MULTIOBJECTIVE OPTIMAL DESIGN OF STRUCTURE AND CONTROL OF A CONTINUOUSLY VARIABLE TRANSMISSION

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Keywords: Mechatronic system, parametric optimal design, continuously variable transmission.

Abstract: An approach to solve the mechatronic design problem is to formulate the problem as a multiobjective dynamic optimization problem (MDOP), where kinematic and dynamic models of the mechanical structure and the dynamic model of the controller are considered besides a set of constraints and a performance criteria. This design methodology can provide a set of optimal mechanical and controller parameters so that the desired dynamic behavior and the performance criteria are satisfied. In this paper a MDOP is proposed and applied to a continuously variable transmission (CVT). Performance criteria are the mechanical efficiency and the minimal controller energy. The goal attainment method and a sequential approach are used to solve the MDOP.

1 INTRODUCTION

Optimization arises by the necessity to design or to improve systems according to the requirement under which systems operate. There are several criteria that can help to quantify the system performance; however, these criteria are often in conflict since frequently the structural objectives of design require hard conditions for the controller. Therefore the design problem is usually considered as a multiobjective design problem in order to obtain better systems. Recent research in the area of mechatronic systems exposes the need of a concurrent design methodology for mechatronic systems. This methodology must produce mechanical, electronical and control flexibility for the designed system (Zhang et al., 1999), (van Brussel et al., 2001).

In (Li et al., 2001) a concurrent method for mechatronic systems design is proposed. There, a simple dynamic model of the mechanical structure is obtained. The dynamic model obtained allows an easier controller design which improves the dynamic performance. However, this concurrent design concept is based on an iterative process. This method obtains the mechanical structure in a first step and the controller design in a second step, if the resulting controller design is very difficult to implement, the first step must be done again.

The main contribution of this paper is to develop and apply an integral methodology to formulate the system design problem in the dynamic optimization framework. In order to do this, the parametric optimal design of a pinion-rack continuously variable transmission (CVT) is stated as a multiobjective dynamic optimization problem (MDOP), where both the kinematic and dynamic models of the mechanical structure and the dynamic model of the controllers are jointly considered besides system performance criteria. The methodology allows us to obtain a set of optimal mechanical and controller parameters in only one step, which can produce a simple system reconfiguration.

In the multiobjective optimization framework, a classical approach is to reduce the original problem into an equivalent single objective problem using a weighted sum of the original objectives. In most of the cases, this single objective problem will be easier to solve than the original multiobjective problem. However, the weakness of the weighted method is that not all of the non dominated solutions can be found unless the problem is convex (Osyczka, 1984).

On the other hand, in spite of the development of many control strategies in the last decades, the proportional, integral and derivative (PID) controller remains as the most popular approach for industrial processes control due to the adequate performance in most of such applications. Many PID design techniques have been developed; these provide a sim-

Alvarez-Gallegos J., A. Cruz-Villar C. and A. Portilla-Flores E. (2005).
MULTIOBJECTIVE OPTIMAL DESIGN OF STRUCTURE AND CONTROL OF A CONTINUOUSLY VARIABLE TRANSMISSION.
In *Proceedings of the Second International Conference on Informatics in Control, Automation and Robotics*, pages 154-160
DOI: 10.5220/0001185001540160

ple tuning process to determinate the PID controller gains. However, these do not provide a good control performance in all cases.

A MDOP can be solved by converting it into a nonlinear programming (NLP) problem (Kraft, 1985), (Goh and Teo, 1988) and using the Goal Attainment method (Liu et al., 2003) for the resulting problem. Two transcription methods exist for the MDOP problem: the sequential and the simultaneous methods (Betts, 2001). In the sequential method, only the control variables are discretized; this method is also known as the control vector parameterization. In the simultaneous method the state and control variables are discretized resulting in a large-scale NLP problem which usually requires special solution strategies.

Current research efforts in the field of power transmission of rotational propulsion systems, are dedicated to obtain low energy consumption with high mechanical efficiency. An alternative solution to this problem is the so called continuously variable transmission (CVT), whose transmission ratio can be continuously changed in an established range. There are many CVT's configurations built in industrial systems, especially in the automotive industry due to the requirements to increase the fuel economy without decreasing the system performance. The mechanical development of CVT's is well known and there is little to modify regarding its basic operation principles. However, research efforts go on with the controller design and the CVT instrumentation side. Different CVT's types have been used in different industrial applications; the Van Doorne belt or V-belt CVT is the most studied mechanism (Shafai et al., 1995), (Setlur et al., 2003). This CVT is built with two variable radii pulleys and a chain or metal-rubber belt. Due to its friction-drive operation principle, the speed and torque losses of rubber V-belt are a disadvantage. The Toroidal Traction-drive CVT uses the high shear strength of viscous fluids to transmit torque between an input torus and an output torus. However, the special fluid characteristic used in this CVT becomes the manufacturing process expensive. A pinion-rack CVT which is a traction-drive mechanism is presented in (De-Silva et al., 1994), this CVT is built-in with conventional mechanical elements as a gear pinion, one cam and two pair of racks. The conventional CVT manufacture is an advantage over other existing CVT's.

In this paper the parametric optimal design of a pinion-rack CVT is stated as a MDOP to obtain a set of optimal mechanical and controller parameters of the CVT and, a higher mechanical efficiency and a minimal energy controller. This paper is organized as follows: The description and the dynamic CVT model are presented in Section 2. The design variables, performance criteria and constraints to be used in the parametric CVT design are established in Section 3. Section 4 presents some optimization results and discuss them. Section 5 presents some conclusions and future work.

2 DESCRIPTION AND DYNAMIC CVT MODEL

In order to apply the design methodology proposed in this paper, the pinion-rack CVT presented in (De-Silva et al., 1994) is used. The pinion-rack CVT, changes its transmission ratio when the distance between the input and output rotation axes is changed. This distance is called "offset" and will be denoted by "*e*". This CVT is built-in with conventional mechanical elements as a gear pinion, one cam and two pair of racks. Inside the CVT an offset mechanism is integrated. This mechanism is built-in with a lead screw attached by a nut to the vertical transport cam. Fig. 1 depicts the main mechanical CVT components.

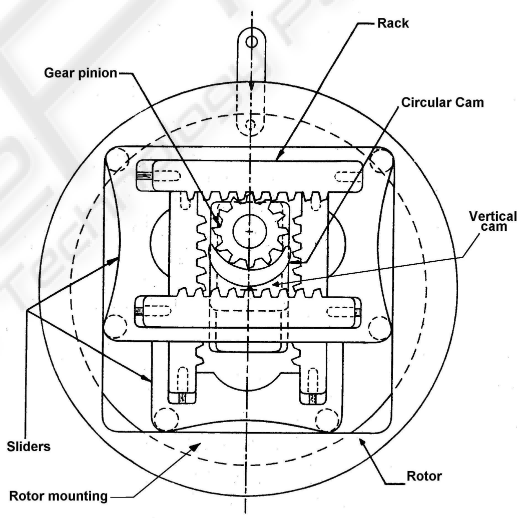

Figure 1: Main CVT mechanical components

The dynamic model of a pinion-rack CVT is presented in (Alvarez-Gallegos et al., 2005). Ordinary differential equations (1), (2) and (3) describe the CVT dynamic behavior. In equation (1): T_m is the input torque , J_1 is the mass moment of inertia of the gear pinion, b_1 is the input shaft coefficient viscous damping, r is the gear pinion pitch circle radius, T_L is the CVT load torque, J_2 is the mass moment of inertia of the rotor, R is the planetary gear pitch circle radius, b_2 is the output shaft coefficient viscous damping and θ is the angular displacement of the rotor. In equations (2) and (3): L, R_m, K_b, K_f and n represent the armature circuit inductance, the circuit resistance, the back electro-motive force constant, the motor torque constant and the gearbox gear ratio of the DC motor, respectively. Parameters r_p, λ_s, b_c and b_l denote the pitch radius, the lead angle, the viscous damping coefficient of the lead screw and the viscous damping coefficient of the offset mechanism, respectively. The control signal $u(t)$ is the input voltage to the DC motor. $J_{eq} = J_{c2} + Mr_p^2 + n^2 J_{c1}$ is the equivalent mass moment of inertia, $J_{c1}$is the mass moment of inertia of the DC motor shaft, J_{c2} is the mass moment of inertia of the DC motor gearbox and $d = r_p \tan\lambda_s$, is a lead screw function. Moreover, $\theta_R(t) = \frac{1}{2}\arctan\left[\tan\left(2\Omega t - \frac{\pi}{2}\right)\right]$ is the rack angle meshing. The combined mass to be translated is denoted by M and $P = \frac{T_m}{r_p}\tan\phi\cos\theta_R$ is the loading on the gear pinion teeth, where ϕ is the pressure angle.

$$\begin{aligned} \left(\frac{R}{r}\right)T_m - T_L &= \left[J_2 + J_1\left(\frac{R}{r}\right)^2\right]\ddot{\theta} \\ &- \left[J_1\left(\frac{R}{r}\right)\frac{e}{r}\sin\theta_R\right]\dot{\theta}^2 \\ &+ \left[\begin{array}{c} b_2 + b_1\left(\frac{R}{r}\right)^2 \\ +J_1\left(\frac{R}{r}\right)\frac{\dot{e}}{r}\cos\theta_R \end{array}\right]\dot{\theta} \end{aligned} \quad (1)$$

$$L\frac{di}{dt} + R_m i = u(t) - \left[\frac{nK_b}{d}\right]\dot{e} \quad (2)$$

$$\left[\frac{nK_f}{d}\right]i - P = \left[M + \frac{J_{eq}}{d^2}\right]\ddot{e} + \left[b_l + \frac{b_c}{r_p d}\right]\dot{e} \quad (3)$$

3 PARAMETRIC OPTIMAL DESIGN

In order to apply the design methodology proposed in this work, two criteria are considered. The first criterion is the mechanical CVT efficiency which considers mechanical parameters and the second criterion is the minimal energy controller which considers the controller gains and the dynamic system behavior.

3.1 Performance criteria and objective functions

The performance of a system is measured by several criteria, one of the most used criteria is the system efficiency because it reflects the energy loss. In this work, the mechanical efficiency criterion of the gear systems is used in the optimization methodology. This is because the racks and the gear pinion are the principal CVT mechanical elements .

The mathematical equation (4) for mechanical efficiency presented in (Spotts, 1964) is used in this work, where μ, N_1, N_2, m, r_1 and r_2 represent the coefficient of sliding friction, the gear pinion teeth number, the spur gear teeth number, the gear module, the pitch pinion radius and the pitch spur gear radius respectively.

$$\eta = 1 - \pi\mu\left(\frac{1}{N_1} + \frac{1}{N_2}\right) = 1 - \frac{\pi\mu}{2m}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad (4)$$

In (Alvarez-Gallegos et al., 2005) the speed ratio equation is stated by (5), where ω is the input angular speed and Ω is the output angular speed of the CVT.

$$\frac{\omega}{\Omega} = \frac{R}{r} = 1 + \frac{e}{r}\cos\theta_R \quad (5)$$

Considering $r_1 \equiv r$ and $r_2 \equiv R$, the CVT mechanical efficiency is given by (6).

$$\eta(t) = 1 - \frac{\pi\mu}{N_1}\left(1 + \frac{1}{1 + \frac{e\cos\theta_R}{r}}\right) \quad (6)$$

In order to maximize the mechanical CVT efficiency, $F(\cdot)$ given by (7) must be minimized.

$$F(\cdot) = \frac{1}{N_1}\left(1 + \frac{1}{1 + \frac{e\cos\theta_R}{r}}\right) \quad (7)$$

Equation (7) can be written as (8) which is used to state the design problem objective function.

$$F(\cdot) = \frac{1}{N_1}\left(\frac{2r + e\cos\theta_R}{r + e\cos\theta_R}\right) \quad (8)$$

The second objective function is stated to obtain the minimal controller energy.

3.2 Constraint functions

The design constraints for the CVT optimization problem are proposed according to geometric and strength conditions for the gear pinion of the CVT.

To prevent fracture of the annular portion between the axe bore and the teeth root on the gear pinion, the pitch circle diameter of the pinion gear must be greater than the bore diameter by at least 2.5 times the module (Papalambros and Wilde, 2000). Then, in order to avoid fracture, the constraint g_1 must be imposed. To achieve a load uniform distribution on the teeth, the face width must be 6 to 12 times the value of the module (Norton, 1996), this is ensured with constraints g_2 and g_3. To maintain the CVT transmission ratio in the range $[2r, 5r]$ constraints g_4, g_5 are imposed. Constraint g_6 ensures a teeth number of the gear pinion equal or greater than 12 (Norton, 1996). A practical constraint requires that the gear pinion face width must be equal or greater than $20mm$, in order to ensure that, constraint g_7 is imposed. To constraint the distance between the corner edge in the rotor and the edge rotor, constraint g_8 is imposed. Finally to ensure a practical design for the pinion gear, the pitch circle radius must be equal or greater than $25.4mm$, then constraint g_9 is imposed.

On the other hand, it can be observed that J_1, J_2 are parameters which are function of the CVT geometry. For this mechanical elements the mass moments of inertia are defined by

$$J_1 = \frac{1}{32}\rho\pi m^4 \left(N+2\right)^2 N^2 h \tag{9}$$

$$J_2 = \rho h \left[\frac{3}{4}\pi r_c^4 - \frac{16}{6}\left(e_{max} + mN\right)^4 - \frac{1}{4}\pi r_s^4\right] \tag{10}$$

where ρ, m, N, h, e_{max}, r_c and r_s are the material density, the module, the teeth number of the gear pinion, the face width, the highest offset distance between axes, the rotor radius and the bearing radius, respectively.

3.3 Design variables

In order to propose a parameter vector for the parametric optimal CVT design, the standard nomenclature for a gear tooth is used.

Equation (11) states a parameter called module m for metric gears, where d is the pitch diameter and N is the teeth number.

$$m = \frac{d}{N} = \frac{2r}{N} \tag{11}$$

The face width h, which is the distance measured along the axis of the gear and the highest offset distance between axes e_{max} are parameters which define the CVT size.

The vector p^i is proposed in order to carry out the parametric optimal CVT design.

$$\begin{aligned} p^i &= [p_1^i, p_2^i, p_3^i, p_4^i, p_5^i, p_6^i]^T \\ &= [N, m, h, e_{max}, K_P, K_I]^T \end{aligned} \tag{12}$$

3.4 Optimization problem

In order to obtain the mechanical CVT parameter optimal values, we propose a multiobjective dynamic optimization problem given by equations (13) to (21). As the objective functions must be normalized to the same scale, the corresponding factors $W = [0.4397, 1126.71]^T$ were obtained using the algorithm of the subsection 3.5 by minimizing each objective function subject to constraints given by equations (14) to (21).

$$\min_{p \in R^6} F(x,p,t) = [F_1, F_2]^T \tag{13}$$

where

$$F_1 = \frac{1}{W_1}\int_0^{10}\left[\frac{1}{p_1}\left(\frac{p_1p_2 + x_3\cos\theta_R}{\frac{p_1p_2}{2} + x_3\cos\theta_R}\right)\right]dt$$

$$F_2 = \frac{1}{W_2}\int_0^{10} u^2 dt$$

subject to

$$\begin{aligned} \dot{x}_1 &= \frac{AT_m + \left[J_1 A \frac{2x_3}{p_1p_2}\sin\theta_R\right]x_1^2 - T_L - \left[b_2 + b_1A^2 + J_1 A\frac{2x_4}{p_1p_2}\cos\theta_R\right]x_1}{J_2 + J_1A^2} \\ \dot{x}_2 &= \frac{u\left(t\right) - \left(\frac{nK_b}{d}\right)x_4 - Rx_2}{L} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{\left(\frac{nK_f}{d}\right)x_2 - \left(b_l + \frac{b_c}{r_pd}\right)x_4 - \frac{T_m}{r_p}\tan\phi\cos\theta_R}{M + \frac{J_{eq}}{d^2}} \end{aligned} \tag{14}$$

$$u(t) = -p_5(x_{ref} - x_1) - p_6\int_0^t (x_{ref} - x_1)dt \tag{15}$$

$$J_1 = \frac{1}{32}\rho\pi p_2^4\left(p_1 + 2\right)^2 p_1^2 p_3 \tag{16}$$

$$J_2 = \frac{\rho p_3}{4}\left[3\pi r_c^4 - \frac{32}{3}\left(p_4 + p_1p_2\right)^4 - \pi r_s^4\right] \tag{17}$$

$$A = 1 + \frac{2x_3}{p_1p_2}\cos\theta_R \tag{18}$$

$$d = r_p \tan\lambda_s \tag{19}$$

$$\theta_R = \frac{1}{2}\arctan\left[\tan\left(2x_1t - \frac{\pi}{2}\right)\right] \tag{20}$$

$$\begin{aligned} g_1 &= 0.01 - p_2\left(p_1 - 2.5\right) \le 0 \\ g_2 &= 6 - \frac{p_3}{p_2} \le 0 \\ g_3 &= \frac{p_3}{p_2} - 12 \le 0 \\ g_4 &= p_1p_2 - p_4 \le 0 \\ g_5 &= p_4 - \frac{5}{2}p_1p_2 \le 0 \\ g_6 &= 12 - p_1 \le 0 \\ g_7 &= 0.020 - p_3 \le 0 \\ g_8 &= 0.020 - \left[r_c - \sqrt{2}(p_4 + p_1p_2)\right] \le 0 \\ g_9 &= 0.0254 - p_1p_2 \le 0 \end{aligned} \tag{21}$$

3.5 Solution algorithm

The resulting problem stated by (22)-(25) is solved using the goal attainment method, which is described below.

Lets consider the problem of minimizing (22)

$$F(x,\theta,t) = [F_1, F_2]^T \tag{22}$$
$$F_i = \int_{t_0}^{t_f} L_i(x,\theta,t)dt \qquad i = 1,2$$

under θ and subject to:

$$\begin{aligned} \dot{x} &= f(x,\theta,t) & (23)\\ g(x,\theta,t) &\leq 0 & (24)\\ h(x,\theta,t) &= 0 & (25)\\ x(0) &= x_0 \\ \theta &\in R^j \end{aligned}$$

The gradient calculation (26) is obtained using the sensitivity equations stated by (27).

$$\begin{aligned} \frac{\partial F_i}{\partial \theta_j} &= \int_{t_0}^{t_f} \left(\frac{\partial L_i}{\partial x}\frac{\partial x}{\partial \theta_j}(t) + \frac{\partial L_i}{\partial \theta_j} \right) dt & (26)\\ \frac{\partial \dot{x}}{\partial \theta_j} &= \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta_j} + \frac{\partial f}{\partial \theta_j} & (27) \end{aligned}$$

Formulating the MDOP in the goal attainment framework, the resulting problem is stated in equations (28) and (29) subject to equations (23) to (25), where $\omega = [w_1, w_2]^T$ is the scattering vector (Osyczka, 1984), $F^d = [1,1]^T$ are the desired goals for each objective function and $F_1(\theta)$ and $F_2(\theta)$ are the evaluated function.

$$\min_{\theta,\lambda} G(\theta,\lambda) \triangleq \lambda \tag{28}$$

subject to:

$$\begin{aligned} g(\theta) &\leq 0 \\ g_{a1}(\theta) &= F_1(\theta) - \omega_1\lambda - F_1^d \leq 0 & (29)\\ g_{a2}(\theta) &= F_2(\theta) - \omega_2\lambda - F_2^d \leq 0 \end{aligned}$$

A vector θ^i which contains the current parameter values is proposed and the NLP problem given by equations (30) and (31) is obtained, where B_i is the BFGS updated positive definite approximation of the Hessian matrix, and the gradient calculation is obtained using sensitivity equations. Hence, if γ^i solves the subproblem given by (30) and (31) and $\gamma^i = 0$, then the parameter vector θ^i is an original problem optimal solution. Otherwise, we set $\theta^{i+1} = \theta^i + \gamma^i$ and with this new vector the process is done again.

$$\min_{\gamma \in R^{j+1}} QP(\theta^i) = G\left(\theta^i\right) + \nabla G^T\left(\theta^i\right)\gamma + \frac{1}{2}\gamma^T B_i \gamma \tag{30}$$

subject to

$$\begin{aligned} g(\theta^i) + \nabla g^T\left(\theta^i\right)\gamma &\leq 0 \\ g_{a1}(\theta^i) + \nabla g_{a1}^T\left(\theta^i\right)\gamma &\leq 0 & (31)\\ g_{a2}(\theta^i) + \nabla g_{a2}^T\left(\theta^i\right)\gamma &\leq 0 \end{aligned}$$

4 OPTIMIZATION RESULTS

This section presents some optimization results when the solution algorithm of section 3.5 is applied to solve the problem stated in section 3.4 under the following conditions. The system parameters used in numerical simulations were: $b_1 = 1.1Nms/rad$, $b_2 = 0.05Nms/rad$, $r = 0.0254m$, $T_m = 8.789Nm$, $T_L = 0Nm$, $\lambda_s = 5.4271$, $\phi = 20$, $M = 10Kg$, $r_p = 4.188E-03m$, $K_f = 63.92E-03Nm/A$, $K_b = 63.92E-03Vs/rad$, $R = 10\Omega$, $L = 0.01061H$, $b_l = 0.015Ns/m$, $b_c = 0.025Nms/rad$ and $n = ((22*40*33)/(9*8*9))$. The initial conditions vector was $[x_1(0), x_2(0), x_3(0), x_4(0)]^T = [7.5, 0, 0, 0]^T$. In order to show the CVT dynamic performance, for all simulations the output reference was considered as $x_{ref} = 7.5$ for $0 \leq t \leq 2$; $x_{ref} = 7.2$ for $t > 2$.

The goal attainment method requires the goal for each one of the objective functions. The goal for F_1 was obtained by minimizing this function subject to equations (14)-(21). The optimal solution vector p^1 is shown in table 1. The goal for F_2 was obtained by minimizing this function subject to equations (14)-(21). The optimal solution vector p^2 for this problem is also shown in table 1.

Varying the scattering vector can produce different non dominated solutions. In table 1, two cases are presented; p_A^* is obtained with $\omega = [0.5, 0.5]^T$, p_B^* is obtained with $\omega = [0.3, 0.7]^T$

Figures 2, 3 and 4 show the mechanical CVT efficiency, the control CVT input and the CVT output respectively, with solutions vectors p_A^*, p^1 and p^2. The solution p_A^* was selected because it has the same over achievement of the proposed goal for each function.

Discussion

Solutions p_A^* and p_B^* in table 1, have an euclidean norm closer to that one associated to the proposed desired vector of goals. These results are according with the structure and control integration approach considered in this work.

It can be observed in figure (2), that the optimal multiobjective solution implies a low sensitivity of the mechanical efficiency with respect to reference

Table 1: MDOP solutions

$[N^*, m^*, h^*, e^*_{max}, K^*_P, K^*_I]$	$F(\bullet) = [F_1(\bullet), F_2(\bullet)]$
$p^1 = [38.1838, 0.0017, 0.02, 0.0636, 10.000, 1.00]$	$F(p^1) = [1.0000, 3.1580]$
$p^2 = [12.0000, 0.0028, 0.02, 0.0880, 5.000, 0.01]$	$F(p^2) = [3.1999, 1.0000]$
$p^*_A = [33.6469, 0.0017, 0.02, 0.0631, 5.000, 0.01]$	$F(p^*_A) = [1.2149, 1.2149]$
$p^*_B = [38.1800, 0.0017, 0.02, 0.0636, 5.001, 0.01]$	$F(p^*_B) = [1.0786, 1.2730]$

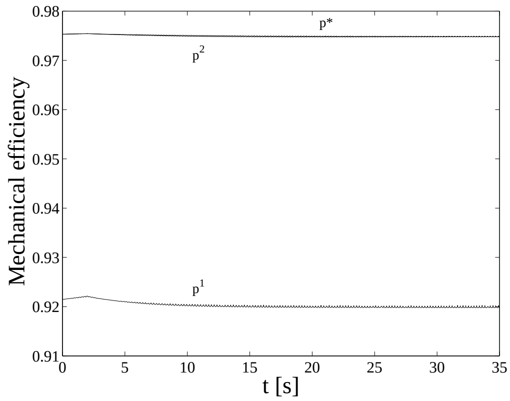

Figure 2: Mechanical efficiency

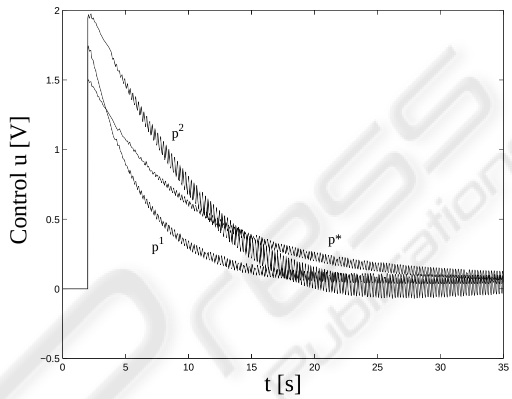

Figure 3: Control input

changes. This is an advantage for CVT's, because the output speed requirements are usually changed.

In figure (3) it can be observed that the p^*_A vector minimizes the initial overshoot of the control input. This fact implies a lower mechanical system wear.

Figure (4) shows the output CVT behavior, it can be observed that with the optimal multiobjective solution a smoother convergence to the reference is obtained.

5 CONCLUSIONS

In this paper, we have developed a suitable parametric optimal design methodology for mechatronic systems where kinematic and dynamic behaviors are jointly considered. This methodology was successfully applied to a traction-drive CVT. Results obtained lead to a higher mechanical efficiency and to a minimal energy controller. The advantage of this design methodology is that the parametric optimal design can be considered as a MDOP. Formulating it in the goal attainment framework, new considerations for the optimization problem are applied to the objective functions. This is a process which does not happen in a weighted approach.

The slow CVT output convergence to the reference shown in figure (4) is due to the small value of the lead angle (λ_s). Further work, will include this parameter as an optimization variable.

Further research includes the proposal of new design constraints. These constraints must consider stress conditions and bounding of the state variables. On the other hand, another objective function of the overall mechanical efficiency of the CVT, including the offset mechanism and lead screw constraints, could be considered in the parametric optimal design. These facts would improve the CVT response.

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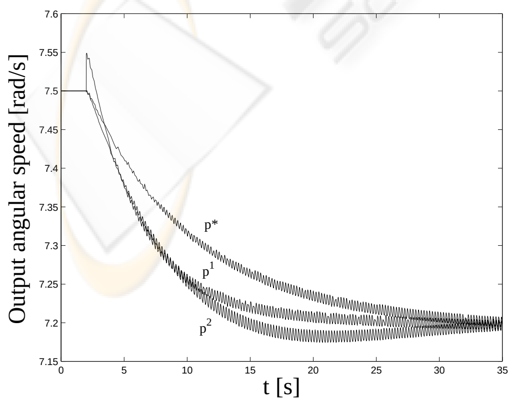

Figure 4: Output CVT behavior