

# SUNSPOT SERIES PREDICTION USING ADAPTIVE IDENTIFICATION

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Abstract: In this paper a parallel and adaptive methodology for optimizing the time series prediction using System Identification is shown. In order to validate this methodology, a set of time series based on the sun activity measured during the 20th century have been used. The prediction precision for short and long term improves with this technique when it is compared with the found results using System Identification with classical values for the main parameters.

## 1 INTRODUCTION

The time series (TS) prediction is a very important knowledge area because the evolution of many processes is represented as a time series: meteorological phenomena, chemical reactions, financial indexes, etc. Although the behaviour of any of these processes may be due to the influence of several causes, in many cases the ignorance of these forces to study the process considering only the time series evolution that represent it. By this reason, numerous methods of time series analysis and mathematical modelling have been developed.

System Identification (SI) techniques (Söderström, 1989) can be used to obtain the TS model. The model precision depends on the assigned values to certain parameters. In SI, a time series is considered as a sampled signal  $y(k)$  with period  $T$  that is modeled with an ARMAX (Ljung, 1999) parametric polynomial description of  $n_a$  dimension. Basically the identification consists in determining the ARMAX model parameters from measured samples. Then it is possible to compute the estimated signal  $y_e(k)$  and to compare it with the real signal, calculating the error  $(y(k)-y_e(k))$ .

The recursive estimation updates the model in each time step  $k$ , thus modeling the system. The more sampled data are processed, the more precision for the model, because it has more information about the system behaviour history. We consider SI performed by the well-known Recursive Least Squares (RLS) algorithm (Ljung, 1999). This algorithm is mainly specified by the constant  $\lambda$  (forgetting factor) and the observed samples  $\{y(k)\}$ . There is not any fixed value for  $\lambda$ , even it is used a

value between 0.97 and 0.995 (Ljung, 1991). The cost function  $F$  we use is defined as the value to minimize in order to obtain the best precision (see equation 1, where  $SN$  is the sample number).

$$F(\lambda) = \sum_{k=k_0}^{k=k_0+SN-1} |y_e(k) - y(k)|$$

Equation 1: The considered cost function

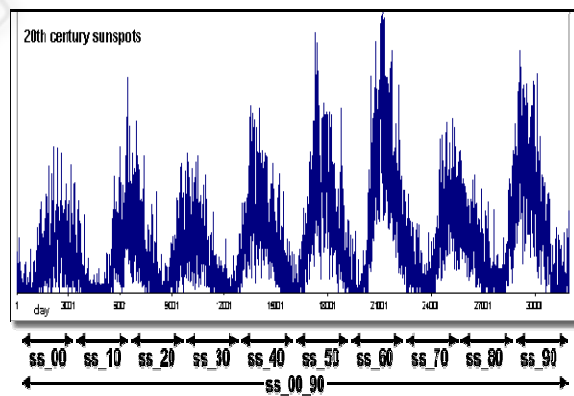


Figure 1: The sunspot time series used in this work

In this paper we use a time series set corresponding to sunspot series obtained from measured observations (ROB, 2004)(NOAA, 2004). We have used 13 time series (Fig. 1) showing daily sunspots: ten series (ss\_00, ss\_10, ss\_20, ss\_30, ss\_40, ss\_50, ss\_60, ss\_70, ss\_80 and ss\_90) corresponding to the sunspot measurements during

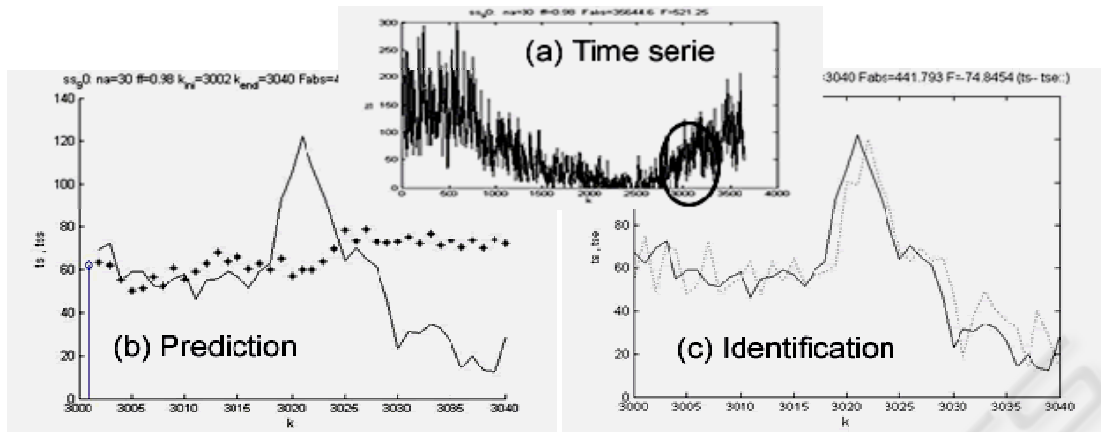


Figure 2: In (a) is displayed the  $ss_{90}$  series. If  $ks=3000$ , we can see in (b) the long-term prediction based on the model obtained up to  $ks$  ( $na=30$  and  $\lambda=0.98$ ). The prediction precision is reduced when we are far from  $ks$ . In (c) we can see the predicted values for the successive  $ks+1$  obtained from the updated models when the identification (and  $ks$ ) advances

ten years each (for example,  $ss_{20}$  compiles the sunspots from 1/1/1920 to 31/12/1929), two series ( $ss_{00\_40}$  and  $ss_{50\_90}$ ) covering 50 years each, and finally one series ( $ss_{00\_90}$ ) that covers all measurements of the 20th century.

## 2 SYSTEM IDENTIFICATION BASED PREDICTION

The recursive identification can be used to predict the following behaviour of the time series (Fig. 2a) from the data observed up to the moment. It is well known that SI allows finding, in sample time, a mathematical model of a system in the  $k$  time from which is possible to predict the system behaviour in  $k+1$ ,  $k+2$  and so on. As identification advances in the time, the predictions improve using more precise models. If  $ks$  is the time until the model is elaborated and from which we carry out the prediction, we can confirm that this prediction will have a larger error while we will be more far away from  $ks$  (Fig. 2b). The predicted value for  $ks+1$  corresponds with the last estimated value until  $ks$ . When we have more data, the model starts to be re-elaborated for computing the new estimated values (Fig. 2c).

The main parameters of the identification are  $na$  y  $\lambda$ . Both parameters have influence on the precision of prediction results, as it shows Fig. 3. As we can see, to establish an adequate value of  $na$  and  $\lambda$  may be critical in order to obtain a good prediction. By this reason, in the strategy of optimizing the prediction, we try, first of all, to establish an adequate dimension of the mathematical model for finding the optimal  $\lambda$  value using an adaptive algorithm.

In order to do that, many experiments have been carried out. In these experiments, the absolute difference between the real and predicted values has been used in order to quantify the prediction precision (see equation 2). So, in Fig. 4 the results of many experiments are shown. In these experiments different measures of the prediction error (for short term DIFA1 and for long term DIFA50) for 200 different values of the model are obtained. In order to establish reliable conclusions, these experiments have been made for different  $ks$  values.

We use our own terminology for the prediction error (DIFAX, where X is an integer) because like this the reader can understand more easily the degree of accuracy that we are measuring.

Provisionally we conclude that the increase of  $na$  does not imply an improvement of the prediction precision (however a considerable increase of the computational cost is spent). This is easily verified using large  $ks$  values (the elaborated models have more information related to the time series) and long-term predictions. From these results and as trade-off between prediction and computational cost, we have chosen  $na=40$  to establishing it as the model size for the future experiments

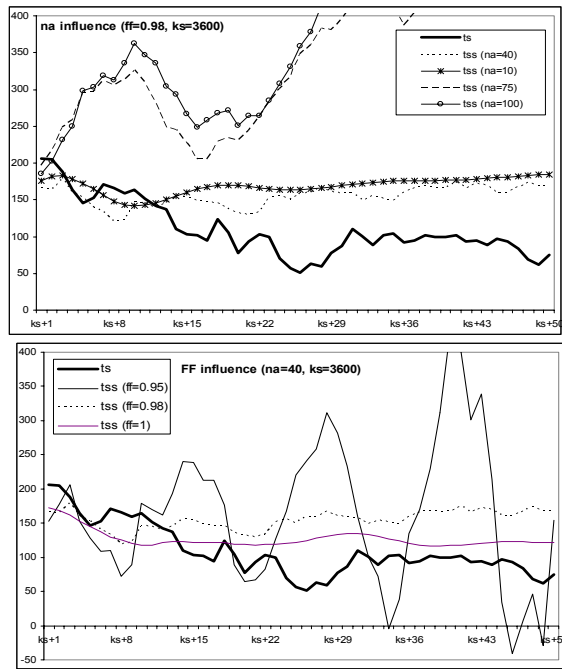


Figure 3: Influence of the model dimension  $n_a$  and forgetting factor  $\lambda$  on the short-term ( $ks$ ) and long-term ( $ks+50$ ) prediction for the  $ss_{90}$  series

$$DIFA(X) = \sum_{i=1}^{i=X} |y_s(ks+i) - y(ks+i)|$$

Eq.2. The cost function in the prediction measurements.

The key parameter for the prediction optimization is  $\lambda$ . Fig. 5 shows, with more detail than in Fig. 3b, an example of the  $\lambda$  influence in the short-term and long-term predictions by representing its precision measure. It is clear that for any  $\lambda$  value is more precise the short-term prediction than the long-term prediction. However, we can observe the chosen value for  $\lambda$  is critical for finding a good predictive model (from the four chosen values,  $\lambda = 1$  produces a better prediction, even in the long term). This analysis has been confirmed making a great number of experiments with the sunspot series, modifying the initial prediction time,  $ks$ .

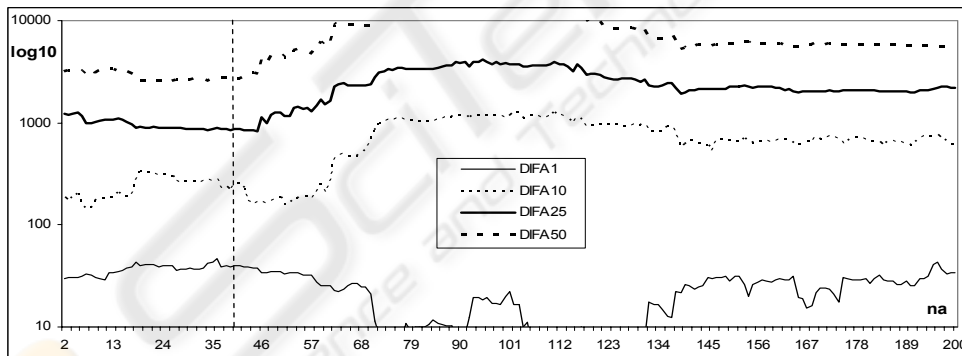


Figure 4: Measures of the prediction precision for the  $ss_{90}$  time series from  $ks=3600$ , using  $\lambda=0.98$ . The measures have been made for models of dimensions between 2 and 200.

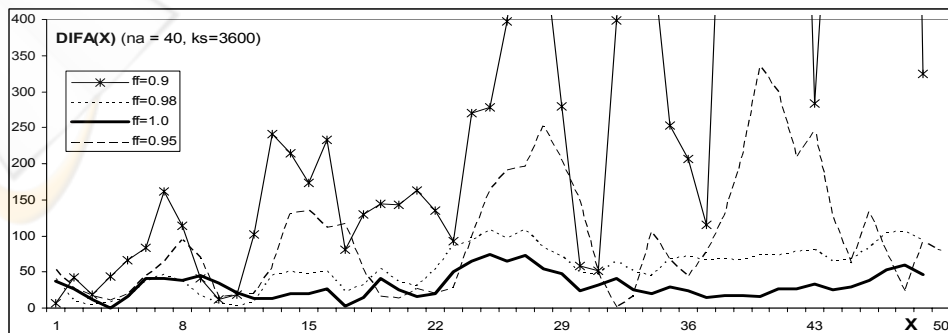


Figure 5: Influence of  $\lambda$  in the prediction precision for  $ss_{90}$  with  $n_a=40$  and  $ks=3600$

### 3 OPTIMIZING THE PREDICTION WITH AN ADAPTIVE STRATEGY

In order to find the optimum value of  $\lambda$ , we propose an adaptive algorithm inspired on the artificial evolution (Goldberg, 1989)(Rechenberg, 1973) and on the simulated annealing mechanism (Kirkpatrick, 1983). This algorithm, named PARLS (Parallel Adaptive Recursive Least Squares), has been implemented using parallel processing units, built with neural networks (Gomez, 2003). In PARLS the optimization parameter  $\lambda$  evolves to predict new situations during the iterations of the algorithm. In other words,  $\lambda$  evolves at the same time that improves the cost function performance.

The evolution mechanism (Fig. 6) is as follows: The first phase starts building a set of  $\lambda$  values covering the interval  $R$  uniformly from its selected middle  $\lambda_c$ . An equal number of parallel processing units (PUN) perform RLS identification with each  $\lambda$  in the interval, for a given number of sampling times (PHS). Then, the optimum  $\lambda$  is that whose corresponding cost function  $F$  is the minimum of all computed  $F$ , and from it a new set of  $\lambda$  values is generated and used in the next phase to perform new identifications during the following PHS samples.  $F$  is defined as the accumulated error of the samples in each phase, and the generation of new  $\lambda$  values is made with a more reduced  $R$  by the factor  $RED$ . Finally, PARLS stops when the number of phases (PHN) is reached, according to the total number of samples (TSN).

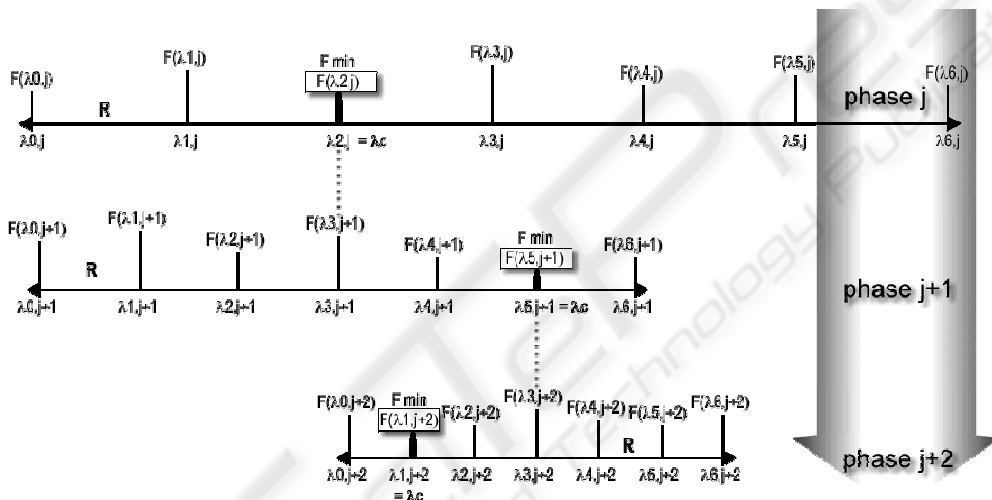


Figure 6: All the  $\lambda$  values in the same phase running in the processing units are generated in the  $R$  interval from the previous phase optimum  $\lambda$  found, corresponding with the smallest  $F$ .

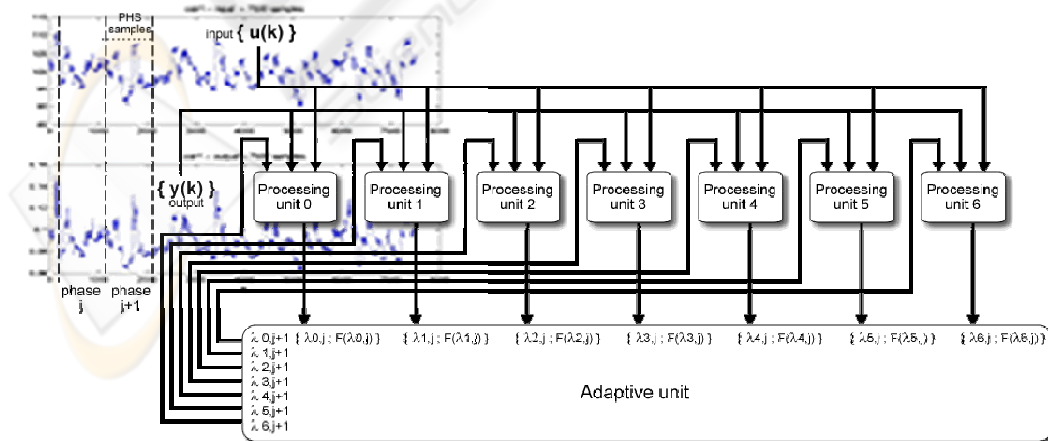


Figure 7: PARLS architecture. A set of processing units generates the next set of  $\lambda$  values to be computed by RLS algorithm

Therefore, PARLS could be considered as a population-based rather than a parallel metaheuristic, because each processing unit is able to operate isolated, as well as the tackled problem itself as only a single real-valued parameter ( $\lambda$ ) that is optimized.

In Fig. 7 we can see a top-level view of the PARLS architecture: a set of processing units that performs system identification in each phase sends the errors found to the adaptive unit. This unit will generate the new search range in a feedback loop.

### 4 EXPERIMENTAL RESULTS

Some results found using the PARLS algorithm for predicting the next values from the last value registered for the time series of the sun activity indexes of the 20th century second half are shown in Table 1. The prediction precision is given by the results DIFA1 (short term) and DIFA50 (long term). These results are compared, for evaluating purposes, with the obtained using RLS identification with some values of  $\lambda$  inside the classical range (Ljung, 1991). We can see how PARLS finds a better precision.

### 5 CONCLUSIONS

The results shown in Table 1 are a part of the great number of experiments carried out using different time series and initial prediction times. In the great majority of the cases, PARLS offers better results than if  $\lambda$  random values are used. However, we are trying to increase the prediction precision. Thus, our future working lines suggest using genetic algorithms or strategies of analogous nature for, on

the one hand, finding the optimum set of values for the parameters of PARLS and, on the other hand, finding the optimum couple of values  $\{na, \lambda\}$ .

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Table 1: A sample of the prediction precision results for short and long term, compared with the ones obtained from three classical values of  $\lambda$ : 0.97, 0.98 and 0.995. The settings for this experiment are: benchmark= ss\_50\_90; na=40; ks=18,210; TSN=18,262;  $\lambda_c=1$ ; R=0.2; RED=2

PUN	PHS	PHN	PARLS $\lambda_{opt}$ found	(A) DIFA1 for $\lambda_{opt}$	(B) DIFA50 for $\lambda_{opt}$	%(A) better than (A1)	%(A) better than (A2)	%(A) better than (A3)	%(B) better than (B1)	%(B) better than (B2)	%(B) better than (B3)
5	3500	5	1.00000	33.61	1,637.4	33%	20%	6%	99%	69%	25%
7	500	36	1.00417	38.15	1,706.4	17%	6%	-7%	91%	62%	20%
7	1000	18	1.00000	33.61	1,637.4	33%	20%	6%	99%	69%	25%
7	2000	9	1.00000	33.61	1,637.4	33%	20%	6%	99%	69%	25%
7	3000	6	0.99895	34.11	1,826.9	31%	18%	4%	78%	51%	12%
7	4000	4	1.00000	33.61	1,637.4	33%	20%	6%	99%	69%	25%
91	500	36	1.00361	38.01	1,674.0	17%	6%	-6%	95%	65%	23%
91	1000	18	0.99889	34.16	1,841.5	31%	18%	4%	77%	50%	11%
91	2000	9	1.00014	34.60	1,640.0	29%	16%	3%	99%	68%	25%
91	3000	6	0.99931	33.66	1,740.9	32%	20%	6%	87%	59%	18%
91	4000	4	1.00000	33.61	1,637.4	33%	20%	6%	99%	69%	25%

(A1) DIFA1 for $\lambda=0.97$ :	44.6	(A2) DIFA1 for $\lambda=0.98$ :	40.3	(A3) DIFA1 for $\lambda=0.995$ :	35.58
(B1) DIFA50 for $\lambda=0.97$ :	3,259.7	(B2) DIFA50 for $\lambda=0.98$ :	2,759.7	(B3) DIFA50 for $\lambda=0.995$ :	2,052.7

(\*) PARLS uses the tuned values of its parameters in [9], where the processing units were implemented with neural networks