

# On Swarm Optimality in Dynamic and Symmetric Environments <sup>\*</sup>

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**Abstract.** The field of multi agents and multi robotics has become increasingly popular during the last two decades. The motivation behind multi agents based systems is that many tasks can be much efficiently completed by using multiple simple autonomous agents (robots, software agents, etc.) instead of a single sophisticated one. However, when examining such systems, one may be concerned of the price-tag attached to the decentralized nature of swarm based approaches. Meaning, while we simplify designs and control mechanisms in order to save costs and computation resources, how far do our systems drift from optimality ? This work examines this issue by constructing an optimal algorithm for the *Dynamic Cooperative Cleaners* problem (presented and analyzed in [2]). The performance of the **SWEEP** algorithm of [2] is compared to this of an optimal algorithm. The results of this comparison show that not only that the swarm algorithm produces close results to the optimal solution, but also as the problem gets harder, the performance of the two converge. In addition, insightful results concerning optimal swarms in symmetric environments are presented.

## 1 Introduction

The fields of multi agents and multi robotics have become increasingly popular during the last two decades. A multi agents system, or a *swarm*, can generally be defined as a decentralized group of multiple autonomous units, either homogenous or heterogenous, such that those units are simple and possess limited capabilities. Many research efforts have been invested examining distributed systems models inspired by biology (behavior based control model — [16, 13], flocking and dispersing models — [22, 18, 19] and predator-prey approach — [14, 21]), physics [12], and economics [7–11]. The motivation behind multi agents and multi robotics systems is that by using multiple simple autonomous agents instead of a single sophisticated one, many tasks can be performed cheaply and easily, while the performance of those system remains satisfactory. In addition, such systems have several other advantages such as high scalability and adaptivity. Application for which multi robotics systems fit successfully include covering, exploration and patrolling [1], construction of complex structures and self-assembly [17, 15, 26], mapping and localizing [20, 27] and transportation [23, 25, 24].

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However, when designing such systems, one must take into account the decrease in performance which is inherent in such approaches, in comparison to an optimal centralized system (albeit much more complex and expensive).

In this work we examine this issue by constructing a centralized optimal algorithm for the *Dynamic Cooperative Cleaners* problem (presented and analyzed in [2]. Similar works appear in [4–6]). This problem assumes a grid, part of which is “dirty”, when the “dirty” part is a connected region of the grid. On this dirty region several agents move, each having the ability to “clean” the place it is located in. A deterministic evolution of the environment is assumed, simulating a spreading *contamination*, or *fire*.

A cleaning algorithm designed to be used by a swarm of cleaning agents is presented and discussed in [2]. In addition, a lower bound over the cleaning time of agents employing *any* algorithm (i.e. a lower bound over an optimal algorithm for the problem) is presented. The performance of the optimal algorithm described in section 4 is compared to those of the sub-optimal **SWEEP** algorithm (described in section 3) and the generic lower bound for the problem of [2].

The results of this comparison surprisingly show that although the **SWEEP** algorithm is fully decentralized and extremely simple, assuming completely autonomous agents and using no explicit form of communication, the improvements which can be achieved by using an optimal algorithm is in no way significant enough to justify the immense costs and complexity of such an algorithm.

In addition, several insightful and counter-intuitive results concerning optimal clean strategies for symmetric environments are presented in section 6.

## 2 The Dynamic Cooperative Cleaners Problem

Let us assume that the time is discrete. Let  $G$  be a two dimensional grid, whose vertices have a binary property of ‘contamination’. Let  $cont_t(v)$  state the contamination state of the vertex  $v$  in time  $t$ , taking either the value “on” or “off”. Let  $F_t$  be the dirty sub-graph of  $G$  at time  $t$ , i.e.  $F_t = \{v \in G \mid cont_t(v) = on\}$ . We assume that  $F_0$  is a single connected component. Let a group of  $k$  agents that can move across the grid  $G$  (moving from a vertex to its neighbor in one time step) be placed in time  $t_0$  on  $F_0$ .

Each agent is equipped with a sensor capable of telling the condition of the square it is currently located in, as well as the condition of the squares in the  $8-Neighbors$  group of the this square. An agent is also aware of other agents which are located in its square, and all the agents agree on a common “north”. Each square can contain any number of agents simultaneously. When an agent moves to a vertex  $v$ , it has the possibility of causing  $cont(v)$  to become *off*. The agents do not have any prior knowledge of the shape or size of the sub-graph  $F_0$  except that it is a single connected component. Every  $d$  time steps the contamination spreads. That is, if  $t = nd$  for some positive integer  $n$ , then  $(\forall v \in F_t, \forall u \in 4-Neighbors(v) : cont_{t+1}(u) = on)$ . The agents’ goal is to clean  $G$  by eliminating the contamination entirely, so that  $(\exists t_{success} : F_{t_{success}} = \emptyset)$ . In addition, it is desired that this  $t_{success}$  will be minimal.

### 3 The SWEEP Cleaning Algorithm

For solving the *Dynamic Cooperative Cleaners* problem the **SWEEP** algorithm was suggested in [2]. The algorithm can be described as follows. Generalizing an idea from computer graphics (which is presented in [3]), the connectivity of the *contaminated* region is preserved by preventing the agents from cleaning what is called *critical points* — points which disconnect the graph of contaminated grid points. This ensures that the agents stop only upon completing their mission. At each time step, each agent cleans its current location (assuming this is not a critical point), and moves to its *rightmost* neighbor — a local movement rule, creating the effect of a clockwise traversal of the contaminated shape. As a result, the agents “peel” layers from the shape, while preserving its connectivity, until the shape is cleaned entirely.

### 4 An Optimal Cleaning Algorithm

In order to find the minimal cleaning time possible for a given contaminated shape  $F_0$ , we have designed a system in which all the cleaning agents are controlled by a central unit (referred to as the *queen*). Upon initialization, the queen is given the complete information regarding the contaminated shape. While the agents are traveling along the grid, the queen is immediately aware of any new information discovered by the agents. The queen’s orders as to the next desired movements of the agents are also immediately transferred to the agents, which carry them out automatically.

Since the agents are completely bound to the desires of the queen, lacking even the slightest amount of autonomy, this mechanism can be thought of as one big robot, equipped with numerous long “cleaning hands”. Implementing such mechanism will of course be very complicated and there is no doubt that such a robot will be very expensive (remembering that the “cleaning hands” are capable of moving to very large distances from one another). However, since we are interested in the optimality issue, costs and complexity aspects are not of interest to us, at the moment.

For calculating the optimal solution to the problem, the queen uses the well known  $A^*$  algorithm [28]. Upon initialization, the queen exhaustively searches for the shortest path within the states space, where the initial state contains  $F_0$  and the initial locations of the agents. Every state can be developed into  $2^k$  states (since every agents can make a single move clockwise or counterclockwise). Since the queen possesses complete information regarding  $F_0$  and knows exactly what the agents are doing, there is no need for the agents to *not* clean certain contaminated squares (note that in the **SWEEP** algorithm for example, sometimes the agents do not clean contaminated squares in order to preserve the connectivity of the contaminated shape). Every  $d$  transitions between states, the queen simulates a contamination spread. The success state is a state in which there are no longer contaminated squares.

Once finding the shortest path within the states space from the initial state to a success state, the queen starts moving the agents according to this path.

The optimality of this algorithm is immediately derived from the optimality of the  $A^*$  algorithm in finding shortest paths in states spaces.

## 5 Experimental Results

A computer simulation of both the optimal algorithm described in section 4 and of the **SWEEP** algorithm of [2] was constructed. Note that the size of the states spaces covered by the optimal algorithm was at least :

$$2^{k \cdot f(F_0)} \quad (1)$$

where  $k$  is the number of agents,  $F_0$  is the initial contaminated shape, and  $f()$  is the lower bound over the cleaning time of [2], defined as :

$$f(F_0) \equiv \min\{t \in \mathbb{N} \wedge S_t \leq 0\} \quad (2)$$

where :

$$S_{t+d} \geq S_t - d \cdot k + (\sqrt{8 \cdot (S_t - d \cdot k) - 4} + 2) \quad (3)$$

$d$  is the number of time steps between contamination spreads and  $S_0$  is the initial area of  $F_0$ , namely —  $S_0 = |F_0|$ .

Three types of shapes were examined, varying in their convexity (i.e. their  $\frac{S_0}{c_0}$  value, where  $c_0$  is the initial circumference of the shape). Shapes whose  $\frac{S_0}{c_0}$  value is high are referred to as *convex* shapes. Similarly, *semi-convex* and *concave* shapes are defined.

For each type of shape, three values for  $S_0$  were chosen. For each value of  $S_0$ , three values for  $k$  were selected. For each set of values, six random simulations were performed, in order to achieve statistical significance.

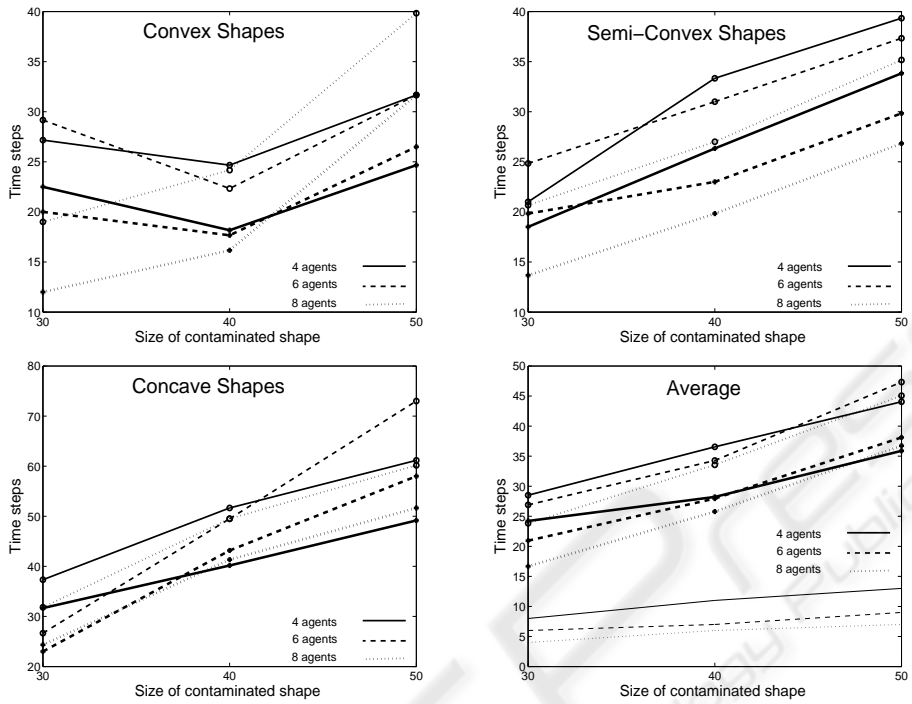
Figure 1 presents the results of the simulations, including the predictions of the lower bound presented in equation 2. Note that although the sub-optimal cleaning algorithm achieves results which are roughly 400% slower than the estimations of the lower bound, the optimal algorithm achieves results which are also relatively far from the lower bound's estimations. This demonstrates the fact that the sub-optimal algorithm's performance are in fact, much closer to the optimal performances than expected initially in [2].

After analyzing the results and trying to find a correlation between the number of agents to the  $\frac{performance_{optimal}}{performance_{sub-optimal}}$  ratio, it seems that such correlation does not exist.

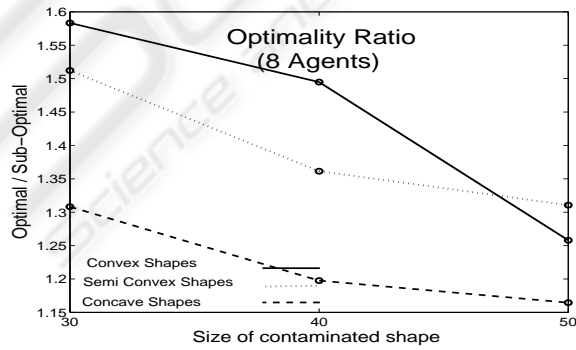
However, when focusing on the largest number of agents examined (8 agents), it can be seen that as the size of  $F_0$  gets bigger, the  $\frac{performance_{optimal}}{performance_{sub-optimal}}$  ratio gets smaller. This holds for all three types of shapes. The meaning of this observation is that as the problem is getting harder (larger initial shapes and more agents), the benefit from using an optimal algorithm reduces (meaning, the sub-optimal algorithm achieves better and better results). This is demonstrated in figure 2.

## 6 Symmetric Environments

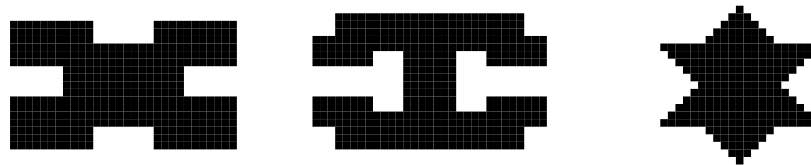
In this section we shall present several comparisons of the **SWEEP** sub-optimal algorithm and our optimal algorithm, in geometrical symmetric environments. Figure 3 presents the symmetric initial shapes chosen for this comparison. Four agents were placed in symmetric places along the shapes, and both algorithms were tested.



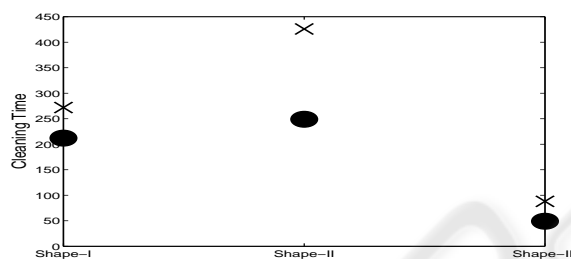
**Fig. 1.** Comparison of sub optimal and optimal algorithms. The lower and thicker three lines represent the cleaning time of the optimal algorithm whereas the upper lines represent the **SWEEP** cleaning algorithm's performance. In the right chart on the bottom, the lower three lines represent the lower bound of the optimal solution, as appears in equation 2.



**Fig. 2.** Comparison of sub optimal and optimal algorithms. The Y-axis represents the  $\frac{performance_{optimal}}{performance_{sub-optimal}}$  ratio for various sizes. Note that as the problem gets bigger, the sub-optimal performance of the algorithm gets closer to those of the optimal algorithm.



**Fig. 3.** Symmetric initial shapes referred to as Shape-I, Shape-II and Shape-III respectively (left shape is Shape-I).



**Fig. 4.** Comparison of sub optimal and optimal algorithms. As can be seen once again, the performance of the sub-optimal algorithm are only roughly 60% worse than those of the optimal algorithm.

The comparison between the cleaning time of the optimal and sub-optimal algorithms are presented in figure 4.

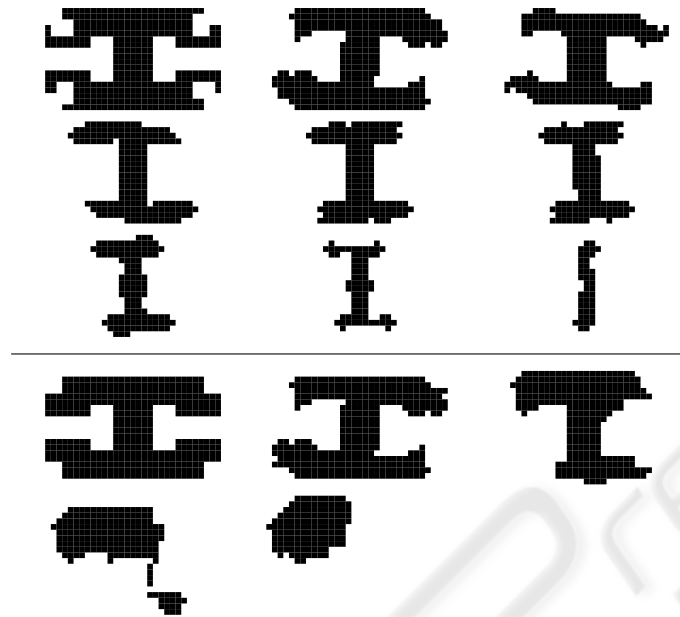
It is interesting to state that although the **SWEEP** algorithm, according to its definition, generates a symmetric behavior of the agents (meaning, the agents are traversing the shape in the same manner, creating symmetric effects over it) this is not the case for the optimal algorithm. Counter-intuitively, the optimal algorithm generates a remarkably non-symmetric behavior. The agents are divided into non-identical and non-stable groups, while the effects the activities of the agents have on the shape transform it quickly into a truly non-symmetric shape. Interestingly enough, this behavior occurs in all three shapes (as well as in other symmetric shapes).

This insightful observation may lead to a change in the concepts which shape the design of swarm algorithms in the future. Hitherto, it seems that most such works tend to be biased by the belief that symmetric behavior generates high quality results while being implemented into swarms' behavior. Nevertheless, by ignoring this miss-belief, an improvement in the performance of such algorithms may be achieved.

The reason for this may rely in low robustness of symmetric behaviors, which are prone to "*error resonance*", meaning — significantly increasing the effect of a small error embedded within the swarm's algorithm. By intentionally avoiding a symmetric behavior, such traps may also be avoided, increasing the swarm's robustness and subsequently, its performance.

A demonstration of the above appears in figure 5.





**Fig. 5.** The upper drawings demonstrate the symmetric nature of the **SWEEP** algorithm, while the lower drawings demonstrate the non-symmetric behavior that was shown to be optimal for Shape-II.

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