CONTROL OF DISCRETE LINEAR REPETITIVE PROCESSES WITH VARIABLE PARAMETER UNCERTAINTY

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Abstract: This paper is devoted to solving the problem of stabilising an uncertain discrete linear repetitive process, where the model uncertainty is a result of the variable along the pass uncertainty of the parameters. The analysis is applied to the engineering example of the material rolling process, which can be modelled as a repetitive process (Rogers and Owens, 1992; Gałkowski et al., 2003b). Due to its analytical simplicity and due to computational effectiveness, the LMI based approach to design a robust state controller for 2D systems has been used here.

1 INTRODUCTION

Repetitive processes are a distinct class of 2D systems of both system theoretic and applications interest. The essential unique characteristic of such a process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. This, in turn, leads to the unique control problem for these processes in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.

The analysis of linear repetitive processes has received considerable attention in the literature — see, for example, (Rogers et al., 2005; Gałkowski and Wood, 2001; Roberts, 2000; Rogers and Owens, 1992). Although these processes are well known, many control design problems for them are still (relatively) open. This stems from the fact that control of these processes using standard (or 1D) systems theory/algorithms fails (except in a few very restrictive special cases) precisely because such an approach ignores their inherent 2D systems structure, i.e. information propagation occurs from pass-to-pass and along a given pass, and also the pass initial conditions are reset before the start of each new pass.

Material rolling is one of a number of physically based problems which can be modelled as a linear repetitive process (Rogers and Owens, 1992). In this paper, we use material rolling as a basis to illustrate numerically algorithms the solution we develop to a currently open robust stability and stabilization problem for the underlying sub-class of so-called discrete linear repetitive processes. The design itself can be executed in terms of a linear matrix inequality (LMI) which, in turn, can be solved with well established effective numerical algorithms (Gahinet et al., 1995; Nesterov and Nemirovskii, 1994).

Inn physical applications, the system or process parameters are most often not known exactly and only some nominal values or admissible intervals are available. Hence, although the nominal process is most often time invariant, the uncertain process can be time variant. This will be the case here and to

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solve the problem we generalize previously reported LMI based design algorithms for uncertain repetitive processes (Gałkowski et al., 2002) and other classes of systems (Daafouz and Bernussou, 2001) with polytopic uncertainty. As the resulting solution requires the presence of a convex uncertainty admissible set, the question then arises as to what can be done in cases where this is not true. Here we propose a solution to this problem by first obtaining the convex hull of the original non-convex uncertainty set.

Throughout this paper, the null matrix and the identity matrix with appropriate dimensions are denoted by 0 and I, respectively. Moreover, a matrix, say $M <$ which is real symmetric and positive definite is denoted by $M > 0$.

2 MATERIAL ROLLING AS A LINEAR REPETITIVE PROCESS

Material rolling is an extremely common industrial process where, in essence, deformation of the workpiece takes place between two rolls with parallel axes revolving in opposite directions.

In practice, a number of models of this process can be developed depending on the assumptions made on the underlying dynamics and the particular mode of operation under consideration. Here, we restrict attention to a linearised model of the dynamics. In particular, following, for example (Gałkowski et al., 2003b), the model considered is a so-called discrete linear repetitive process whose state space model is of the form

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p) + Bu_{k+1}(p) + B_0 y_k(p) \quad (1)\\ y_{k+1}(p) &= Cx_{k+1}(p) + Du_{k+1}(p) + D_0 y_k(p) \end{aligned}$$

Here on pass k, $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the pass profile vector, $u_k(p) \in \mathbb{R}^l$ is the vector of control inputs. To detail the structure for our material rolling example, first introduce

$$u_{k+1}(p) = F_M(p)$$

$$x_{k+1}(p) = \begin{bmatrix} y_{k+1}(p-1) & y_{k+1}(p-2) & y_k(p-1) & y_k(p-2) \end{bmatrix}^T$$

$$A = \begin{bmatrix} a_1 & a_2 & a_4 & a_5 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} b \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad B_0 = \begin{bmatrix} a_3 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad (2)$$

$$C = \begin{bmatrix} a_1 & a_2 & a_4 & a_5 \end{bmatrix} \quad D = b \quad D_0 = a_3$$

where

$$a_1 = \frac{2M}{\lambda T^2 + M} \qquad a_2 = \frac{-M}{\lambda^2 T + M}$$

$$a_3 = \frac{\lambda}{\lambda T^2 + M}\left(T^2 + \frac{M}{\lambda_1}\right) \qquad a_4 = \frac{-2\lambda M}{\lambda_1(\lambda T^2 + M)}$$

$$a_5 = \frac{\lambda M}{\lambda_1(\lambda T^2 + M)} \qquad b = \frac{-\lambda T^2}{\lambda_2(\lambda T^2 + M)}$$

The systems variables in above expressions are: $y_{k+1}(p)$ and $y_k(p)$, which denote thickness of the material on the current and previous pass respectively, M is the lumped mass of the roll-gap adjusting mechanism, λ_1 is the stiffness of the adjustment mechanism spring, λ_2 is the hardness of the material strip, $\lambda = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$ is the composite stiffness of the material strip and the roll mechanism. Finally, $F_M(p)$ is the force developed by the motor and T is the sampling period.

To complete the process description it is necessary to specify the pass length and the initial, or boundary, conditions, i.e. the pass state initial vector sequence and the initial pass profile. Here these are taken to be of the form

$$\begin{aligned} x_{k+1}(0) &= d_{k+1}, \ k \geq 0 \\ y_0(p) &= f(p), \ 0 \leq p \leq \alpha - 1 \end{aligned} \qquad (3)$$

where d_{k+1} is an $n \times 1$ vector with constant entries and $f(p)$ is an $m \times 1$ vector whose entries are known functions of p. For ease of presentation, we make no further explicit reference to the boundary conditions except in Section 5 where a numerical example is given.

3 STABILITY AND STABILIZATION OF DISCRETE LINEAR REPETITIVE PROCESSES

The stability theory (Rogers and Owens, 1992) for linear repetitive processes consists of two distinct concepts but here it is the strongest of these, termed stability along the pass, which is of interest. In essence, this property (recall the unique control problem for these processes) demands bounded-input bounded-output stability (defined in terms of the norm on the underlying function space) uniformly, i.e. independent of the pass length. Several sets of necessary and sufficient conditions for its existence but in this paper it is the following result which is the starting point. Even though this condition is sufficient but not necessary, it forms a basis for control law design, a feature which is not present in currently available necessary and sufficient conditions.

Theorem 1 *(Gałkowski et al., 2003a) A discrete linear repetitive process described by* (1) *is stable along the pass if* $\exists$ *matrices* $W_1 > 0$ *and* $W_2 > 0$ *such that the Lyapunov inequality*

$$\widehat{A}^T W \widehat{A} - W < 0 \qquad (4)$$

holds and $W = \operatorname{diag}\{W_1, W_2\} > 0$, *where*

$$\widehat{A} = \begin{bmatrix} A & B_0 \\ C & D_0 \end{bmatrix} \qquad (5)$$

To provide a Lyaunov interpretation of this result (which will be used extensively in the analysis to follow in this paper), introduce the candidate Lyapunov function as

$$\mathcal{V}(k,p) = x_{k+1}^T(p)W_1x_{k+1}(p) + y_k^T(p)W_2y_k(p) \quad (6)$$

where $W_1 > 0, W_2 > 0$, with associated increment

$$\Delta\mathcal{V}(k,p) = x_{k+1}^T(p+1)W_1x_{k+1}(p+1) + y_{k+1}^T(p)W_2 y_{k+1}(p) - x_{k+1}^T(p)W_1x_{k+1}(p) - y_k^T(p)W_2y_k(p). \quad (7)$$

Then it is easy to show that

$$\Delta\mathcal{V}(k,p) < 0 \quad (8)$$

is equivalent to (4). For this reason, and the quadratic structure of the Lyapunov function, stability along the pass is also referred to as quadratic stability.

An extensively analyzed control law for the processes considered here has the following form over $0 \le p \le \alpha - 1, k \ge 0$

$$u_k(p+1) = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x_k(p) \\ y_{k-1}(p) \end{bmatrix} \quad (9)$$

where K_1 and K_2 are appropriately dimensioned matrices to be designed. In effect, this control law is composed of the weighted sum of current pass state feedback and feedforward of the previous pass profile.

The LMI of (4) extends in a natural manner to the design of (9) for stability along the pass (or quadratic stability), but here we will use the approach based on (Peaucelle et al., 2000) and first adopted for repetitive processes in (Gałkowski et al., 2003a). This will prove to be of particular use in the analysis of the case when there is uncertainty in the model.

Theorem 2 *(Gałkowski et al., 2003a) Suppose that a control law of the form* (9) *is applied to a discrete linear repetitive process of the form described by* (1)*. Then the resulting process is stable along the pass if* $\exists$ *matrices* $W = \operatorname{diag}\{W_1, W_2\}$, $W_1 > 0, W_2 > 0$, $G = \operatorname{diag}\{G_1, G_2\}$, *and* $N = \operatorname{diag}\{N_1, N_2\}$, *such that*

$$\begin{bmatrix} -G - G^T + W & (\widehat{A}G + \widehat{B}N)^T \\ \widehat{A}G + \widehat{B}N & -W \end{bmatrix} < 0 \quad (10)$$

If this condition holds, stabilizing K_1 *and* K_2 *in the control law* (9) *are given by* $K = NG^{-1}$, *where* $K = \operatorname{diag}\{K_1, K_2\}$ *and* $\widehat{B}$ *is given by*

$$\widehat{B} = \operatorname{diag}\{B, D\} \quad (11)$$

4 ROBUST STABILITY AND STABILIZATION OF DISCRETE LINEAR REPETITIVE PROCESSES

The design of control laws for discrete linear repetitive processes has been the subject of much research effort (Gałkowski et al., 2003b; Gałkowski et al., 2003a; Gałkowski et al., 2002) and here we continue the development of this general area by giving new results relating to the practical case when there is uncertainty associated with the process (state space model) description. In particular, we consider the case when the model matrices $\{\widehat{A}, \widehat{B}\}$ are not precisely known, but belong to a convex bounded (polytope type) uncertain domain $\mathcal{D}$. This, in turn, means that any uncertain matrix can be written as a convex combination of the vertices of the polytope $\mathcal{D}$ defined as follows

$$\mathcal{D} = \Big\{ \Big[\widehat{A}(\xi(k,p)), \widehat{B}(\xi(k,p))\Big] : \Big[\widehat{A}(\xi(k,p)), \widehat{B}(\xi(k,p))\Big] = \sum_{i=1}^{v} \xi_i(k,p) \Big[\widehat{A}_i, \widehat{B}_i\Big]; \sum_{i=1}^{v} \xi_i(k,p) = 1; \; \xi_i(k,p) \ge 0; \; k \ge 0; \; 0 \le p \le \alpha - 1 \Big\} \quad (12)$$

where v denotes the number of vertices. Note also that the uncertainty here is variable in both independent directions of information propagation, i.e. along the pass (depends on p) and pass-to-pass (depends on k).

Now we can write the following linear parameter dependent system describing the process dynamics

$$\begin{aligned} x_{k+1}(p+1) &= A\big(\xi(k,p)\big)x_{k+1}(p) + B\big(\xi(k,p)\big)u_{k+1}(p) \\ &\quad + B_0\big(\xi(k,p)\big)y_k(p) \\ y_{k+1}(p) &= C\big(\xi(k,p)\big)x_{k+1}(p) + D\big(\xi(k,p)\big)u_{k+1}(p) \\ &\quad + D_0\big(\xi(k,p)\big)y_k(p) \end{aligned} \quad (13)$$

and also the parameterized candidate Lyapunov function

$$\begin{aligned} \mathcal{V}(k,p,\xi(k,p)) &= x_{k+1}^T(p)^T W_1(\xi(k,p))x_{k+1}(p) \\ &\quad + y_k^T(p)W_2(\xi(k,p))y_k(p) \end{aligned} \quad (14)$$

with

$$W_1(\xi(k,p)) = \sum_{i=1}^{v} \xi_i(k,p)W_{1i}$$

$$W_2(\xi(k,p)) = \sum_{i=1}^{v} \xi_i(k,p)W_{2i} \quad (15)$$

Also $\mathcal{V}(0,0,\xi(0,0)) < \infty$ and the Lyapunov function increment is given by

$$\begin{aligned} \Delta\mathcal{V}(k,p,\xi(k,p)) &= x_{k+1}^T(p+1)W_1\big(\xi(k,p+1)\big) x_{k+1}(p+1) + y_{k+1}^T(p)W_2\big(\xi(k+1,p)\big)y_{k+1}(p) - x_{k+1}^T(p) \\ &\quad W_1\big(\xi(k,p)\big)x_{k+1}(p) - y_k^T(p)W_2\big(\xi(k,p)\big)y_k(p) \end{aligned} \quad (16)$$

Hence we can define the so-called poly-quadratic stability for the repetitive processes considered here (see (Daafouz and Bernussou, 2001) for the 1D systems case).

Definition 1 *A discrete linear repetitive process described by* (1) *with uncertainty defined by* (12) *is said to be poly-quadratically stable provided*

$$\Delta\mathcal{V}\big(k,p,\xi(k,p)\big) < 0 \quad (17)$$

$\forall\, k \ge 0, 0 \le p \le \alpha - 1$

The requirement of (17) is easy seen to be equivalent to

$$\widehat{A}\big(\xi(k,p)\big)^T \operatorname{diag}\left\{W_1\big(\xi(k,p+1)\big), W_2\big(\xi(k+1,p)\big)\right\} \widehat{A}\big(\xi(k,p)\big) - \operatorname{diag}\left\{W_1\big(\xi(k,p)\big), W_2\big(\xi(k,p)\big)\right\} < 0 \tag{18}$$

where $\widehat{A}$ of (5) now becomes

$$\widehat{A}\big(\xi(k,p)\big) = \begin{bmatrix} A\big(\xi(k,p)\big) & B_0\big(\xi(k,p)\big) \\ C\big(\xi(k,p)\big) & D_0\big(\xi(k,p)\big) \end{bmatrix} = \sum_{i=1}^{v} \xi_i(k,p)\widehat{A}_i \tag{19}$$

and $\widehat{A}_i$ is a polytope vertex, see (12).

Remark 1 *When* $\operatorname{diag}\{W_1\big(\xi(k,p+1)\big), W_2\big(\xi(k+1,p)\big)\} = \operatorname{diag}\{W_1\big(\xi(k,p)\big), W_2\big(\xi(k,p)\big)\} = W$ *then poly-quadratic stability reduces to quadratic stability as in Corollary 1.*

Now we have the following result from (Cichy et al., 2005) which (drawing on the work in (Daafouz and Bernussou, 2001)) aims to minimize the conservativeness present from the use of a sufficient but not necessary stability condition.

Theorem 3 *A discrete linear repetitive process of the form described by* (1) *with uncertainty defined by* (12) *is poly-quadratically stable if* $\exists$ *block diagonal matrices* $\widehat{S}_i$, $i = 1,2,\ldots,v$, *i.e.* $\widehat{S}_i = \operatorname{diag}\{S_{i1}, S_{i2}\}$, *and matrices* $\widehat{G} = \operatorname{diag}\{G_1, G_2\}$ *such that*

$$\begin{bmatrix} \widehat{G} + \widehat{G}^T - \widehat{S}_i & \widehat{G}^T \widehat{A}_i^T \\ \widehat{A}_i \widehat{G} & \widehat{S}_j \end{bmatrix} > 0 \tag{20}$$

for all $i, j = 1,2,\ldots,v$.

With the control law (9) applied, (20) becomes

$$\begin{bmatrix} \widehat{G} + \widehat{G}^T - \widehat{S}_i & \widehat{G}^T (\widehat{A}_i + \widehat{B}_i K)^T \\ (\widehat{A}_i + \widehat{B}_i K)\widehat{G} & \widehat{S}_j \end{bmatrix} > 0 \tag{21}$$

where $K = \operatorname{diag}\{K_1, K_2\}$. The following result now gives a sufficient condition for the existence of a poly-quadratically stabilizing control law of the form (9) for the case under consideration.

Theorem 4 *Suppose that a control law of the form* (9) *is applied to a discrete linear repetitive process described by* (1) *with uncertainty defined by* (12). *Then the resulting process is poly-quadratically stabilizable if* $\exists$ *symmetric matrices* $\widehat{S}_i > 0$, $i = 1,2,\ldots,v$, *i.e.* $\widehat{S}_i = \operatorname{diag}\{S_{i1}, S_{i2}\}$ *and* $\widehat{G} = \operatorname{diag}\{G_1, G_2\}$, $\widehat{N} = \operatorname{diag}\{N_1, N_2\}$, *such that the following LMI is feasible*

$$\begin{bmatrix} \widehat{G} + \widehat{G}^T - \widehat{S}_i & \widehat{G}^T \widehat{A}_i^T + \widehat{N}^T \widehat{B}_i^T \\ \widehat{A}_i \widehat{G} + \widehat{B}_i \widehat{N} & \widehat{S}_j \end{bmatrix} > 0 \tag{22}$$

for all $i, j = 1,\ldots,v$. *If this condition holds then stabilizing* K_1 *and* K_2 *in the control law are given by* (9) *with*

$$K = \widehat{N}\widehat{G}^{-1} \tag{23}$$

where $K = \operatorname{diag}\{K_1, K_2\}$.

Proof: Follows immediately from (21) on setting $K\widehat{G} = \widehat{N}$.

5 APPLICATION TO THE MATERIAL ROLLING EXAMPLE

In this section we apply our new design to the material rolling model of Section 2 when the model parameters $T, M, \lambda_1, \lambda_2$ are uncertain. In order to avoid a control law with very large entries in the defining matrices, we limit attention to solutions of the LMI (22) where the matrix N is diagonal. The boundary conditions are $x_{k+1}(p) = 0, k \geq 0$ and $y_0(p) = 1, 0 \leq p \leq 14$, and in the simulations given below $x_k^1(p)$ denotes the first entry of the state vector $x_k(p)$, the number of passes is 26, and the number of points along the pass is 15.

Consider first the case when the discretization period belongs to the interval

$$T \in [\underline{T},\ \overline{T}] = [0.21, 0.25] \tag{24}$$

and the rest of parameters satisfy

$$M \in [\underline{M},\ \overline{M}] = [90, 110], \quad \lambda_1 \in [\underline{\lambda_1},\ \overline{\lambda_1}] = [430, 600]$$
$$\lambda_2 \in [\underline{\lambda_2},\ \overline{\lambda_2}] = [1970, 2070]. \tag{25}$$

Here the uncertainty domain has 16 vertices, but is not convex and hence the new design procedure developed in this paper cannot be applied. To overcome this difficulty, we first use the Geometric Bounding Toolbox (GBT) to numerically estimate the minimum convex domain (i.e. convex hull) which covers the original non-convex one. (This, of course, introduces extra conservativeness.)

The resulting convex domain which now be used to execute our new design algorithm for the example considered has the following 6 vertices

Vertex 1

$$A = \begin{bmatrix} 1.7502 & -0.0041151 & -1.4491 & 0.72457 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1.7502 & -0.0041151 & -1.4491 & 0.72457 \end{bmatrix}$$

$$B = \begin{bmatrix} -6.0343e-5 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad B_0 = \begin{bmatrix} 0.84948 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$D = -6.0343e-5 \qquad D_0 = 0.84948$$

Vertex 2

$$A = \begin{bmatrix} 1.5159 & -0.001699 & -1.162 & 0.58098 \\ & * & & \end{bmatrix}$$

$$C = \begin{bmatrix} 1.5159 & -0.001699 & -1.162 & 0.58098 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.00012288 \\ * \end{bmatrix} \qquad B_0 = \begin{bmatrix} 0.82305 \\ * \end{bmatrix}$$

$$D = -0.00012288 \qquad D_0 = 0.82305$$

Vertex 3

$$A = \begin{bmatrix} 1.6886 & -0.0024702 & -1.2944 & 0.6472 \\ * & & & \end{bmatrix}$$

$$C = \begin{bmatrix} 1.6886 & -0.0024702 & -1.2944 & 0.6472 \end{bmatrix}$$

$$B = \begin{bmatrix} -7.9026e-5 \\ * \end{bmatrix} \qquad B_0 = \begin{bmatrix} 0.80288 \\ * \end{bmatrix}$$

$$D = -7.9026e-5 \qquad D_0 = 0.80288$$

Vertex 4

$$A = \begin{bmatrix} 1.6035 & -0.0028319 & -1.3277 & 0.66386 \\ * & & & \end{bmatrix}$$

$$C = \begin{bmatrix} 1.6035 & -0.0028319 & -1.3277 & 0.66386 \end{bmatrix}$$

$$B = \begin{bmatrix} -9.5766e-5 \\ * \end{bmatrix} \qquad B_0 = \begin{bmatrix} 0.8621 \\ * \end{bmatrix}$$

$$D = -9.5766e-5 \qquad D_0 = 0.8621$$

Vertex 5

$$A = \begin{bmatrix} 1.5117 & -0.001661 & -1.172 & 0.58599 \\ * & & & \end{bmatrix}$$

$$C = \begin{bmatrix} 1.5117 & -0.001661 & -1.172 & 0.58599 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.00011795 \\ * \end{bmatrix} \qquad B_0 = \begin{bmatrix} 0.83015 \\ * \end{bmatrix}$$

$$D = -0.00011795 \qquad D_0 = 0.83015$$

Vertex 6

$$A = \begin{bmatrix} 1.5159 & -0.001699 & -1.162 & 0.58098 \\ * & & & \end{bmatrix}$$

$$C = \begin{bmatrix} 1.5159 & -0.001699 & -1.162 & 0.58098 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.00012288 \\ * \end{bmatrix} \qquad B_0 = \begin{bmatrix} 0.82305 \\ * \end{bmatrix}$$

$$D = -0.00012288 \qquad D_0 = 0.82305$$

where $*$ denotes an entry equal to that of the corresponding value for Vertex 1.

The parameters $T, M, \lambda_1, \lambda_2$ vary on each pass k stochastically with p within the constant intervals (24) and (25) respectively and are denoted by $T(k,p), M(k,p), \lambda_1(k,p)$, and $\lambda_2(k,p)$ respectively. Note also that the functions $T(k,p), M(k,p), \lambda_1(k,p)$, and $\lambda_2(k,p)$ can be different on each pass k.

Applying Theorem 4 now gives the stabilizing control law matrices

$$\begin{aligned} K_1 &= \begin{bmatrix} 18161.073 & -31.212 & -10010.633 & 7335.353 \end{bmatrix} \\ K_2 &= 9425.674. \end{aligned} \tag{26}$$

Figures 1 and 2 show the pass profile sequences generated by the uncontrolled and controlled processes respectively (with zero input sequence).

To conclude this section, it is instructive to give some comments on the steps necessary to apply the new design algorithm developed in this paper. The first point is that we have 4 parameters which can vary between given maximum and minimum values

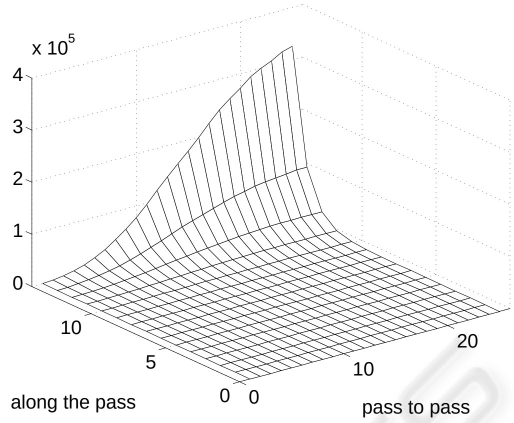

Figure 1: The process pass profile sequence with no control action applied (plot is $x_k^1(p)$).

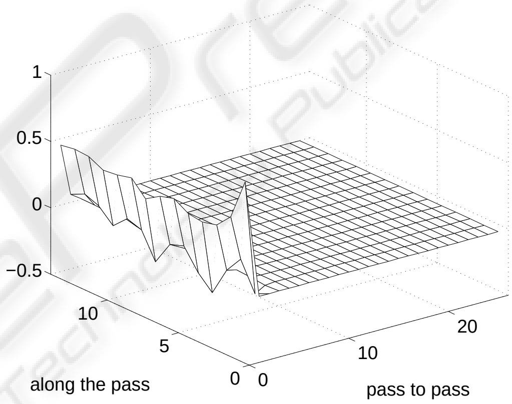

Figure 2: The process pass profile sequence with control action applied (plot is $x_k^1(p)$).

and hence there are 16 combinations for the uncertainty domain vertices, and the uncertainty domain is required to be convex. If, as in the numerical example above, convexity is not present, then the most obvious idea is to numerically estimate the convex hull of the vertices where here we have used (GBT). This, of course, introduces extra conservativeness into the design and how to reduce this is clearly a subject for further research.

Suppose now that a control law has been successfully designed and we wish to simulate the response of the process under control action. Then we first have to determine $\xi_i(p) \quad \forall i = 1, 2, \ldots, v$ and $0 \le p \le \alpha - 1$, where v denotes number of vertices. We can do this by applying *Matlab* function `fmincon` to solve the following problem: determine $\xi_i(k,p) \in \mathbf{R}^+$, $i = 1, 2, \ldots, v$ such that

$$\sum_{i=1}^{v} \xi_i(k,p) V_i = P(k,p) \tag{27}$$

where

$$\sum_{i=1}^{v} \xi_i(k,p) = 1; \quad \xi_i(k,p) \geq 0;$$

$$k \geq 0; \quad 0 \leq p \leq \alpha - 1$$

Here the V_i denote the minimal convex domain vertices and $P(k,p)$ denotes a matrix within the obtained polytope for point p on pass k, i.e the corresponding process state space model matrix computed from the corresponding $T(k,p), M(k,p), \lambda_1(k,p), \lambda_2(k,p)$. Repeating this procedure $\forall\ k \geq 0$ and $\forall\ p$, $0 \leq p \leq \alpha - 1$, enables the process response with or without control action applied

6 CONCLUSION

In this paper, we have extended previous results on the stability and control of discrete linear repetitive processes to the case when the defining state space model matrices are subject polytopic uncertainty. This has led to a design algorithm which can be implemented using well tested software. Also we have attempted to minimize the conservativeness introduced by the use of sufficient only conditions for stability.

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