OPTIMAL CONTROL APPLIED TO OPTIMIZATION OF MOBILE SWITCHING SURFACES PART I: ALGORITHM

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Abstract: Following (Boccadoro *et al.*, 2004) and (Wardi *et al.*, 2004), we consider hybrid dynamical systems with parameterized switching surfaces. The goal is to optimize the choice of parameters in relation with a criterion. In an optimal control framework we deepen and generalize results of these authors. We get that thanks a known algorithm, usually not totally explicit, that can be here specified up to obtain an efficient one. Ideas of some new or classical applications are given. They will be developped in a second paper, enforcing the theoric results expanded here.

1 INTRODUCTION

Authors of (Boccadoro *et al.*, 2004, 2005) and (Wardi *et al.*, 2004), have pointed out and studied an optimization problem of switching surfaces in hybrid dynamical systems (h.d.s. – for general notions see, for example, (Bensoussan *et al.*, 1997), (Van Der Schaft and Schumacher, 1999), (Zaytoon *et al.*, 2001)). Here, drawing our inspiration from the classical reference book (Bryson and Ho, 1969), we get the results of (Boccadoro *et al.*, 2004) and (Wardi *et al.*, 2004), by another method. This one uses the variational calculus with an augmented criterion. It readily gives the searched relations. As opposed to the more technical method used by the previous authors, here the meaning of the costate in the framework of optimal control becomes clear. Moreover, our results are more general, including mobility of switching surfaces and specific terms in the criterion at switching instants. An important result is the determination of the optimal switching instants.

First, the problem is stated. Varitional calculus is then applied to an augmented criterion. This supplies a method for the criterion gradient calculus which reduces the optimization problem to the use of a classical steepest descent algorithm. In our conclusion we give ideas of classical or new applications. They are developped in a second paper (Quémard *et al.*, 2005d), enforcing the theoric results expanded here.

2 PRESENTATION OF THE OPTIMIZATION PROBLEM

Let t_0, $x_0 = x(t_0) \in \mathbb{R}^n$ be a given initial instant and a given initial state. At the beginning the considered h.d.s. follows a given (classical) dynamical system $\dot{x} = f_1(x,t)$ up to a switching instant t_1. This one corresponds to the first instant at which the trajectory hits a given mobile (or fixed) switching surface of equation $\psi_1(x_1,t_1,a_1)=0$ for state $x_1 = x(t_1)$. This surface depends on a parameter $a_1 \in \mathbb{R}^{r_1}$. Then the h.d.s. follows $\dot{x} = f_2(x,t)$ up to t_2 such that $\psi_2(x_2,t_2,a_2)=0$ for $x_2 = x(t_2)$ and $a_2 \in \mathbb{R}^{r_2}$. By induction, this defines $t_1,\ldots,t_N,t_{N+1}$ an increasing sequence of switching instants linked to some given switching surfaces of equations

$$\psi_i(x_i,t_i,a_i)=0, \quad i=1,\ldots,N+1, \qquad (1)$$

for states $x_i = x(t_i)$ and some parameters $a_i \in \mathbb{R}^{r_i}$. In $[t_0,t_{N+1}]$ state $x(t)$ is supposed to be continuous,

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and in $\left[t_{i-1},t_i\right], i=1,\ldots,N+1$ state $x(t)$ complies with some given dynamical systems

$$\dot{x}=f_i\left(x,t\right). \tag{2}$$

Functions ψ_i and f_i are from C^1 class with values in $\mathbb{R}$ and $\mathbb{R}^n$ respectively. Instant t_{N+1} is interpreted as a final instant and ψ_{N+1} as a final constraint.

Notations — *For a composed function as $u\left(v(\alpha),\alpha\right)$ we can note more simply $u\big|_\alpha$. For example, $f_i\left(x(t),t\right)$ and $\psi_i\left(x(t_i),t_i,a_i\right)$ can be noted $f_i\big|_t$ and $\psi_i\big|_{t_i}$.*

We make the following assumptions:

Assumption A1 (consistency) — *For $i=1,\ldots,N+1$, if parameters $a_1,\ldots,a_{i-1}$ are given, then t_i is the smallest instant $t, t>t_{i-1}$ such that $\dot{x}(t)=f_i\left(x(t),t\right)$ and $\psi_i\left(x(t),t,a_i\right)=0$. This defines t_i as a function of $a_1,\ldots,a_i$. Moreover, we assume that such a sequence $t_1,\ldots,t_{N+1}$ exists for all $\left(a_1,\ldots,a_{N+1}\right)$ belonging to an open set of $\mathbb{R}^{r_1}\times\cdots\times\mathbb{R}^{r_{N+1}}$.*

Assumption A2 (transversality) — *For $i=1,\ldots,N+1$, if parameters $a_1,\ldots,a_{i-1}$ are given, then t_i as partial function of a_i is one from C^1 class obtained by application of the implicit theorem to constraints $\dot{x}(t_i)=f_i\left(x(t_i),t_i\right)$, $\psi_i\left(x(t_i),t_i,a_i\right)=0$. In particular, we have* (3)

$$\frac{\partial\psi_i}{\partial x_i}f_i\big|_{t_i}+\frac{\partial\psi_i}{\partial t_i}\neq 0, \frac{\partial t_i}{\partial a_i}=-\left(\frac{\partial\psi_i}{\partial x_i}f_i+\frac{\partial\psi_i}{\partial t_i}\right)^{-1}_{t_i}\frac{\partial\psi_i}{\partial a_i}$$

Remark — *For A1 as for A2, case where t_i is specified, i.e. $\psi_i=t_i-cst$, is possible.*

Relation (3) comes from $0=d\psi_i=\frac{\partial\psi_i}{\partial x_i}dx_i+\frac{\partial\psi_i}{\partial t_i}dt_i+\frac{\partial\psi_i}{\partial a_i}da_i$, $dx_i=f_i\big|_{t_i}dt_i$ and given assumptions.

Criterion — *Let J^0 be a criterion, to minimize or maximize, in the form*

$$J^0=\sum_{i=1}^{N+1}J_i^0$$

where $$J_i^0=\phi_i\left(x_i,t_i,a_i\right)+\int_{t_{i-1}}^{t_i}L_i\left(x,t\right)dt \tag{4}$$

with ϕ_i and L_i from C^1 class.

Optimization problem — *Assuming A1, A2, we consider t_i as a function of $a_1,\ldots,a_i$, $i=1,\ldots,N+1$. We search values for $a_1,\ldots,a_{N+1}$ which optimize criterion J^0 under constraints* (1) *and* (2).

In this paper, we limit ourselves to the search of a calculus method for the variation $dJ^0=\sum_{i=1}^{N+1}\frac{dJ^0}{da_i}da_i$.

Here, notation $\frac{dJ^0}{da_i}$ means that the variation of J^0 according to a_i is to be considered both directly through a_i, namely $\frac{\partial J^0}{\partial a_i}$, and indirectly through all t_j and $x_j=x\left(t_j\right)$, $j=i,\ldots,N+1$. The use of a classical steepest descent algorithm (Polak, 1997) permits then pursuing the resolution.

3 VARIATIONAL CALCULUS

For $i=1,\ldots,N+1$, we introduce a costate variable $\lambda_i=\lambda_i(t)$ from C^1 class in $\left[t_{i-1},t_i\right]$ and a control parameter $\nu_i\in\mathbb{R}$ that define an augmented criterion J_i by $J_i=J_i^0+J_i^1$ with J_i^0 like in (4) and

$$J_i^1=\nu_i\psi_i\left(x_i,t_i,a_i\right)+\int_{t_{i-1}}^{t_i}\lambda_i\left(f_i\left(x,t\right)-\dot{x}\right)dt.$$

We define a global augmented criterion J by $J=\sum_{i=1}^{N+1}J_i$. In accordance with paragraph 2, we have also $J^0=\sum_{i=1}^{N+1}J_i^0$. Defining $J^1=\sum_{i=1}^{N+1}J_i^1$, we have therefore $J=J^0+J^1$.

Let $F_i\left(x,t,\dot{x}\right)=f_i\left(x,t\right)-\dot{x}$. Constraints (1), (2) give $\psi_i=0, d\psi_i=0, F_i=0, dF_i=0$. We can deduce that $J_i^1=\nu_i\psi_i+\int_{t_{i-1}}^{t_i}\lambda_i^T F_i dt$ satisfies:

$$dJ_i^1 = d\nu_i \cdot \psi_i + \nu_i \cdot d\psi_i + \lambda_i^T F_i\big|_{t_i} dt_i$$
$$-\lambda_i^T F_i\big|_{t_{i-1}} dt_{i-1} + \int_{t_{i-1}}^{t_i} \left\{ d\lambda_i^T \cdot F_i - \lambda_i^T \cdot dF_i \right\} dt = 0.$$

Hence, we have $dJ_i^0 = dJ_i - dJ_i^1 = dJ_i$ and therefore $dJ^0 = dJ$. In the sequel we will enforce conditions on $\lambda_i(t)$ and ν_i in order to obtain an expression for $dJ = \sum_{i=1}^{N+1} \frac{dJ}{da_i} da_i$ as explicit as possible.

With $\Phi_i = \phi_i + \nu_i \psi_i$, we have

$$J_i = \Phi_i + \int_{t_{i-1}}^{t_i} \left(H_i - \lambda_i^T \dot{x} \right) dt.$$

For variations $\delta x, \delta\dot{x}, dx_{i-1}, dx_i, dt_{i-1}, dt_i$ satisfying $\delta\dot{x} = \dot{\widehat{\delta x}}$ and $\delta x_{i-1} = dx_{i-1} - \dot{x}(t_{i-1}) dt_{i-1}$, $\delta x_i = dx_i - \dot{x}(t_i) dt_i$ (see Bryson and Ho, 1969, fig. 2.7.1) we calculate variation dJ under constraints (1) and (2): $dJ = \sum_{i=1}^{N+1} dJ_i$,

$$dJ_i = \frac{\partial \Phi_i}{\partial x_i} dx_i + \frac{\partial \Phi_i}{\partial t_i} dt_i + \frac{\partial \Phi_i}{\partial a_i} da_i$$
$$+ L_i\big|_{t_i} dt_i - L_i\big|_{t_{i-1}} dt_{i-1} + \int_{t_{i-1}}^{t_i} \left(\frac{\partial H_i}{\partial x} \delta x - \lambda_i^T \delta\dot{x} \right) dt.$$

Integrating by parts yields

$$\int_{t_{i-1}}^{t_i} \left(\frac{\partial H_i}{\partial x} \delta x - \lambda_i^T \delta\dot{x} \right) dt$$
$$= \int_{t_{i-1}}^{t_i} \left(\frac{\partial H_i}{\partial x} + \dot{\lambda}_i^T \right) \delta x dt - \left[\lambda_i^T \delta x \right]_{t_{i-1}}^{t_i},$$

and therefore

$$dJ_i = \left(\frac{\partial \Phi_i}{\partial x_i} - \lambda_i^T \right)_{t_i} dx_i + \left(\frac{\partial \Phi_i}{\partial t_i} + L_i + \lambda_i^T f_i \right)_{t_i} dt_i$$
$$+ \frac{\partial \Phi_i}{\partial a_i} da_i + \lambda_i^T (t_{i-1}) dx_{i-1} - \left(L_i + \lambda_i^T f_i \right)_{t_{i-1}} dt_{i-1}$$
$$+ \int_{t_{i-1}}^{t_i} \left(\frac{\partial H_i}{\partial x} + \dot{\lambda}_i^T \right) \delta x \, dt.$$

We can deduce

$$dJ = \sum_{i=1}^{N+1} dJ_i = \sum_{i=1}^{N+1} \left\{ \left(\frac{\partial \Phi_i}{\partial x_i} - \lambda_i^T \right)_{t_i} dx_i + \left(\frac{\partial \Phi_i}{\partial t_i} + H_i \right)_{t_i} dt_i \right.$$
$$+ \frac{\partial \Phi_i}{\partial a_i} da_i + \lambda_i^T (t_{i-1}) dx_{i-1} - H_i\big|_{t_{i-1}} dt_{i-1}$$
$$\left. + \int_{t_{i-1}}^{t_i} \left(\frac{\partial H_i}{\partial x} + \dot{\lambda}_i^T \right) \delta x \, dt \right\}$$

$$= \sum_{i=1}^{N+1} \left\{ \left(\frac{\partial \Phi_i}{\partial x_i} - \lambda_i^T \right)_{t_i} dx_i + \left(\frac{\partial \Phi_i}{\partial t_i} + H_i \right)_{t_i} dt_i + \frac{\partial \Phi_i}{\partial a_i} da_i \right.$$
$$\left. + \int_{t_{i-1}}^{t_i} \left(\frac{\partial H_i}{\partial x} + \dot{\lambda}_i^T \right) \delta x \, dt \right\} + \sum_{i=0}^{N} \left\{ \lambda_{i+1}^T (t_i) dx_i - H_{i+1}\big|_{t_i} dt_i \right\}$$
$$= \lambda_1^T (t_0) dx_0 - H_1\big|_{t_0} dt_0 + \left(\frac{\partial \Phi_{N+1}}{\partial x_{N+1}} - \lambda_{N+1}^T \right)_{t_{N+1}} dx_{N+1}$$
$$+ \left(\frac{\partial \Phi_{N+1}}{\partial t_{N+1}} + H_{N+1} \right)_{t_{N+1}} dt_{N+1} + \sum_{i=1}^{N} \left\{ \left(\lambda_{i+1}^T - \lambda_i^T + \frac{\partial \Phi_i}{\partial x_i} \right)_{t_i} dx_i \right.$$
$$\left. + \left(-H_{i+1} + H_i + \frac{\partial \Phi_i}{\partial t_i} \right)_{t_i} dt_i + \frac{\partial \Phi_i}{\partial a_i} da_i \right\}. \quad (5)$$

Let us *choose* to compel $\lambda_i(t), \nu_i$ to comply with

$$\frac{\partial H_i}{\partial x} + \dot{\lambda}_i^T = 0 \text{ in } t_{i-1} \le t \le t_i, \quad (6)$$

$$\lambda_i^T (t_i) = \lambda_{i+1}^T (t_i) + \frac{\partial \Phi_i}{\partial x_i}, \quad (7)$$

$$H_i\big|_{t_i} = H_{i+1}\big|_{t_i} - \frac{\partial \Phi_i}{\partial t_i}. \quad (8)$$

Those relations are given for $i = N+1, \ldots, 1$ and with the definition for notation convenience that

$$\lambda_{N+2}(t_{N+1}) = 0,\ L_{N+2}\big|_{t_{N+1}} = 0,\ H_{N+2}\big|_{t_{N+1}} = 0. \quad (9)$$

One can find jump relations (7), (8) in (Bryson and Ho, 1969), or (El Bagdouri *et al.*, 2005). Combination (8) – (7). f_i gives

$$\left(H_i - \lambda_i^T f_i \right)_{t_i} = \left(H_{i+1} - \lambda_{i+1}^T f_i - \frac{\partial \Phi_i}{\partial t_i} - \frac{\partial \Phi_i}{\partial x_i} f_i \right)_{t_i}$$

$$L_i\big|_{t_i} = \left(L_{i+1} + \lambda_{i+1}^T (f_{i+1} - f_i) - \left(\frac{\partial \phi_i}{\partial t_i} + \frac{\partial \phi_i}{\partial x_i} f_i \right) \right)_{t_i}$$
$$- \nu_i \left(\frac{\partial \psi_i}{\partial t_i} + \frac{\partial \psi_i}{\partial x_i} f_i \right)_{t_i}.$$

We can deduce

$$\nu_i = \left(L_{i+1} - L_i + \lambda_{i+1}^T (f_{i+1} - f_i) - \left(\frac{\partial \phi_i}{\partial t_i} + \frac{\partial \phi_i}{\partial x_i} f_i \right) \right)_{t_i}$$
$$\cdot \left(\frac{\partial \psi_i}{\partial t_i} + \frac{\partial \psi_i}{\partial x_i} f_i \right)_{t_i}^{-1}. \quad (10)$$

Thus ν_i is explicitly determined as a function of $\lambda_{i+1}(t_i)$. According to $\Phi_i = \phi_i + \nu_i \psi_i$, substituting this expression of ν_i in (7) gives explicitely $\lambda_i(t_i)$ as a function of $\lambda_{i+1}(t_i)$:

$$\lambda_i^T(t_i) = \lambda_{i+1}^T(t_i) + \frac{\partial \phi_i}{\partial x_i} + \nu_i \frac{\partial \psi_i}{\partial x_i}. \qquad (11)$$

By this way system (6) with limit condition (9) or (11) can be solved backwards starting from $i = N+1$ up to $i = 1$. This resolution is efficient because condition

$$\left(\frac{\partial \psi_i}{\partial t_i} + \frac{\partial \psi_i}{\partial x_i} f_i \right)_{t_i} \neq 0$$

that ensure existence of ν_i given by (10) follows from (3) of our assumption A2.

Let $t_0, x_0 = x(t_0)$ be fixed initial conditions. We have $dt_0 = 0, dx_0 = 0$. According to (5), preceding choices made for ν_i, $\lambda_i(t)$ give

$$dJ = \sum_{i=1}^{N+1} \frac{\partial \Phi_i}{\partial a_i} da_i.$$

As we have established $dJ^0 = dJ$, the searched expression for our gradient calculus is then

$$\frac{dJ^0}{da_i} = \frac{dJ}{da_i} = \frac{\partial \Phi_i}{\partial a_i}.$$

Using $\Phi_i = f_i + \nu_i \psi_i$ and constraint (1), we can precise

$$\begin{aligned} \frac{dJ^0}{da_i} &= \frac{\partial \phi_i}{\partial a_i} + \nu_i \frac{\partial \psi_i}{\partial a_i} + \frac{d\nu_i}{da_i}\psi_i \\ &= \frac{\partial \phi_i}{\partial a_i} + \nu_i \frac{\partial \psi_i}{\partial a_i}, \; i = 1, \ldots, N+1. \end{aligned} \qquad (12)$$

We have obtained the following algorithm:

Algorithm — *Our aim is to compute the gradient linked to the optimization problem set in paragraph 2. Let* $a_1, \ldots, a_{N+1}$ *be fixed parameters and let* $t_0, x_0 = x(t_0)$ *be specified initial conditions. A calculus algorithm for variation*

$$dJ^0 = \sum_{i=1}^{N+1} \frac{dJ^0}{da_i} da_i$$

at $a_1, \ldots, a_{N+1}$ *is the following one. For given* $t_{i-1}, x_{i-1} = x(t_{i-1})$*, the resolution of forward system (2) with constraint (1) gives* $t_i, x_i = x(t_i)$*. Starting from* $t_0, x_0 = x(t_0)$ *this gives by forward induction sequences* $t_i, x_i = x(t_i), i = 1, \ldots, N+1$*. For given* $\lambda_{i+1}(t_i)$*, we get* ν_i *and* $\lambda_i(t_i)$ *from* (10) *and* (11)*. The resolution of backward system* (6) *with final condition* $\lambda_i(t_i)$ *gives* $\lambda_i(t_{i-1})$*. Starting from* $\lambda_{N+2}(t_{N+2}) = 0$ *in* (9) *this gives by backward induction sequence* $\nu_i, i = N+1, \ldots, 1$*. We get then* $\frac{dJ^0}{da_i}$ *from* (12)*. All this is obtained under assumptions A1, A2.*

Remark — *A special case is the one where* t_i *is not free, that is to say* $\psi_i = t_i - cst$*. We have then* $\lambda_i^T(t_i) = \lambda_{i+1}^T(t_i) + \frac{\partial \phi_i}{\partial x_i}$, $\frac{dJ^0}{da_i} = \frac{\partial \Phi_i}{\partial a_i} = \frac{\partial \phi_i}{\partial a_i}$*. The value of* ν_i *has no effect on the one of* $\frac{dJ^0}{da_i}$*. Taking it equal to zero simplifies the calculuses.*

This result has been established by authors of (Boccadoro *et al.*, 2004) and (Wardi, Y., 2004), but limited to the case where $\dot{x} = f_i(x)$, $\psi_i(x_i, a_i) = 0$, and $J_i^0 = \int_{t_{i-1}}^{t_i} L_i(x)\,dt$. Moreover, counter to their demonstration more technical, here we make clear the interpretation of costate $\lambda(t)$ in terms of control (they do not consider an augmented criterion *J*). In (Boccadoro, 2004) one can find a nice application to the optimization of switching rules for a fixed obstacle avoidance problem in robotics. Our more general algorithm enables us to extend this application to the case of a mobile obstacle.

4 CONCLUSION

A natural generalization would be to consider the case where there is an additional continuous control term $u(t)$ between switching times. This is studied in (Bryson and Ho, 1969), or (El Bagdouri *et al.*, 2005) but whithout taking into account dependency of switching surfaces on parameters $a_1, \ldots, a_{N+1}$. It appears that an explicit determination of ν_i, $\lambda_i(t_i)$ as done in (10), (11) is not always possible. In particular, it is subject to some transversality conditions more difficult to specify (Bryson and Ho, 1969, p. 59, p. 103 and p. 164). We can read p. 102: “However, finding solutions to such problems is, in general, quite involved”.

The question of optimization of switching surfaces for a hybrid dynamical system is due to authors of (Boccadoro *et al.*, 2004) and (Wardi *et al.,* 2004). Our contribution is about deepening, simplification and generalization of their works. It uses the idea of an augmented criterion as in (Bryson and Ho, 1969) or (El Bagdouri *et al.*, 2005). Like in these references, at cost of more complexity and more place, it would be easy to generalize our algorithm to the case of controlled jumps for state variable x at switching instants t_i (Bryson and Ho, 1969, p. 106-107).

Diversity of possible applications for our theoretical results motivates a second communication (Quémard, 2005d). Let us mention the applications, classical or new, for which a resolution is performed:

- Optimization of limit cycles. Application to a thermal device with hysteresis phenomenon (Quémard *et al.*, 2005a, 2005b, 2005c).
- Optimization of switching instants for a minimum time problem for a car with two gears
- Optimization of switching rules for a mobile obstacle avoidance problem in robotics.

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