat{A}x_{k+1}(t) + \hat{B}u_{k+1}(t) + \hat{B}_0 y_k(t) \\ y_{k+1}(t) &= \hat{C}x_{k+1}(t) + \hat{D}u_{k+1}(t) + \hat{D}_0 y_k(t). \end{aligned} \quad (4)$$

Initial or boundary conditions are defined similarly as in the discrete case and due to space limitations we omit them here.

To simulate (4) one must first build a discrete equivalent of (4), hence the problem considered now is as follows: given a repetitive process of the form (4), construct a discrete approximation of the proper form over $p = 0, 1, \dots, \alpha - 1$, $k = 0, 1, \dots$ i.e. the model as in (1) and (2).

The matrices in the discrete case (1) are to be computed from those of (4) by formulas determined by a particular numerical approximation method used (Gramacki et al., 2002), (Gramacki, 2000), (Rogers et al., 2002), (Gałkowski et al., 1999). The approximate solution generated by (1) and (2), should be as close as possible (in a well defined sense) to the exact solution obtained from (4) (assuming that it is known or may be calculated with negligible errors). Moreover, crucial system properties of (4) such as stability, should be preserved in (1) and (2) or conditions (which can be verified numerically) under which this is true should be given.

Figure 1 gives schematic illustration of the evolution of the dynamics of a repetitive process.

2 THE TOOLBOX

In this section we present some main features of the toolbox (Szumacher, 2004) and describe some selected technical details of its implementation.

2.1 Functionality

In its current state, the toolbox can, amongst other tasks, simulate and display the response of differential and discrete linear repetitive processes and compute, using a user specified numerical integration technique, a discrete approximation to the dynamics of a differential process. Using the toolbox in both Java and Matlab versions we may:

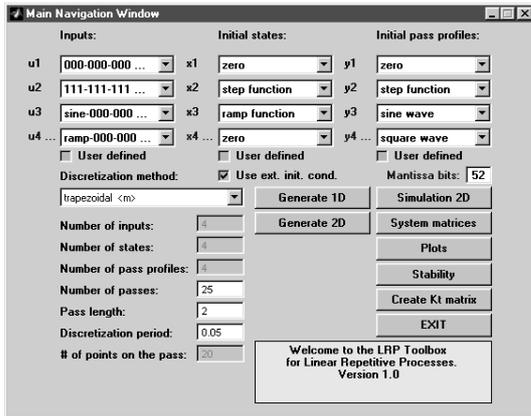


Figure 2: The Main Navigation Window of the Matlab based version of the toolbox. A MIMO (4 inputs, 4 states, 4 outputs), continuous process (in a state ready for simulation) is shown. Some elements do not appear in the Java based version. See Table 1 for details of those differences.

- define a discrete or continuous repetitive process in the form of system matrices in (1) or (4),
- in the case of a continuous process (4), choose a discretization method (e.g. trapezoidal 2D approximation). For a discrete case we simply choose "no discretization",
- based on process dimensions determined from system matrices, define inputs u , initial states $x_{k+1}(0)$ and initial pass profile(s) $y_0(p)$ (2) from a number of predefined values,
- define inputs u , initial states $x_{k+1}(0)$ and initial pass profile(s) $y_0(p)$ from scratch (matrix variables of proper dimensions, taken directly from a Matlab's workspace). We may also use extended version of state initial condition (3)(available only in a Matlab based version),
- define "spatial" characteristics of a process, i.e. how many passes are to be simulated and how long each pass is. For a discrete case, the last value is defined as a number of points on the pass. For a continuous case it is defined as two numbers: pass length (in a unit of measure) and discretization period,
- simulate a described process,

- analyze simulation results as 3D and/or 2D (Matlab version only) plots of state(s) x and pass profile(s) y . If necessary, one may easily narrow up and down to a required subset of passes and a required subset of points on a given pass,
- analyze simulation results by inspecting numerical outputs in a dedicated Java based window or by inspecting Matlab's workspace variables,
- analyze stability conditions of a repetitive process (Matlab based version only).

Figures 2 to 6 show some selected windows of both versions of toolboxes. The Main Navigation Window of the Java based version, due to space limitations and its availability on the Internet has been omitted.

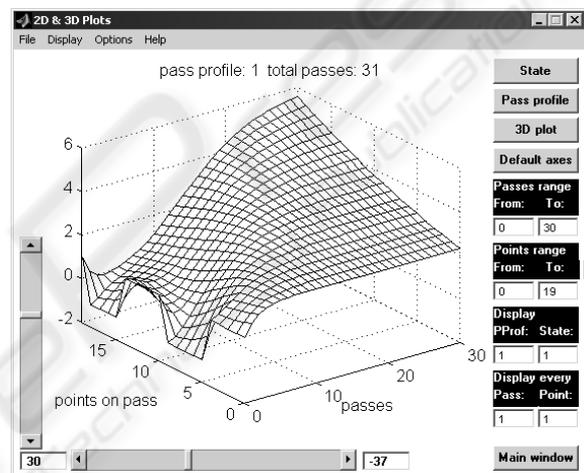


Figure 3: The Matlab based version. A 3D plot of results of simulation of a given LRP process. Note the elements which help 'visualization' of the resulted plots (vertical and horizontal strollers and edit boxes on the right side of the window). The LRP process is the same as on Figure 4

2.2 Data Format Specification

Here we describe the data structures specification as well as some other related tasks necessary to simulate a discrete model defined by (1) and (2). The basic user supplied data required is as follows:

- the matrices which define the LRP model,
- the pass length α ,
- the number of passes, say K , over which the simulation is to be run,
- the sequence of input vectors $u_k(p)$, $k = \{0, 1, \dots, K\}$, $0 \leq p \leq \alpha$,
- the initial state vector sequence $x_k(0)$, $k = \{0, 1, \dots, K\}$,

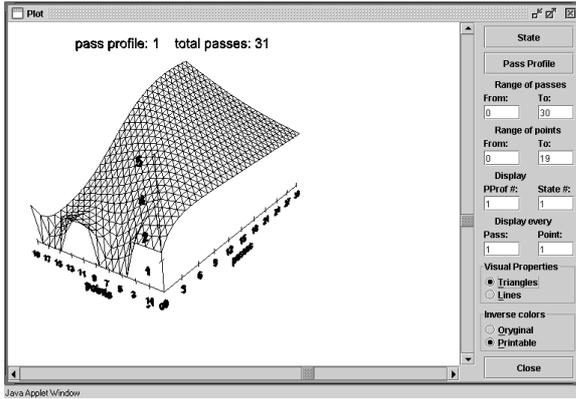


Figure 4: The Java based version. A 3D plot of results of simulation of a given process. Note the elements which help 'visualization' of the resulted plots (vertical and horizontal strollers ,edit boxes and radio buttons on the right side of the window). The LRP process is the same as on Figure 3

- the initial pass profile $y_0(p)$, $0 \leq p \leq \alpha$,
- the sampling period T .

Note: According to the convention adopted in the development stage, the first pass is numbered 0 (zero).

Assuming this data has been supplied, the toolbox calculates:

- the initial state vector for each pass $x_k(p)$, $k = \{0, 1, \dots, K\}$, $0 \leq p \leq \alpha$,
- the pass profile at each instant along each pass $y_k(p)$, $k = \{0, 1, \dots, K\}$, $0 \leq p \leq \alpha$.

Given T and α , the number of points P is calculated according the following formula: $P = (\alpha/T) + 1$ (rounded to integer value if necessary).

Consider now the storage of the sequence of control vectors u for each pass. A natural approach would be to store values of control sequence for a given pass in an array of r rows (number of inputs) and P columns (number of points). Hence there are K passes and one should simply add a third dimension to the array. Unfortunately, when the first release of the toolbox appeared, MATLAB (version 4.2) did not supported multidimensional (that is for $n \geq 3$) arrays. Hence the control sequences for each pass are stored in one (potentially 'large') two-dimensional array where each pass occupies P respective columns. The same method is used for the state initial state sequence x_0 , the initial pass profile y_0 , the computed sequence of state vectors x , and the computed sequence of pass profiles y . Figure 5 gives a schematic illustration of the format of these matrices.

To illustrate the computations, consider a discrete-time process (1) defined by the following matrices

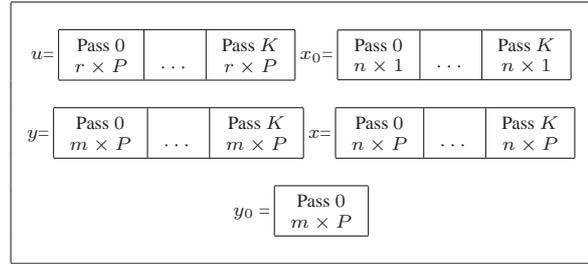


Figure 5: Format details of input and output vectors for linear repetitive processes (vectors u , x_0 , y_0 , x , y). r – number of inputs, n – number of states, m – number of pass profiles (outputs), P – number of points on a given pass, K – number of passes.

$$\begin{aligned}
 A &= \begin{bmatrix} 1 & -2 & -1 \\ 3 & -5 & 1 \\ 1 & -2 & 0 \end{bmatrix} & B &= \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 3 & -1 \end{bmatrix} \\
 B_0 &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ -1 & -1 & -1 & -1 \end{bmatrix} & C &= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\
 D &= \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ -1 & -2 \\ -1 & -2 \end{bmatrix} & D_0 &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \end{bmatrix}
 \end{aligned} \tag{5}$$

here $r = 2$, $n = 3$, $m = 4$. Suppose also that $\alpha = 2$, $T = 1$ and hence $P = (\alpha/T) + 1 = 3$. The inputs are as follows.

$$\begin{aligned}
 x_0 &= \left[\begin{array}{ccc|cc} NaN & & & 1 & 2 \\ NaN & & & 1 & 2 \\ NaN & & & 1 & 2 \end{array} \right] \leftarrow \begin{array}{l} x_0^1 \\ x_0^2 \\ x_0^3 \end{array} \\
 y_0 &= \left[\begin{array}{ccc|c} 1 & 1 & 1 & \leftarrow y_0^1 \\ 1 & 1 & 1 & \leftarrow y_0^2 \\ 1 & 1 & 1 & \leftarrow y_0^3 \\ 1 & 1 & 1 & \leftarrow y_0^4 \end{array} \right] \\
 u &= \left[\begin{array}{ccc|ccc|ccc} NaN & NaN & NaN & 1 & 1 & 1 & 2 & 2 & 2 \\ NaN & NaN & NaN & 1 & 1 & 1 & 2 & 2 & 2 \end{array} \right] \leftarrow \begin{array}{l} u^1 \\ u^2 \end{array}
 \end{aligned} \tag{6}$$

where the superscripts are used to denote the entries in the corresponding vector. Here we have 3 states, 4 pass profiles and 2 inputs. Then the resulting state and pass profile vectors for the case of $K = 3$ are as follows

$$\begin{aligned}
 x &= \left[\begin{array}{ccc|ccc|ccc} NaN & NaN & NaN & 1 & 7 & 16 & 2 & 7 & 19 \\ NaN & NaN & NaN & 1 & 2 & 10 & 2 & 5 & 4 \\ NaN & NaN & NaN & 1 & -4 & 0 & 2 & -7 & 12 \end{array} \right] \\
 y &= \left[\begin{array}{ccc|ccc|ccc} 1 & 1 & 1 & 12 & 14 & 35 & 17 & 20 & 68 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -5 & -47 \\ 1 & 1 & 1 & 2 & 4 & 25 & 7 & 10 & 58 \\ 1 & 1 & 1 & -9 & -9 & -9 & -11 & -15 & -57 \end{array} \right]
 \end{aligned} \tag{7}$$

where *NaN* (*not a number*) denotes entries in the relevant matrices which are only necessary for computational information purposes (the control sequence u and initial state x_0 are not defined for pass number 0 for (4).

2.3 Implementation

The toolbox has been fully implemented in Java (Java applets) technology and hence it can be started from practically any WWW browser currently in use. Although to use it, it is necessary to install some 'additional to standard' software on a client machine. See section *Instalations* below for details.

For a graphical user interface we used a standard Java AWT (Abstract Window Toolkit) – the standard API for providing graphical user interfaces (GUIs) for Java programs (Java-AWT, 2005). For present-

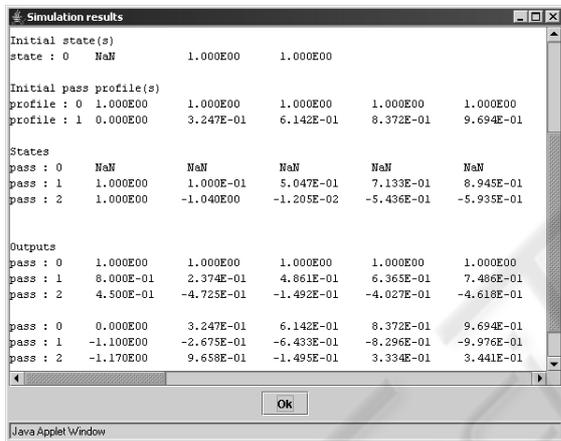


Figure 6: The Java based version. One may see, in numerical format, both input data (Initial state(s), Initial pass profile(s)) and results of simulation (States, Outputs). For visible reasons, unnatural small number of passes = 2 was used. A discrete MIMO process (2 inputs, 1 state, 2 outputs) was simulated. Initial pass profiles are a square function and a sine function.

ing graphical results of simulations we used a Java3D library – the library for creation of three-dimensional graphics applications and Internet-based 3D applets (Java 3D, 2005).

For matrix operations we use JAMA package (JAMA, 2005). JAMA is a basic linear algebra package for Java. It provides user-level classes for constructing and manipulating matrices. It seems that it is intended to serve as the standard matrix class for Java.

Due to space limitations we omit Java code examples. They are however available on request from the authors.

In the Java based version only the basic discretization methods for converting a continuous process to

its discrete-time equivalent have been implemented. The following methods have been implemented: forward, backward, trapezoidal, which seems to be enough for educational purposes. See (Gramacki et al., 2002) for more details and methods.

3 COMPARISON OF JAVA AND MATLAB VERSIONS

Although the Java version of the toolbox seems to be enough for student's needs, some additional possibilities are accessible only in Matlab based version. In Table 1 we enumerate those that are important from user's point of view. Some limitations are due to Java applets technology used.

Table 1: Comparison of Java and Matlab based versions of the toolbox

Feature	Java version	Matlab version
Maximum number of simulated passes	limited	unlimited
Maximum pass length	limited	unlimited
Support for extended initial conditions (3)	no	yes
Support for user defined control, initial and boundary conditions	no, only predefined values	yes, predefined values and/or any values taken from the Matlab's workspace
Predefined system matrices	yes	no, but user may read them from matfiles and/or from the Matlab's workspace
Stability analysis	no	yes, for SISO case only
Number of discretization methods	4	14
Plot modes	3D, useful in analysis of a whole process	3D and 2D, 2D case useful in analysis of a single pass
Plot of a subset of simulated points / passes	yes	yes
Calculation of a horizontal and vertical pass profiles	no	yes
Results in numerical form	yes, available in Java applet's window	yes, available in the Matlab's workspace
Precision of calculations	fixed, 52 bits of mantissa	variable, 1 to 52 bits of mantissa

4 CONCLUSIONS, FUTURE WORKS AND INSTALLATION

4.1 Conclusions

The paper shortly presents two computer tools successfully used in teaching activities of rather complex multidimensional systems - repetitive processes. After an introduction of repetitive processes, the main functionality of these tools have been given.

The two versions, based on Matlab and Java, differ rather considerably. This was intentionally assumed. A Java based version is intended for a wider family of users. It is freely available from the Internet and although shows only basic features of repetitive processes, it seems to be enough for a starting point.

The Matlab based version is intended for a more professional family of users. It seems to be a proper tool in assisting of research in a field of repetitive process and nD systems in general.

4.2 Future Works

At the moment, the Matlab based version of the toolbox supplies much more functionality than its Java based equivalent. It may be purposeful to enrich somehow the last. However due to some real Java applets limitation, not all changes will be possible to implement. A plain Java program may also be considered. They are also works going on to migrate the Matlab based version to non-commercial and free package for scientific and numerical computations Scilab (Scilab, 2005).

4.3 Instalation and Availability

In order to work with the applet it is necessary to install *locally* a Java Runtime Environment (JRE) which allows end-users to run Java applications.

It is also necessary to install the Java3D package which enables the creation of three-dimensional graphics and Internet-based 3D applets. One may download it for free for Windows (Java 3D, 2005) (first look for something like *Download Java 3D x.y.z software* where x.y.z is a release number and then look for something like *Java 3D for Windows (OpenGL Version) Runtime for the JRE*). A version for Linux is also available for free.

Due to Java applets properties / limitations, to use for example a system clipboard sometimes it is necessary to change your *java.policy* settings. See your browser documentation for details. A reader may familiarize with the toolbox by visiting the page

<http://www.uz.zgora.pl/~jgramack/LRP/lrp.html>.

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