

# MULTIDIMENSIONAL SELECTION MODEL FOR CLASSIFICATION

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**Abstract:** Recent research efforts dedicated to classifier fusion have made it clear that combining performance strongly depends on careful selection of classifiers. Classifier performance depends, in turn, on careful selection of features, which on top of that could be applied to different subsets of the data. On the other hand, there is already a number of classifier fusion techniques available and the choice of the most suitable method relates back to the selection in the classifier, feature and data spaces. Despite this apparent selection multidimensionality, typical classification systems either ignore the selection altogether or perform selection along only single dimension, usually choosing the optimal subset of classifiers. The presented multidimensional selection sketches the general framework for the optimised selection carried out simultaneously on many dimensions of the classification model. The selection process is controlled by the specifically designed genetic algorithm, guided directly by the final recognition rate of the composite classifier. The prototype of the 3-dimensional fusion-classifier-feature selection model is developed and tested on some typical benchmark datasets.

## 1 INTRODUCTION

There is a large evidence of various selection models applied to classification (Roli and Giacinto, 2002), (Ruta and Gabrys, 2005). Even within a single classification model it was observed that selection of certain subset of features, rather than taking them all, could improve the performance of a classifier (Zenobi and Cunningham, 2001). It is also well known that not all the data are suitable for training of the classification model. Quite often data need some sort of cleaning, for example to discard outliers believed to be a faulty data (H. Ishibuchi and Nii, 2001). When the multiple classifier systems (MCS) emerged as a new source of further improvement of classification performance, almost immediately emerged the classifier selection methods which proved that some combinations of classifiers perform better than others (Kuncheva, 2004). At present, given quite a few well-performing classifier fusion methods to choose from, the issue of the combiner selection reemerges this time on the classifier fusion abstraction level. Effectively, what is being observed on this course is that emerging new dimensions of classification systems are always paired with the relevant selection methodology. The issue of selection in the context of classi-

fication has clearly multidimensional nature. What is surprising however, is that when more and more complex classification systems are developed, they seem to be blind to the opportunities of multidimensional selection. Such systems typically ignore the strength of selection methodology and achieve a moderate performance improvement by combining as many relevant components as is allowed by the computational power capabilities. The other group of composite classification systems acknowledges the usefulness of selection. However the selection is usually applied to at most single dimension of the classification process (Giacinto and Roli, 1999). To the best knowledge of the author there were only few attempts to select simultaneously the best subsets of features and classifiers, as in (Kuncheva and Jain, 2000) or (Kuncheva and Whitaker, 2001). In other examples it was shown that data instances and features can be jointly selected by means of genetic algorithm (H. Ishibuchi and Nii, 2001) or nearest neighbour rule (Kuncheva and Jain, 1999). On the other hand as shown in (Ruta and Gabrys, 2005) some combiners like majority voting can be used to further combine its outputs or in other words further combine the combiners.

The presented multidimensional selection system (MSS) attempts to perform selection on many dif-

ferent dimensions at the same time. Moreover as it uses adjusted genetic algorithm guided directly by the combiner performance, selection in all dimensions is coordinated and optimised to jointly produce best overall performance of the composite classification system. The prototype of the 3-dimensional selection system is developed and tested on the two benchmark datasets in the comparative experiments. The remainder of this paper is organised as follows. Section 2 discusses dimensions of selection in the classification process covering in detail selection in data, classifiers and fusion systems. The following section introduces the multidimensional selection model and discusses its representation and selection algorithm. Extensive experimental results are shown in Section 4. Finally conclusions and recommendations for future work are briefly drawn in Section 5.

## 2 DIMENSIONS OF SELECTION

### 2.1 Data

Until recently there was a common belief that all the available data should be used to build a classification model. This belief although theoretically genuine was being gradually relaxed by extensive experimental findings related to the feature selection (H. Ishibuchi and Nii, 2001), (Kuncheva and Jain, 1999). It was uncovered that realistic learning systems can not fully distinguish between good or representative data and bad data due to their lack of mechanisms of accumulative and non-conflicting exploitation of all the data. On practical grounds it turned out that to avoid performances losses, the simplest thing to do is to filter out bad data and use only good data to build a classification model. Finding the most suitable training data breaks down into a variety of ways these data can be selected. Direct selection of the optimal data points is usually referred to as data editing (Kuncheva and Jain, 1999) where the aim could be either to attain the compact data sample that retains maximum representativeness of the original data structure, or simply to fit the best input to the learning mechanism. The selection restrictions can be specified in many other ways beyond just direct selection of samples. As the data is mapped onto the input space, the selection rules can be attributed to the space rather than to the data forming it. The input space can be simply segmented into many differently shaped subspaces. The shapes of subspaces may be formed in various generic forms, or can be dictated by the classification methodology. In dynamic classifier selection methodology (Giacinto and Roli, 1999), the shape of the subspace is dictated by the k-nearest neighbour rule, while in Error Correcting Codes (ECOC) method (Dietterich and Bakiri,

1995) the shape of the subspace is fully determined by the structure of classes in the data. In the most common scenario, the input space is divided along parallel or perpendicular space boundaries, which means that selection applies to features and some particular ranges of their variability, respectively. Labelled character of the data for classification adds an additional dimension for potential selection.

All the features have typically open domains allowing for unlimited variability of  $(-\infty, +\infty)$ . However there could be many reasons for limiting these domains by selecting the narrow range of valid feature variability. One of such reasons could be filtering out the outliers - samples laying far from the areas with high data concentrations. To accommodate outliers, the classification model has to stretch model parameters such that a single distant data point has much greater influence on the model than many points within dense regions of the input space. The domain can be limited by a single or multiple ranges or valid variability for each feature. In the special case the domain range can be reduced to none which is equivalent to the exclusion of such feature.

As mentioned above feature selection is a special case of domain selection but due to its simplicity deserves separate treatment. Feature selection has two attractive aspects to consider. First of all selecting some instead of all features significantly reduces computational costs of classification algorithms which are typically at least quadratically complex with respect to the number of features. Secondly, in practice many features are noncontributory to the classifier performance and sometimes due to imperfect learning algorithms can even cause deterioration. Features can be selected along with their variability range limits. Such scenario is equivalent to selection of particular clusters or subspaces in the input data, such as selection of classes of data.

The emergence of classes of data adds another degree of freedom in selection process related to data. However rather than another dimension of selection it appears to be a form of restriction on how the domains in each features should be restricted. Selection of classes of data is used in Error Correcting Output Coding (ECOC) where the N-class problem is converted into a large number of 2-class problems. Selection with respect to classes is particularly attractive if there are expert classifiers which specialise in recognising particular class or classes but are very weak in recognising other classes, in which case it makes sense to decompose the problem rather than aggregate performance over all classes.

### 2.2 Classifiers

Classifier selection is probably the most intuitive form of selection with respect to classifier fusion. There are

generally two approaches to classifier selection leading to further combining by a fusion method. According to one approach, combiner is first picked arbitrarily and then classifiers are selected in such a way that the combiner results in maximum performance. Alternatively, it is the combiner that is adjusted so as to fuse given classifiers in a best possible way.

### 2.3 Combiners

The selection process does not have to end on classifiers. Given a set of optimised classifiers it is reasonable to test a number of available fusion methods and then to select the best performing one. On the other hand if classifier selection is applied to each combiner separately, the best system may turn out to be different. Such top-down decomposition could be drilled further down and it may show that normally inferior pair of classifier-combiner suddenly shows the best results if built on a different subset of features. Such doubts are imminent in any multiple classifier system unless the selection is carried out simultaneously on many dimensions of classification process.

## 3 MULTIDIMENSIONAL MODEL

As mentioned above the weaknesses of previous classifier selection models stem from only a single dimension along which the selection was carried out. The choices related to other dimensions were made arbitrarily based on some general optimality measures. The challenge undertaken in this work is to construct a multidimensional selection method in which data, classifiers and fusion methods are selected simultaneously and cooperatively to maximise classification performance of the system. The common doubt of processes operating along multiple degrees of freedom is the exploding computational complexity. To realise the significance of this problem let us consider a system with  $f$  features,  $c$  classifiers and  $b$  combiners. Let further assumption be that the combiners are selected as singletons only as we can not combine further fusion methods at this stage. In such case number of different systems to examine is:

$$N = (2^f - 1) \cdot (2^c - 1) \cdot b \sim b2^{f+c} \quad (1)$$

Such high complexity means that for a system with 10 features, 10 classifiers and 10 combiners one needs more than  $10^7$  evaluations to pick the best design.

### 3.1 Representation

In a response to such huge computational demands the presented system employs efficient adjustment of a genetic algorithm (Holland, 1975). To handle this

algorithm along many dimensions the chromosomes are designed as incidence cubes dimensions of which correspond to the selection dimensions of features, classifiers and combiners as shown in Figure 1. The meaning of "1" ("0") in each small cube is that the corresponding feature is (not) included in the corresponding classifier, and combiner of the system. The cube matches the hierarchical structure of the dimensions in which combiners are built on many classifiers which are built on many features. Note that such hierarchical structure means that the classifier can only be dropped if it does not have any features selected. Likewise combiner can be excluded only if it corresponds to a whole layer of zeros, corresponding to the lack of any classifiers selected. Another important aspect of the incidence cube is that it is not fully operational along the combiners dimension. The reason for that is the inability to further combine classifier fusion methods at this stage and hence the chromosome is a collection of selection solutions associated with layers of the incidence cube.

### 3.2 Selection algorithm

The actual process of multidimensional selection is consistent with the standard genetic algorithm (Holland, 1975), with some adjustments of mutation and cross-over operators, which have to accommodate the cube representation of the chromosome. The genetic algorithm was developed in 1970s by Holland (Holland, 1975) as an effective evolutionary optimisation method. Since that time, intensive research has been dedicated to GAs, bringing lots of applications in machine learning domain (Davis, 1991), (Cho, 1999), (Ruta and Gabrys, 2001), (Kuncheva and Jain, 2000) including classifier selection. Despite many varieties of GAs, its underlying principles remain unchanged. Chromosomes represent binary encoded solutions to the optimisation problem. A randomly initialised population of chromosomes is then evaluated according to the required fitness function and assigned a probability of survival proportional to their fitness. The best chromosomes are most likely to survive and are allowed to reproduce themselves by recombining their genotype and passing it on to the next generation. This is followed by a random mutation of some bits, which was designed to avoid premature convergence and enables the search to access different regions of a search space. The whole process is repeated until the population converges to a satisfactory solution or after a fixed number of generations. The GA is inspired by an explicit imitation of biological life, in which the strongest (fittest) units survive and reproduce further constantly adjusting to the variable conditions of living. In our case, cube representation of a chromosome imposes changes in mutation and cross-over operations. Mutation is quite straightforward as

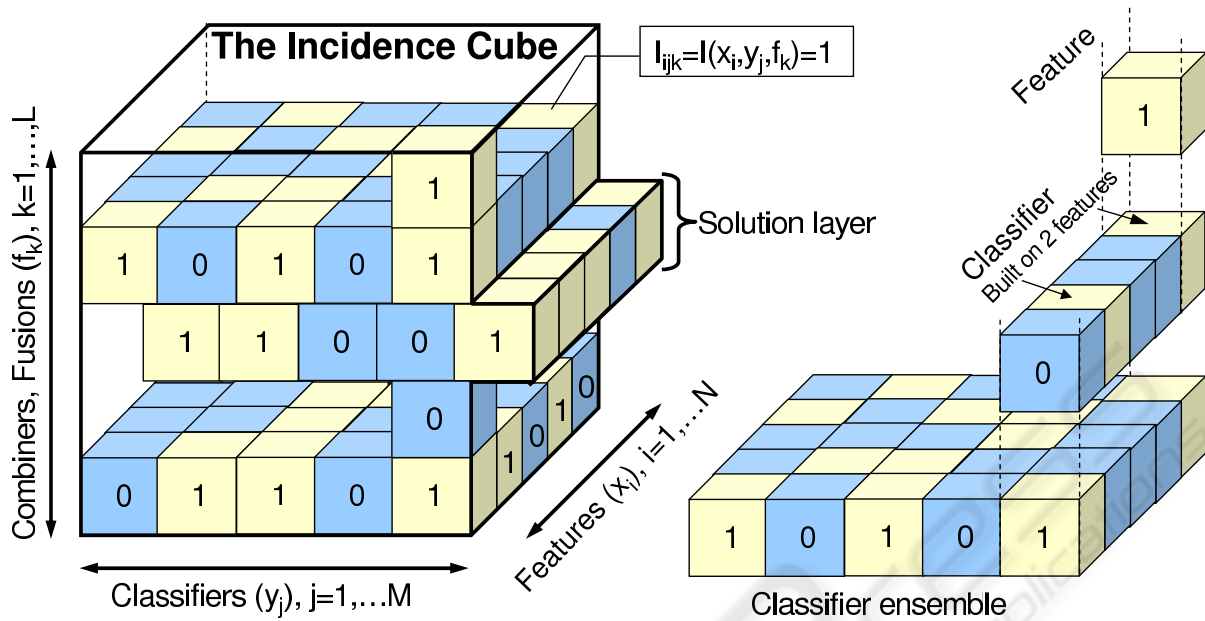


Figure 1: Incidence cube representation of the 3-dimensional fusion-classifier-feature selection model. Small cubes correspond to the triplet combiner-classifier-fusion and take values of 1 (light colour) if the corresponding feature is included in a classifier, which is then included in the corresponding combiner or 0 (dark colour) if the particular triplet is not selected

it only involves sampling from a mutation probability applied to all genes (small cubes). The crossing-over operation is more complicated as there is many degrees of freedom by which the chromosomes can be recombined. Moreover each chromosome actually describes in itself a number of solutions. The presented model uses a two-stage cross-over operation. First the chromosomes recombine internally by exchanging subsets of classifiers and features among randomly selected pairs of combiners as shown in Figure 2. Then the whole chromosomes recombine among each other by swapping parts split by the randomly oriented plane cutting through the incidence cube. Due to multiplicity of solutions within single chromosome the evaluation process is carried out on the basis of an average from the classification performances from each solution layer of the incidence cube. The algorithm uses also the elitism operation but realised through the natural selection process in which the best from both parents and offsprings are passed on to the next generation.

1. Collect and fix the selection space with  $f$  features  $c$  classifiers and  $b$  combiners.
2. Initialise a random population of  $n$  chromosomes ( $f \times c \times b$  binary incidence cubes).
3. Perform mutation and two-stage crossover, .
4. Pool offspring and parents together and calculate the fitness for all
5. Select  $n$  best chromosomes for the next generation.
6. If convergence then finish, else go to step 2.

Note that this particular implementation of GA represents a hill-climbing algorithm, as it guarantees that the average performance will not decrease in the subsequent generations. Mutation along with two-stage crossover ensure sufficient exploration ability of the algorithm. The convergence condition can be associated with the case when no change in the average fitness is observed for an arbitrarily large number of generations. Previous comparative experiments with real classification datasets confirmed the superiority of the presented version of the GA to its standard definition (Ruta and Gabrys, 2001).

## 4 EXPERIMENTS

A number of experiments have been carried out to test the performance of the presented 3-dimensional classifier selection model. Throughout the experiments a fixed sets of 10 different classifiers and 5 combiners were being applied to 2 known datasets from UCI repository<sup>1</sup>. Details of datasets, classifiers and combiners are shown in Table 1. To limit the computational complexity for each dataset the selection algorithm used a population of only 10 incidence cubes. The mutation rate was set to  $p = 0.1$

<sup>1</sup>University of California Repository of Machine Learning Databases and Domain Theories, available free at: <ftp.ics.uci.edu/pub/machine-learning-databases>

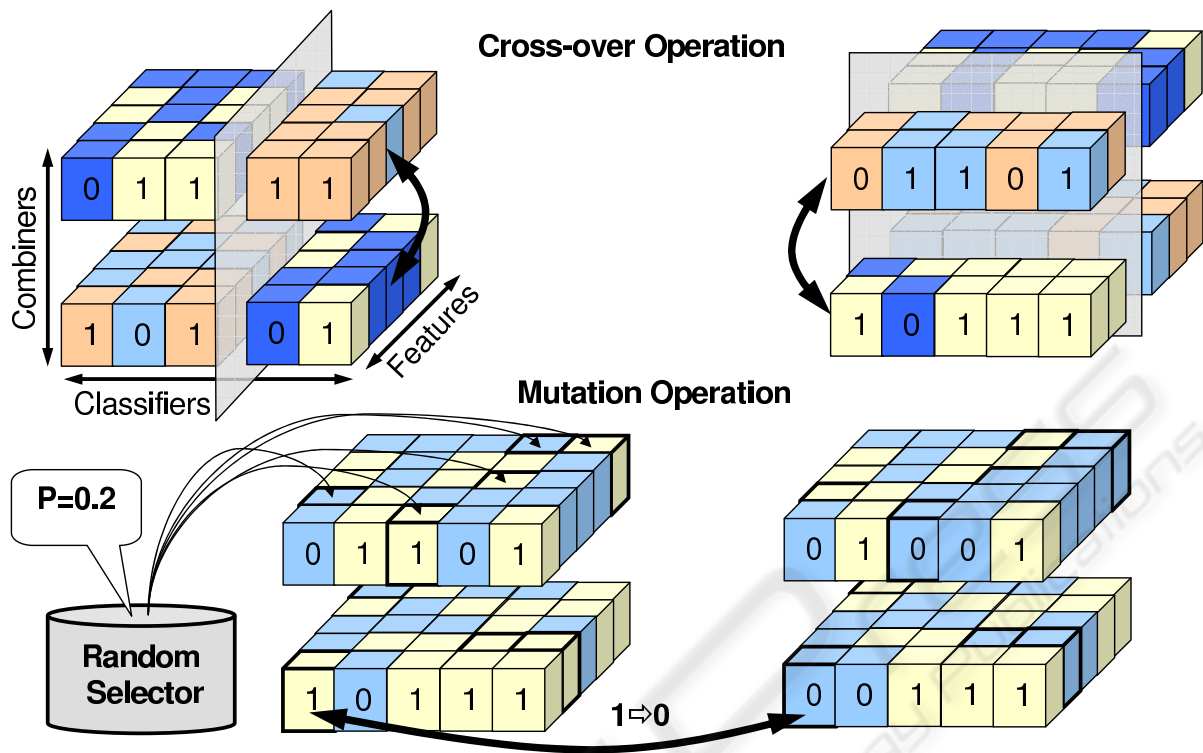


Figure 2: Visualisation of genetic algorithms operations of mutation and cross-over carried out on the incidence cube representation of 3-dimensional selection for classification.

while specific selection technique described in previous section ensured non decreasing convergence of the GA in the average classification performance. The chromosomes were built along 3 dimensions capturing features, classifiers and combiners incidence. They have been evaluated by the average misclassification rate obtained for all layers (combiners) separately. To preserve generalisation abilities of the system, the classifiers and hence the combiners were built on the separate training sets and tested on parts of the dataset which have not been used during training. Then the training and testing sets were swapped such that an equivalent of 2-fold cross-validation rule has been used for chromosome evaluation. For simplicity the GA was stopped after 100 generations for all datasets, despite the fact that for some cases convergence was achieved earlier. Figure 3 illustrates the dynamics of the testing performance characteristics during selection process carried out by the GA algorithm. The typical observation is that the algorithm relatively quickly finds the best performing system and then in subsequent generations it keeps improving other solutions in the population. The algorithm showed the capacity to get out of local minima which effectively means discovery of significantly better solution spreading swiftly in many variations during subsequent generations. The following Figure 4 de-

picts the evaluation of a final population of chromosomes for both datasets. For *Iris* dataset the Min combiner showed the best average performance including the absolute best performing system with only 1.33% misclassification rate. Majority voting showed on average best performance for *Liver* dataset, including the absolute best performing system with 27.8% error rate. The best systems for both datasets were then further uncovered by illustrating the structure of the classifiers and features selected as shown in Figure 5. Interestingly, for each selected classifier the algorithm selected at least two features. One classifier for both datasets was excluded. Other than that there is nothing significant about the selection structures shown in Figure 5. This could only prove that it is very difficult to find the best performing systems as they do not exhibit any visible distinctiveness but are simply lost among large number of system designs embodying huge selection complexity as shown in (1). Finally the last experiment intends to compare the performances of systems designed by means of 3-dimensional selection process (MCSS-3D) with the traditional systems of single best classifiers (SB) or multiple classifier system with GA-based classifier selection only (MCSS-1D). Table 2 shows the error rates of the best system found in the aforementioned design groups. The presented MCSS-3D clearly outperformed both

SB and MCSS-1D systems. The benefits of the selection carried out on many dimensions of the classification process are hereby confirmed.

Table 1: Description of datasets, classifiers and combiners used in experiments.

No	Classifier	Description
1	klclc	Linear with KL expansion
2	loglc	Logistic linear classifier
3	ldc	Linear discriminant classifier
4	qdc	Quadratic with normal density
5	pfsvc	Pseudo-Fisher SVM classifier
6	lmnc	Levenberg-Marquardt neural net
No	Combiner	Description
1	meanc	Mean combiner
2	minc	Minimum rule combiner
3	maxc	Maximum rule combiner
4	prodc	Product rule combiner
5	majorc	Majority voting combiner
Name	Classes	Samples × Features
Iris	3	150 × 4
Liver	2	345 × 6

## 5 CONCLUSION

This work considers the broad issue of selection applied to the multiple classifier system in order to improve its performance. It has been noted that various selection methods are being used along only single dimension - like classifier selection applied for the ensemble of classifiers or feature selection applied to the set of features. In the novel multidimensional selection system proposed in this paper the classification can be potentially handled simultaneously and cooperatively along all possible degrees of freedom including data, features, classes, classifiers and classifier fusion methods. Due to extremely high computational complexity of such systems only a 3-dimensional selection system was implemented and tested on benchmark datasets. This system applies adjusted genetic algorithm to select the optimal configuration of features, classifiers and combiners. The experimental results confirmed anticipated superiority of the MSS model compared to single-best classifier method and even to multiple classifier system

Table 2: Comparison of the error rates obtained for the best systems using: SB, MCS-1D, MCS-3D.

Dataset	SB (classifier)	MCSS-1D	MCSS-3D
iris	2.47 (klclc)	2.13	1.33
liver	32.35 (loglc)	29.06	27.78

with the GA-based selection of the optimal subset of classifiers. The MSS system opens yet another source for further performance improvement in classification once discovered for classifier fusion systems. It also points at the necessity of simultaneous and cooperative optimisation of all components of the classification process, which is gradually being made available by the rapidly increasing computational power.

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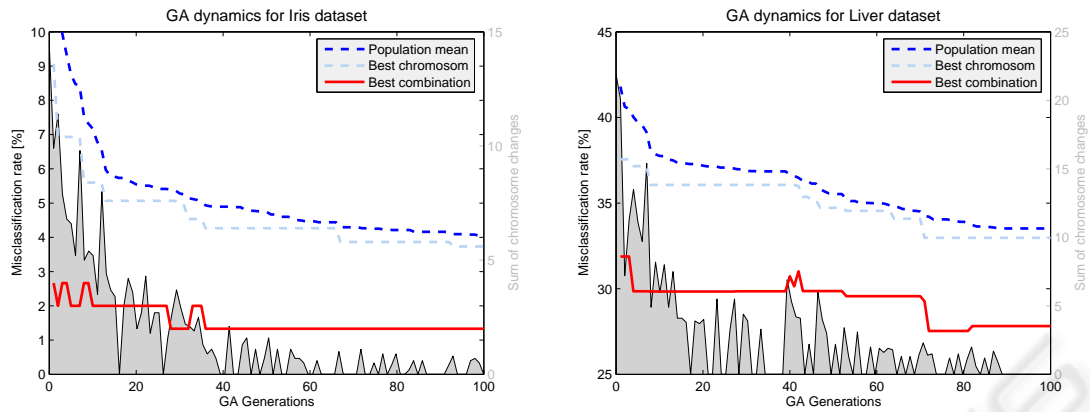


Figure 3: Performance characteristics evolution during GA selection.

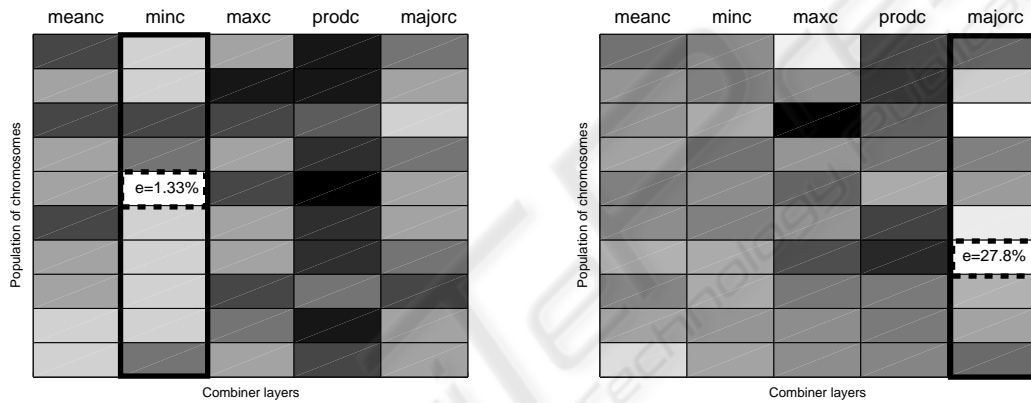


Figure 4: Diagrams showing the misclassification rates of the final population of chromosomes returned by GA. The lighter the field the lower error rate of the corresponding classifier fusion system. Thick frame indicates the best combiner for particular dataset while dashed line shows the performance of the overall best system found.

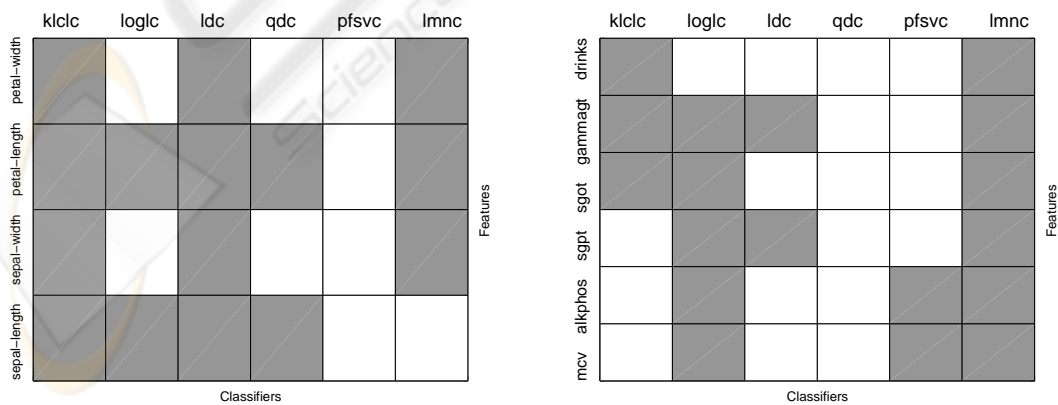


Figure 5: Diagrams showing the subsets of features and classifiers selected by the GA for the best performing combinations fused by *Min* combiner for Iris dataset and *Majority Voting* for the Liver dataset. Dark field indicate inclusion of the corresponding pairs feature-classifier in the final system.