IMPROVEMENT ON THE POLE-PLACEMENT CONTROL SCHEME BY USING GENERALIZED SAMPLED-DATA HOLD FUNCTIONS

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Abstract: This paper studies the benefits the use of GSHF can afford to the pole-placement control scheme. The GSHF makes possible to locate the zeros of the discretized plant arbitrarily in the Z plane. This property can be taken advantage of to improve the performance of the pole-placement control. In this article a new design method is suggested and a simulations-based application example is carried out. In the application example the improvements this method involves with respect to the classical design method are noticed.

1 INTRODUCTION

Usually, most sampled data control systems use the Zero Order Hold (ZOH). However, some authors ((Kabamba, 1987) (Bai and Dasgupta, 1990) (Er and Anderson, 1994) (Yan et al., 1994) (Rossi and Miller, 1999) (Barcena abd De la Sen, 2003)) have proved that using hold patterns which differ from the zero order extrapolation, that the ZOH carries out, can improve the discrete performance of the hybrid system.

The GSHF (Generalized Sampled-data Hold Function), device which has been widely studied (Kabamba, 1987) (Bai and Dasgupta, 1990) (Er and Anderson, 1994) (Yan et al., 1994), has a generic hold function which can be tuned to obtain some advantages. For example, (Kabamba, 1987) proves that the zeros of the discretized plant can be placed arbitrary in the Z plane by tuning the hold function of the GSHF.

Several hybrid control schemes are based on the cancellation of the zeros of the discretized plant with the controller poles. The pole-zero cancellation cannot be done if there is any unstable zero in the discretized plant, because it would make the system internally unstable (Jury, 1956). This cancellation would not be advisable either if there is any insufficiently damped zero, since the cancellation of this zero could cause intersample ripple (Clarke, 1984). It is always possible, by using the GSHF, to stabilize the zeros of the discretized plant, when are unstable, or to increase the stability degree of the critically damped ones, in order to make possible a safe cancellation. Therefore, this device makes possible the use of mentioned control schemes.

On the other hand, it is well known (Kuo, 1992) that the relative location of the zeros from the poles influences in the system response. Therefore, the GSHF can be used to place the zeros of the discretized plant in more favourable locations from the viewpoint of the control strategy and, in that way, to improve the performance achieved with the ZOH.

Although the mentioned advantages, both concerning the discrete performance of the hybrid system, some authors ((Feuer and Goodwin, 1994) (Freudenberg et al., 1995) (Freudenberg et al., 1997)) have proved that the use of the GSHF can cause intersample difficulties which do no appear when the ZOH is used. Nevertheless, this happens in designs in which the intersample ripple these devices can cause has not been taken into account. However, when this possibility is taken into account by the design method, it is possible to get somewhat degree of improvement in the discrete performance of the hybrid systems, without incurring in a too large deterioration of the intersample performance. An example of this appears in (Hjalmarsson and Braslavsky, 1999).

Using the GSHF carries another adverse effect: its static hold function requires that the control signal varies even during the steady-state. This can cause the actuator fatigue and accelerate its wear. In (Chan, 2002) it is suggested an alternative to the static hold pattern of the GSHF. The device suggested in that

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paper converges asymptotically toward a ZOH input pattern as the controlled system response tends to the steady-state step response. This eliminates the unceasing magnitude changes and the ripple the control signal suffers during the steady-state step response when the GSHF is employed. The analysis of this device, variable GSHF from now on, and the benefits its use can afford to the pole-placement control will be the objectives of this paper.

This paper is organized in the following manner. In section 2 the most important characteristics of the variable GSHF are described and the pole-placement control scheme structure is commented. In the section 3 a new design method for this controller is suggested. This design methodology takes into account the variable GSHF properties so that the benefits that this hold device can afford are taken advantage of. In this way the pole-placement control scheme performance is improved. In section 4 a simulations-based application example is carried out and the outcomes are compared with the obtained ones by the classical design method. To finish, the conclusions are drawn in the section 5.

2 PRELIMINARIES

2.1 Variable GSHF (Chan, 2002)

The variable GSHF contains m discrete filters which process the incoming discrete signal with the sample time T_m. A MISO discrete filter of m inputs which works with a sampling time of T_m/m processes the output signals of these m filters. The output signal of this filter is rebuilt with a ZOH which works at the same sampling rate. The Fig. 1 shows the structure of the variable GSHF.

The variable GSHF divides the sampling time T_m in m subintervals and in each one of these subintervals the hold function is kept constant. The amplitude associated to each one subinterval depends on the state of the MISO filter variable GSHF contains. Therefore, the gains of each one subinterval may vary from a sampling time to another. For that reason we call it variable GSHF.

Using a variable GSHF with $m=2$ and a suitable selection of the parameters of the discrete filters it is possible to locate the zeros of any strictly proper transfer function of arbitrary order which no contains zeros, arbitrarily in the Z plane (Chan, 2002) (Chan, 1998). This is the case that we will study in this paper.

The $v_1(z),\dots,v_m(z)$ filters are introduced to produce the redundancy in the discrete control signal necessary for zero placement. These filters are defined by (1) where $c_l(z)$ and $d(z)$ are polynomial of the same degree. The MISO filter is defined by equation system (2) where s_l, with $l=1,2,\dots,m$, and p are scalars.

$$v_l(z) = \frac{c_l(z)}{d(z)} r(z) \quad ,l = 1,2,\dots,m \tag{1}$$

$$\begin{cases} w\left(k+\frac{1}{2}\right) = pw(k) + \sum_{l=1}^{m} s_l v_l(k) \\ u(k) = w(k) + \sum_{l=1}^{m} v_l(k) \\ w(0) = 0 \end{cases} \tag{2}$$

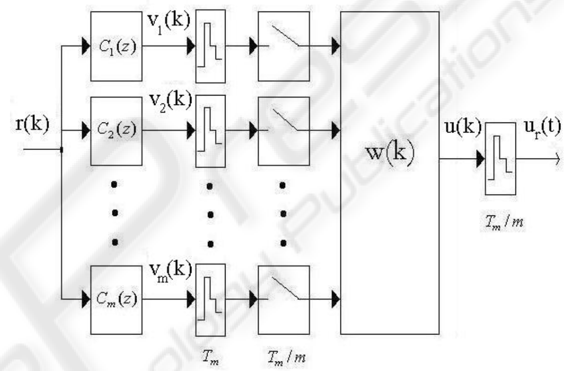

Figure 1: Variable GSHF

If the process to be controlled, discretized by the ZOH contained in the GSHF, has the following discrete-time

$$\begin{cases} x(k+1) = A_d x(k) + B_d u(k) \\ y(k) = C_d x(k) \end{cases} \tag{3}$$

then, the state space representation of the system composed of MISO filter and the discrete system represented by (3) has the discrete time representation described by the system (4).

$$\begin{cases} z(k+1) = Fz(k) + \sum_{l=1}^{m} G_l v_l(k) \\ y(k) = (C_d \quad 0) z(k) \end{cases} \tag{4}$$

where

$$z(k) = \begin{pmatrix} x(k) \\ w(k) \end{pmatrix} \tag{5}$$

$$F = \begin{pmatrix} A_d & B_d \\ 0 & p^2 \end{pmatrix}^m \tag{6}$$

$$G_l = \sum_{i=1}^{m} \begin{pmatrix} A_d & B_d \\ 0 & p \end{pmatrix}^{i-1} \begin{pmatrix} B_d \\ s_l \end{pmatrix} \quad ,l \in [1,\dots,m] \tag{7}$$

In the equation system (2) it is noticed that, when the MISO settles, the signal which is reconstructed by the ZOH at each multiple of T_m/m begin to update only a time per sampling period. Therefore, the variable GSHF behaves as a GSHF during transient response and as a ZOH in the steady state. This eliminates the unceasing changes of magnitude and achieves to reduce the ripple that the static hold pattern of the GSHF produces.

Using two subintervals ($m=2$), the assignment of the discrete transfer function numerator is obtained resolving the following diophantine equation:

$$\sum_{l=1}^{m} g_l(z)c_l(z) = g_1(z)c_1(z) + g_2(z)c_2(z) = num(z) \quad (8)$$

where $c_l(z)$ are the numerator polynomial of the m discrete filters which precede the MISO filter, $num(z)$ is the polynomial required as numerator of the discretized plant transfer function and $g_l(z)$ is the polynomial of the numerator of each one of the m discrete systems obtained when the equation system (4) is particularized to a concrete value of l.

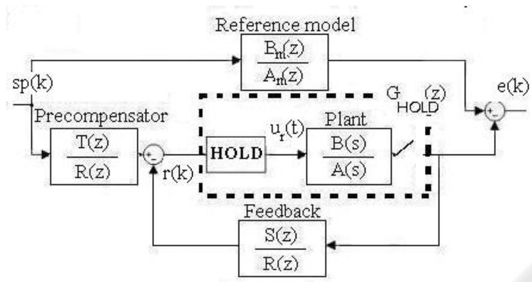

Figure 2: Reference model control scheme

It is important to point out that the implementation of the variable GSHF involves no additional hardware since the discrete filters are implemented in the computer and, therefore, the only hardware needed to implement this device is a ZOH.

2.2 Control scheme

In this paper the improvement the use of GSHF can contribute to the pole-placement discrete control, which is vastly studied in (Aström and Wittenmark, 1990), is analyzed.

In this control scheme, which structure is represented in Fig. 2, the polynomial R(z), S(z) and T(z) of the discrete filters of the feedback loop and precompensator are calculated to match the behaviour of the closed-loop with the reference model. This is carried out by solving the following equations

$$\begin{aligned} T(z)B(z) &= B_m(z)A_0(z) \\ A(z)R(z) + B(z)S(z) &= A_m(z)A_0(z) \end{aligned} \quad (9)$$

where $A_0(z)$ is a polynomial introduced to ensure the solvability of the second equation of (9). The discretized plant zeros are transmitted to the reference model unless they are cancelled with the controller poles. It is important to point out that only stable zeros can be cancelled. For this reason the $B(z)$ polynomial is factorized into the following polynomials: $B^+(z)$ and $B^-(z)$. Where B^+ contains the stable zeros of the discretized plant the designer wants to cancel and $B^-(z)$ contains the unstable zeros and the stable zeros the designer decides to transmit to the reference model. The zeros of the $B^-(z)$ are necessarily roots of the numerator of the reference model and, therefore, when $B^-(z)$ contains any zero, the reference model cannot be chosen totally freely. For this reason the polynomial $B_m(z)$ is factorized in the following way

$$B_m(z) = B^-(z)B'_m(z) \quad (10)$$

where $B'_m(z)$ is the polynomial contained in B_m that can be chosen freely. In other respect, the polynomial $B^+(z)$ is cancelled with the controller poles and therefore the roots of $B^+(z)$ must be roots of the polynomial $R(z)$

$$R = B^+(z)R'(z) \quad (11)$$

Using (10) and (11) in (9) the equations obtained are

$$\begin{aligned} T(z) &= B'_m(z)A_0(z) \\ A(z)R'(z) + B^-(z)S(z) &= A_m(z)A_0(z) \end{aligned} \quad (12)$$

By solving the equations (12) the discretized plant behaviour is matched with the reference model described by the equation (13).

$$G_M(z) = \frac{B'_m(z)B^-(z)}{A_m(z)} \frac{A_0(z)}{A_0(z)} \frac{B^+(z)}{B^+(z)} \quad (13)$$

3 DESIGN METHOD

This paper accomplishes the analysis of the improvements that the GSHF can contribute to pole placement control scheme. In this control scheme, the first step is to locate the poles of the reference model in terms of the required behaviour. Then, two possibilities exist. It is possible to cancel the zeros with controller poles or to transmit them to the reference model. Is well known (Kuo, 1992) that the relative position of the zeros from the poles influences on the closed loop performance. In a generic manner, to be able to relocate the zeros of the closed loop anywhere in the Z plane supposes an advantage. However, it is not always possible, since, if the zeros of the plant are unstable or not sufficiently damped, it is not advisable to cancel them. Therefore, when the ZOH is the device used for the reconstruction and the discrete transfer function has this kind of zeros, we are forced to transmit them to the reference model.

The variable GSHF, however, allows us to relocate the zeros arbitrarily in the Z plane. Therefore, this device permits to avoid these problems. Besides, two possibilities exist.

The first method lies in using the variable GSHF to locate the zero of the discretized transfer function of the plant in the position where the reference model has its zero. This allows achieving the model matching without carrying out a cancellation.

The second way to match the closed-loop discrete behaviour with the reference model using the GSHF lies in positioning the zero of the discretized plant transfer function in a place where the discrete zeros are suitable to be cancelled without incurring in intersample ripple. If the zero is located in a place with this characteristic, then it is possible to cancel this with the controller and to place the zero of the system in the prescribed position.

4 APPLICATION EXAMPLE

In this section it is carried out a comparative study between the performance attained in pole-placement scheme by the ZOH and the attained one by a variable GSHF, tuned as is described in section 3.

Both methods are applied for accurate positioning of a computer hard disk read/write head. The model of the read/write head used (Franklin et al., 1992) is described by de following differential equation

$$I\ddot{\theta}(t) + C\dot{\theta}(t) + K\theta(t) = K_i i(t) \tag{14}$$

where I is the inertia of the head assembly, C is the viscous damping coefficient of the bearings, K is the return spring constant, K_i, is the motor torque constant, $i(t)$ is the input current and $\theta(t)$ is the angular position of the head. With the parameters suggested in (Franklin et al., 1992) ($I=0.01\ Kgm^2$, $C=0.004\ Nm/rad$, $K=10Nm/rad$ y $K_i=0.005Nm/A$) the transfer function that describes the dynamic of the plant is

$$G(s) = \frac{5}{s^2 + 0.4s + 1000} \tag{15}$$

To discretize a process, a usual agreement suggests choosing the sampling time 20 times higher than the continuous plant bandwidth (Kuo, 1992). To accomplish with this agreement the sampling time is fixed to *0.006 s*. Pole-placement controller is the scheme used in this section. The discrete behaviour that is wished to transmit to the plant is described by the following transfer function:

$$G_M(z) = \frac{0.30417\left(z-0.4\right)}{z^2 - 1.2z + 0.3825} \tag{16}$$

When the control objective is reached, the equivalent damping coefficient of the closed-loop system is about 0.89. This reference model has been chosen to obtain the closed-loop system with the minimal settling time. This selection is carried out due to the fact that the settling time is one of the more important specifications that the step response of a hard disk needs to improve since the read/write operation cannot start until the read/write head of the hard disk places correctly in the position the reference signal requires.

However, if the ZOH is used the obtained discrete transfer function is

$$G_{ZOH}(z) = \frac{8.9659 \cdot 10^{-5}\left(z+0.9992\right)}{z^2 - 1.962z + 0.9976} \tag{17}$$

This transfer function has its zero located almost on the unit circle and it is not advisable to cancel this by the controller. Fig. 3 shows the obtained result when necessary cancellation to catch up with the discrete reference model is carried out.

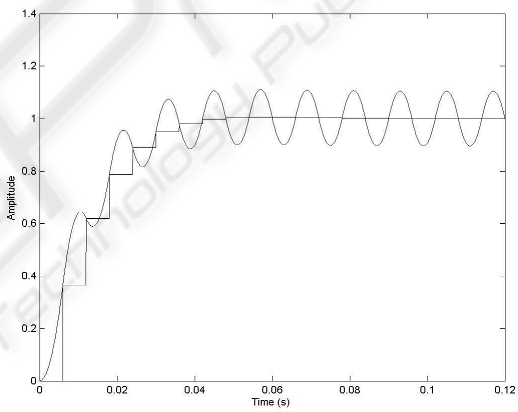

Figure 3: Unit-step response of the compensated system by using ZOH device when discrete controller cancels the discretization zero.

Hence, it is necessary to transmit the zero of the process to the reference model. When the zero is transmitted to the reference model the system has the step response shown in Fig. 4.

However, using GSHF it is possible to move the zero from the place that it presents when ZOH is used. On the one hand it is possible to use the GSHF to locate the zero of the discrete transfer function of the plant in the position where the reference model has its zero. This allows achieving the model matching without carrying out a cancellation. The variable GSHF parameters used to locate the discretization zero in $z=0.4$ are $p=0$, $s_1=1$, $s_2=0.5$, $c_1(z)=-13.8528z - 1.1708$, $c_2(z) = 18.3565z + 2.3416$ and $d(z)=z$. In Fig. 5 is depicted the step response of the system when the related design method is achieved.

As is shown in Fig. 5 with the use of GSHF the matching with the reference model (16) is attained without incurring in the generation of the intersample ripple that the system was having when in the control scheme the reconstruction device employed was the ZOH (compare with Fig. 3). Comparing Fig. 4 and Fig. 5 it is noticed that to be able to fit the model reference model (16) when the GSHF is used supposes an improvement: No one of both responses present overshoot, but while the system which employs ZOH settles in *0.52 s.* the system that uses GSHF needs only *0.41 s.*. This supposes an improvement of 21% in the settling time.

On the other hand, if the zero is located with the GSHF at $z=-0.2$ ($p=0$, $s_1=1$, $s_2=0.5$, $c_1(z)=-4.6707z - 0.3344$, $c_2(z) = 6.6713z + 0.6687$ and $d(z)=z$) and then is cancelled with a controller pole, the step response of the closed loop is almost the same to the obtained one in the case where the zero was directly located in $z=0.4$.

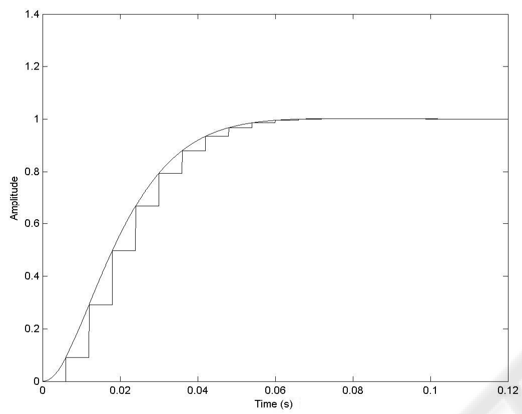

Figure 4: Unit-step response of compensated system by using ZOH device, when discrete plant zero is transmitted to reference model.

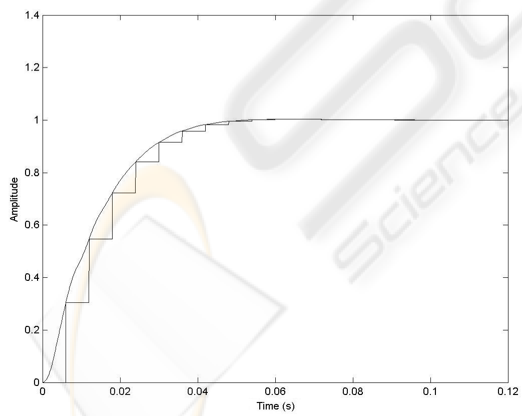

Figure 5: Unit-step response of compensated system by using GSHF device to locate the zero in z=0.4 and this is transmitted to reference model.

The control signals of the three designs are depicted in Fig. 6, Fig. 7 and Fig.8. In that figures it is noticed that the reduction of the settling time is obtained at expense of amplification of the control signal. The amplification of the control signal is quite big, and therefore, it is important to decide, depending on the application, if the improvement obtained justifies the amplification the control signal suffers respect the ZOH case, or not. It is important to notice that in this paper only two subintevals *(m=2)* are taken into account and may be possible, with the use of more subintervals, to reduce the control signal amplitude during the transient response. However, if the number of subintervals used is very high the ZOH contained in the GSHF is forced to work at high rate.

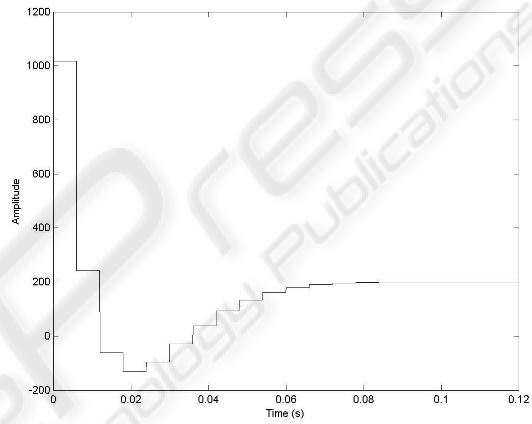

Figure 6: Control signal when the discrete plant zero is transmitted to the reference model and ZOH is used.

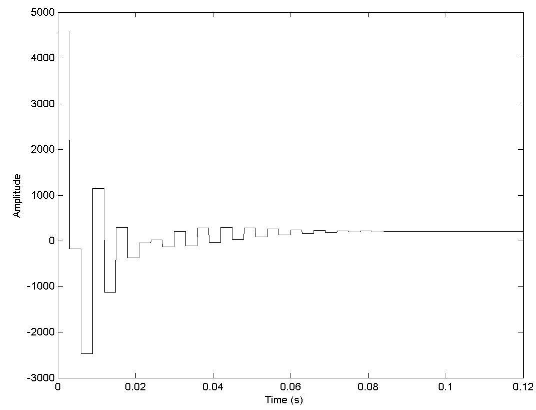

Figure 7: Control signal of the design that uses the GSHF to locate the zero in z=0.4 and this zero is transmitted the discrete controller.

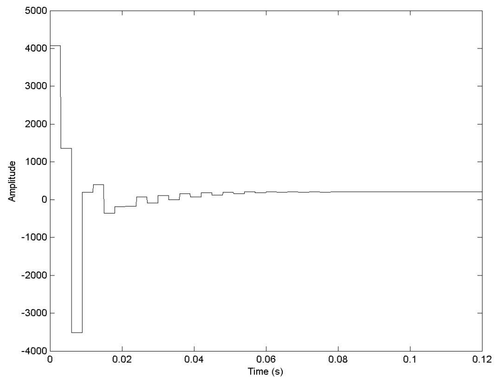

Figure 8: Control signal of the design that uses the GSHF to locate the zero in *z=-0.2* and this zero is cancelled by the discrete controller.

5 CONCLUSION

In this paper, it has been noticed by means of an application example that the variable GSHF can improve the performance of the pole-placement control scheme. The variable GSHF allowed us to place the discretization zero of a second order continuous plant in more beneficial location from the viewpoint of the control strategy, improving in that way the performance of the closed loop.

On the one hand, when the ZOH discretization zero is sufficiently damped, it is possible to cancel it with one of the controller poles and to locate the closed-loop zero in the place where the reference model has its zero. In such situations, the ZOH discretization zero does not impose limitations to the attainable performance and, therefore, the possibility of relocating it that GSHF provides does not suppose any advantage. On the other hand, when the ZOH discretization zero is unstable or poorly damped, which often happens when the sampling time used is small enough (Aström et al., 1984), it is not advisable to cancel it and, therefore, the performance that can be attained by the classical design method is limited given that the designer is forced to transmit such a zero to the reference model in order to avoid intersample ripple. This is the case studied in this paper and it has been noticed that it is possible to match the closed-loop discrete behaviour to the reference model by the variable GSHF, without generating intersample ripple.

From the carried out study it is concluded that the GSHF ability to move the zeros can be used to improve the transient response of a pole-placement control. It has been also noticed that this improvement is obtained at expense of the amplification of the control signal during transient state. It is important to point out that in this study it has been used a variable GSHF with two subintervals and it may be possible to reduce the control amplitude by using more subintervals.

During the study, the possible deterioration of the sensitivity functions, both discrete and hybrid ones, have not been taken into account. That is one of the possible drawbacks that the use of the GSHF can generate (Freudenberg et al., 1997) and future investigations on this device should integrate the analysis of such functions.

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