CLUSTERING INTERESTINGNESS MEASURES WITH POSITIVE CORRELATION

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Abstract: Selecting interestingness measures has been an important problem in knowledge discovery in database. A lot of measures have been proposed to extract the knowledge from large databases and many authors have introduced the interestingness properties for selecting a suitable measure for a given application. Some measures are adequate for some applications but the others are not, and it is difficult to capture what the best measures for a given data set are. In this paper, we present a new approach implemented in a tool to select the groups or clusters of objective interestingness measures that are highly correlated in an application. The final goal relies on helping the user to select the subset of measures that is the best adapted to discover the best rules according to his/her preferences.

1 INTRODUCTION

Many interestingness measures have been proposed in the literature to select the most significant knowledge from a large database in knowledge discovery in database research such as *support, confidence, causal support, laplace, lift, cosine, gini-index, conviction, loevinger, yule's y, intensity of implication entropy's version, ...* in order to reduce the enormous amount of rules discovered under the form of association rules introduced by Agrawal (Agrawal et al., 1993). Each measure is used with its own characteristics of a given domain of application so that it is not adequate for a given domain of application that is strongly different. There are two types of measures (Freitas, 1999): subjective and objective. Subjective measures (Padmanabhan and Tuzhilin, 1998) (Liu et al., 1999) depend on the user who examines the data with his/her experiences while objective measures depend only on the data structure. The problem of finding interesting patterns leads to the design of a suitable measure and the definition of a set of principles or properties (Silberschartz and Tuzhilin, 1996)(Gavrilov et al., 2000) (Klemettinen et al., 1994) (Brin et al., 1997) (Bayardo and Agrawal, 1999) (Hilderman and Hamilton, 2001) (Tan et al., 2004). In our works, we focus on the objective interestingness measures.

The paper is organized as follows. Section 2 gives the definition and approach with the concepts of intensity of implication. Section 3 introduces an overview of interestingness properties proposed in the literature and determines some mathematical relation of the measures. Section 4 presents our approach on cluster analysis. Section 5 gives the first results of our works in finding the clusters of interestingness measures. Finally, we conclude and introduce some future research works.

2 INTENSITY OF IMPLICATION

Gras (Gras, 1996) introduced the theory of statistical implication, with the concepts of examples and negative examples (contra-example). We consider the quality of the implication and determine this quality with the change of negative examples.

An association rule $a \Rightarrow b$ is an implication that b tends to be true once we know a. But it is difficult to see this case in reality. There is always some negative examples that b is false when a is true. Intensity of implication (Gras, 1996) is a measure $\varphi(a \Rightarrow b)$ based on the probability model of association rule and giving the user some thing interesting by considering the effects of the number of negative examples on the decision. Intensity of implication is also a robustness approach for the evaluation of the data set by just tak-

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ing in account a small number of negative examples.

Each association rule $a \Rightarrow b$ must be associated to 4 cardinalities $(n, n_a, n_b, n_{a\bar{b}})$. More precisely, n is the number of transactions, n_a (resp. n_b) the number of transactions satisfying the itemset a (resp. b), and $n_{a\bar{b}}$ is the number of transactions satisfying $a \wedge \bar{b}$ (negative examples).

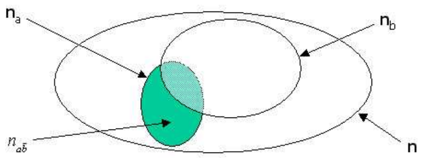


Figure 1: Cardinalities of a rule $a \Rightarrow b$.

With these notions, $p(a), p(b), p(a \wedge \bar{b}), p(a \wedge b)$ are the probabilities of the premise, conclusion, negative examples, and examples of the rule respectively computed as: $\frac{n_a}{n}, \frac{n_b}{n}, \frac{n_{a\bar{b}}}{n}, \frac{n_{ab}}{n}$.

With all the studied interestingness measures, we are going to calculate as one function f with four arguments $n, n_a, n_b, n_{a\bar{b}}$. Then, all the quality measures will be transformed in terms of n, n_a, n_b, and $n_{a\bar{b}}$. *See Appendix A.*

3 INTERESTINGNESS PROPERTIES

Objective measure is a data-driven, domain-independent approach to evaluate the quality of discovered patterns. Piatetsky-Shapiro (Piatetsky-Shapiro, 1991) proposed three principles for a suitable rule-interest measure (RI) on a rule $a \Rightarrow b$: (P1) $RI = 0$ if a and b are statistically independent; the rule is not interesting, (P2) RI *monotonically increases* with $p(a \wedge b)$ when $p(a)$ and $p(b)$ remain the same, (P3) RI *monotonically decreases* with $p(a)$ or $p(b)$ when the rest of the parameters ($p(a \wedge b)$ and $p(b)$ or $p(a)$) remain unchanged.

There are many authors who have extended these principles or proposed new principles:

(Bayardo and Agrawal, 1999) concluded that the best rules according to any interestingness measures must reside along a support/confidence border. The work allows for improved insight into the data and supports more user-interaction in the optimized rule-mining process.

(Hilderman and Hamilton, 2001) proposed five principles for ranking summaries generated from databases, and performed a comparative analysis of sixteen diversity measures to determine which ones satisfy the proposed principles. The objective of this work is to gain some insight into the behavior that can be expected from each of the measures in practice.

(Kononenco, 1995) analyses the biases of eleven measures for estimating the quality of multi-valued attributes. The values of information gain, j-measure, gini-index, and relevance tend to linearly increase with the number of values of an attribute.

(Tan et al., 2004) introduced twenty-one interestingness measures using Pearson's correlation and has found two situations in which the measures may become consistent with each other, namely, the support-based pruning or table standardization are used. In addition, he also proposed five new interestingness properties: (1) symmetry under variable permutation, (2) row/column scaling invariance, (3) anti-symmetry under row/column permutation, (4) inversion invariance, and (5) null invariance, to capture the utility of an objective measure in terms of analyzing k-way contingency tables.

Our approach is different because we consider the data set values with the intensity of implication measure, taking into account the number of negative examples in the data sets, while the other authors have investigated the properties of interestingness on the basis of their own approaches, not on the data set.

Furthermore, we have found some relations in mathematical formulae of the quality measures. This work is useful and interesting for reducing the quantity of measures. If one measure strongly depends on the other measures, we will not consider it any more. For example: $TauxDeLiaison = Lift - 1$, we will only select $Lift$ for both of them. *See Appendix B.*

4 CLUSTER ANALYSIS

4.1 Correlation

We extended the definition of similarity between two associations patterns introduced by Tan (Tan et al., 2004):

Let $R(D) = \{r_1, r_2, ..., r_p\}$ denote input data as a set of p association rules derived from a data set D. Each rule $a \Rightarrow b$ is described by its itemsets (a, b) and its cardinalities $(n, n_a, n_b, n_{a\bar{b}})$. Let M be the set of q available measures for our analysis $M = \{m_1, m_2, ..., m_q\}$. Each measure is a numerical function on rule cardinalities: $m(a \Rightarrow b) = f(n, n_a, n_b, n_{a\bar{b}})$.

For each measure $m_i \in M$, we can construct a vector $m_i(R) = \{m_{i1}, m_{i2}, ..., m_{ip}\}, i = 1..q$, where m_{ij} corresponds to the calculated value of the measure m_i for a given rule r_j.

The correlation value between any two measures $m_i, m_j \{i, j = 1..q\}$ on the set of rules R will be calculated by using a Pearson's correlation coefficient

CC (Saporta, 1990), where $\overline{m_i}, \overline{m_j}$ are the average calculated values of vector $m_i(R)$ and $m_j(R)$ respectively.

$$CC(m_i, m_j) = \frac{\sum_{k=1}^{p}[(m_{ik} - \overline{m_i})(m_{jk} - \overline{m_j})]}{\sqrt{[\sum_{k=1}^{p}(m_{ik} - \overline{m_i})^2][\sum_{k=1}^{p}(m_{jk} - \overline{m_j})^2]}}$$

Definition 1. *Strongly positive/negative correlated measures*. Two measures m_i and m_j are strongly positive (resp. negative) correlated to each other with respect to the data set D if the correlation value between m_i and m_j is greater (resp. lower) than or equal to a threshold t_g (resp. t_l).

Definition 2. *Uncorrelated measures*. Two measures m_i and m_j are uncorrelated to each other with respect to the data set D if the absolute value of the correlation between them is lower than or equal to a critical value t_u.

In our experiment, we use $t_g = 0.85, t_l = -0.85$, and $t_u = 1.960 \times \frac{1}{\sqrt{p}}$ (a level of significance of the test $\alpha = 0.05$ for hypothesis testing) in a population p because of their wide acceptation in the literature.

Definition 3. *Positively correlated cluster*. A connected component of the graph that is based on the similarity matrix in which each strongly positive correlation between two measures represents an edge, it is a positively correlated cluster.

4.2 A quick description of data and measures used

We have applied our experiments on the rule set of 120000 association rules. The rules have been extracted from the "Mushroom" data set by using the Apriori algorithm (Agrawal and Srikant, 1994). The Mushroom data set is issued from one of the categorical data set from Irvine machine-learning database repository (Blake and Merz, 1998).

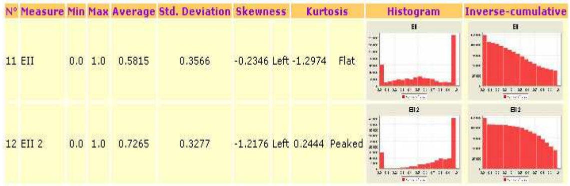
In our experiment, we compared and analyzed 34 interestingness measures (*see Appendix A for complete definitions*).

As announced, we will present some interesting results of the Mushroom data set, focusing on cluster results.

5 FIRST RESULTS ON CLUSTER ANALYSIS

5.1 Distributions

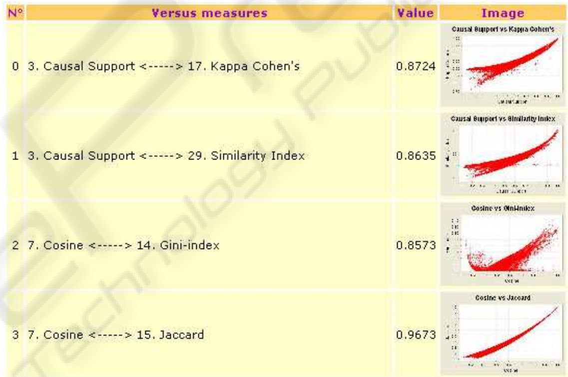
Frequent and inverse-cumulative histograms are introduced to present the frequency of values calculated from the measures in one cluster. The minimum, maximum, average, skewness and kurtosis values are computed in order to allow the user to have a first view of all the measures in the cluster (Fig. 2).



N°	Measure	Min	Max	Average	Std. Deviation	Skewness		Kurtosis		Histogram	Inverse-cumulative
11	EII	0.0	1.0	0.5815	0.3566	-0.2346	Left	-1.2974	Flat	EII	EII
12	EII 2	0.0	1.0	0.7265	0.3277	-1.2176	Left	0.2444	Peaked	EII 2	EII 2

Figure 2: Distribution of one cluster from Mushroom data set.

We have determined all the scatter plots that can be generated from every pair of measures, and it takes a lot of time to draw the images. Some scatter plots (Fig. 3) are illustrated from the Mushroom data set.



N°	Versus measures	Value	Image
0	3. Causal Support <-----> 17. Kappa Cohen's	0.8724	Causal Support vs Kappa Cohen's
1	3. Causal Support <-----> 29. Similarity Index	0.8635	Causal Support vs Similarity Index
2	7. Cosine <-----> 14. Gini-index	0.8573	Cosine vs Gini-index
3	7. Cosine <-----> 15. Jaccard	0.9673	Cosine vs Jaccard

Figure 3: The strongly positive correlation of one cluster from Mushroom data set (extract).

5.2 Strongly positive correlation

The strongly positive correlations in each cluster of the data set are introduced in order to find the most interesting patterns according to the reason why the measure is proposed. For example, the correlation value between Cosine and Jaccard is 0.9673, so we can see the correlation image is close to linear correlation. But in reality, it is not true that a high correlation value always has a linear correlation, the conclusion also depends on the form of the image obtained (Fig. 3).

5.3 Weighted graph representation

To have an intuitive view of a cluster, a weighted graph is introduced. A vertex is based on the measure name and an edge is the correlation value between the two measures. The value assigned for the edge is given from the strongly positive correlation value (see Fig. 4).

Based on the strongly positive correlation and constructed with the weighted graph, we have found eleven positively correlated clusters (Fig. 4):

- (C0) Least Contradiction, Causal Confirm, Laplace, Confidence, Descriptive Confirm, Example & Contra-Example, Causal Confidence, Causal Confirmed-Confidence, Descriptive Confirmed-Confidence. These measures have formed the cluster because most of these measures are issued from confidence measure.
- (C1) Cosine, Gini-index, Phi-Coefficient, Similarity Index, Dependence, Lift, Putative Causal Dependency, Causal Support, Kappa Cohen's, Pavillon, Jaccard, Rule Interest, TIC, Klosgen.
- (C2) Yule's Q and Yule's Y are the measures that form a cluster, the same result as Tan (Tan et al., 2004) with the effect of the second property, row/column scaling invariance. This conclusion is not very surprising because of our demonstration in Section 3 these measures are functionally dependent (*See Appendix B for more details*).
- (C3) EII and EII 2, the two entropic versions of the measure Intensity of Implication. These two measures are issued from the change of one parameter in their formula ($\alpha = 1$ or $\alpha = 2$) (Blanchard et al., 2003), so this cluster is naturally and strongly within-related.
- (C4..C10) Seven clusters with a single measure: Collective Strength, Odds Ratio, J-measure, Conviction, Loevinger, Support, and Sebag & Schoenauer, which are not strongly correlated to other ones.

5.4 Best rules

5.4.1 Intersection of the ten best rules

We choose the ten best rules of each positive cluster and give the user an overview of their intersection. The rank of each rule is used to validate the result. The Y-axis holds the rank of the rule for the corresponding measure. Each rule is represented with parallel coordinates among interestingness measure values (see Fig. 5). We can see the intersection in a horizontal line and if we obtain many rules having the same rank value so we will print these rules with only one line. If the user want to capture a small group of the best rules for making their decision, he/she can use these rules for their first choice.

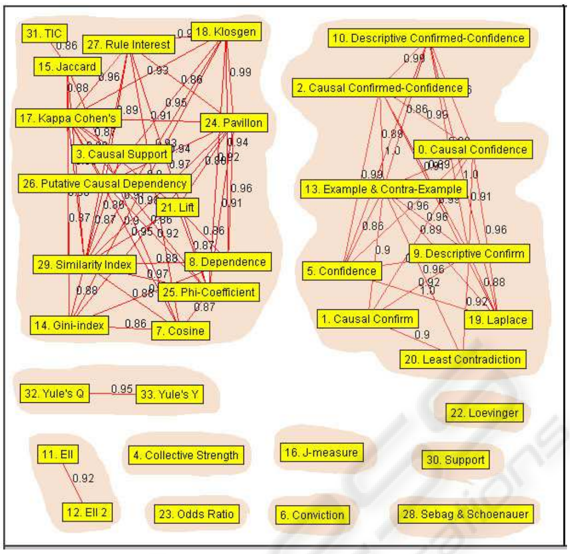


Figure 4: Weighted graph from Mushroom data set.

5.4.2 Union of the ten best rules

With the same technique as above, we introduce the union of the ten best rules in each cluster, in order to give the user a more specific view in the cluster . The measures have the set of highest ranks (more interesting) rely on the low value of the Y-axis. With the concentration lines on low rank values, we can capture 3 measures: Confidence(5), Descriptive Confirmed-Confidence (10), and Example Contra-Example (13) that are suitable for all of the best rules in this cluster (see Fig. 5).

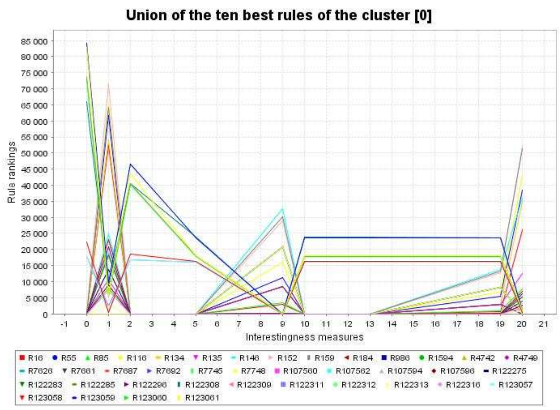


Figure 5: Within-cluster union of one positive cluster from Mushroom data set.

5.5 Clusters in relation to a selected cluster

With the same technique used in the Section 5.4, we introduce the two following views :

5.5.1 Union of all the clusters in relation to the current cluster

Based on the ten best rules in the current cluster, we draw the parallel coordinates of each rule on other clusters. The user can see the zone that is the most interesting with the highest value. The effect of the ten best rules on the other clusters gives the user a general sampling of the entire cluster. With the union approach, many best rules may be presented and compared.

5.5.2 Intersection of all the clusters in relation to the current cluster

By decreasing the quantity of best rules in one cluster, we will observe the rank distribution. The intersection is less interesting than the union because we generally do not have any interesting zone. Using the intersection in relation to the current cluster is important when the user just finds a small set of interesting and close rules.

6 CONCLUSION

This work has led to the implementation of a tool embedding more than 8000 lines of Java codes for the analysis of data set characteristics, quality measure sensitivity, correlations, clustering and ranking.

We have found and identified eleven clusters from all the quality measures studied on the Mushroom data set. We have also proposed a way to study the ten best rules of each cluster, the union and intersection of the ten best rules of all the cluster in relation to the current cluster. The union of the ten best rules for all the clusters is also presented for the user's choice. For the first presentation of our results, we just use thirty four measures for implementation.

Our future research will focus on improving the clustering of measures: (1) in designing a better similarity measure than the linear correlation, (2) in selecting the best representative measure in a cluster.

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Appendix B. Relation between measures

N°	Formulae
0	$Laplace = \frac{Confidence \times (n \times Support + 1)}{n \times Support + 2 \times Confidence}$
1	$RuleInterest = \frac{Support}{Confidence} \times Pavillon$
2	$Lift = \frac{Confidence}{Confidence - Pavillon}$
3	$Wang = \frac{Support}{Confidence} \times (Confidence - \alpha)$
4	$Gray\&Orlowska =$ $(Lift^k - 1) \times (Support - RuleInterest)^m$
5	$J - measure = Support \times \log_2(Lift) + (Support$ $- DescriptiveConfirm) \times \log 2(\frac{1}{Conviction})$
6	$J - measurevariant = Support \times \log_2(Lift)$
7	$Jaccard =$ $\frac{Support}{\frac{Support}{Confidence} + Confidence - Pavillon - Support}$
8	$Loevinger = 1 - \frac{1}{Conviction}$
9	$Consine = \frac{Support}{\sqrt{Support - RuleInterest}}$
10	$CausalConfirm = CausalSupport - 2 \times Support +$ $2 \times DescriptiveConfirm$
11	$CausalConfirmed - Confidence =$ $DescriptiveConfirmedConfidence - Confidence +$ $\frac{CausalConfidence}{2}$
12	$Consine^2 = Lift \times Support$
13	$RuleInterestvariant = \|RuleInterest\|$
14	$TauxDeLiaison = Lift - 1$
15	$Yule'sQ = \frac{OddsRatio - 1}{OddsRatio + 1}$
16	$Yule'sY = \frac{\sqrt{OddsRatio} - 1}{\sqrt{OddsRatio} + 1}$
17	$Example\&Contra - Example = \frac{DescriptiveConfirm}{Confidence - Pavillon}$
18	$LeastContradiction = \frac{DescriptiveConfirm}{}$
19	$Klosgen = \sqrt{Support} \times Pavillon$
20	$Sebag\&Schoenauer = \frac{Support}{Support - DescriptiveConfirm}$

Appendix A. Quality measures

N°	Interestingness Measure	$f(n, n_a, n_b, n_{a\bar{b}})$
0	Causal Confidence	$1 - \frac{1}{2}(\frac{1}{n_a} + \frac{1}{n_{\bar{b}}})n_{a\bar{b}}$
1	Causal Confirm	$\frac{n_a + n_{\bar{b}} - 4n_{a\bar{b}}}{n}$
2	Causal Confirmed-Confidence	$1 - \frac{1}{2}(\frac{3}{n_a} + \frac{1}{n_{\bar{b}}})n_{a\bar{b}}$
3	Causal Support	$\frac{n_a + n_{\bar{b}} - 2n_{a\bar{b}}}{n}$
4	Collective Strength	$\frac{(n_a - n_{a\bar{b}})(n_{\bar{b}} - n_{a\bar{b}})(n_a n_{\bar{b}} + n_b n_{\bar{a}})}{(n_a n_b + n_{\bar{a}} n_{\bar{b}})(n_b - n_a + 2n_{a\bar{b}})}$
5	Confidence	$1 - \frac{n_{a\bar{b}}}{n_a}$
6	Conviction	$\frac{n_a n_{\bar{b}}}{n n_{a\bar{b}}}$
7	Cosine	$\frac{n_a - n_{a\bar{b}}}{\sqrt{n_a n_b}}$
8	Dependence	$\|\frac{n_{\bar{b}}}{n} - \frac{n_{a\bar{b}}}{n_a}\|$
9	Descriptive Confirm	$\frac{n_a - 2n_{a\bar{b}}}{n}$
10	Descriptive Confirmed-Confidence	$1 - 2\frac{n_{a\bar{b}}}{n_a}$
11	EII ($\alpha = 1$)	$\sqrt{\varphi \times I^{\frac{1}{2\alpha}}}$
12	EII ($\alpha = 2$)	$\sqrt{\varphi \times I^{\frac{1}{2\alpha}}}$
13	Example & Contra-Example	$1 - \frac{n_{a\bar{b}}}{n_a - n_{a\bar{b}}}$
14	Gini-index	$\frac{(n_a - n_{a\bar{b}})^2 + n_{a\bar{b}}^2}{n n_a} + \frac{(n_b - n_a + n_{a\bar{b}})^2 + (n_{\bar{b}} - n_{a\bar{b}})^2}{n n_{\bar{a}}} - \frac{n_{\bar{b}}^2}{n^2} - \frac{n_{\bar{b}}^2}{n^2}$
15	Jaccard	$\frac{n_a - n_{a\bar{b}}}{n_b + n_{a\bar{b}}}$
16	J-measure	$\frac{n_a - n_{a\bar{b}}}{n} log_2 \frac{n(n_a - n_{a\bar{b}})}{n_a n_b} + \frac{n_{a\bar{b}}}{n} log_2 \frac{n n_{a\bar{b}}}{n_a n_{\bar{b}}}$
17	Kappa Cohen’s	$\frac{2(n_a n_{\bar{b}} - n n_{a\bar{b}})}{n_a n_{\bar{b}} + n_{\bar{a}} n_b}$
18	Klosgen	$\sqrt{\frac{n_a - n_{a\bar{b}}}{n}}(\frac{n_{\bar{b}}}{n} - \frac{n_{a\bar{b}}}{n_a})$
19	Laplace	$\frac{n_a + 1 - n_{a\bar{b}}}{n_a + 2}$
20	Least Contradiction	$\frac{n_a - 2n_{a\bar{b}}}{n_b}$
21	Lift	$\frac{n(n_a - n_{a\bar{b}})}{n_a n_b}$
22	Loevinger	$1 - \frac{n n_{a\bar{b}}}{n_a n_{\bar{b}}}$
23	Odds Ratio	$\frac{(n_a - n_{a\bar{b}})(n_{\bar{b}} - n_{a\bar{b}})}{n_{a\bar{b}}(n_b - n_a + n_{a\bar{b}})}$
24	Pavillon	$\frac{n_{\bar{b}}}{n} - \frac{n_{a\bar{b}}}{n_a}$
25	Phi-Coefficient	$\frac{n_a n_{\bar{b}} - n n_{a\bar{b}}}{\sqrt{n_a n_b n_{\bar{a}} n_{\bar{b}}}}$
26	Putative Causal Dependency	$\frac{3}{2} + \frac{4n_a - 3n_b}{2n} - (\frac{3}{2n_a} + \frac{2}{n_{\bar{b}}})n_{a\bar{b}}$
27	Rule Interest	$\frac{1}{n}(\frac{n_a n_{\bar{b}}}{n} - n_{a\bar{b}})$
28	Sebag & Schoenauer	$1 - \frac{n_{a\bar{b}}}{n_a - n_{a\bar{b}}}$
29	Similarity Index	$\frac{n_a - n_{a\bar{b}} - \frac{n_a n_b}{n}}{\sqrt{\frac{n_a n_b}{n}}}$
30	Support	$\frac{n_a - n_{a\bar{b}}}{n}$
31	TIC	$\sqrt{TI(a \to b) \times TI(\bar{b} \to a)}$
32	Yule’s Q	$\frac{n_a n_{\bar{b}} - n n_{a\bar{b}}}{n_a n_{\bar{b}} + (n_b - n_{\bar{b}} - 2n_a)n_{a\bar{b}} + 2n_{a\bar{b}}^2}$
33	Yule’s Y	$\frac{\sqrt{(n_a - n_{a\bar{b}})(n_{\bar{b}} - n_{a\bar{b}})} - \sqrt{n_{a\bar{b}}(n_b - n_a + n_{a\bar{b}})}}{\sqrt{(n_a - n_{a\bar{b}})(n_{\bar{b}} - n_{a\bar{b}})} + \sqrt{n_{a\bar{b}}(n_b - n_a + n_{a\bar{b}})}}$