

FREE SOFTWARE FOR DECISION ANALYSIS

A Software Package for Data Envelopment Models

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Abstract: Data Envelopment Analysis is based on linear programming problems (LPP) in order to find the efficiency of Decision Making Units (DMUs). This process can be computationally intense, as a LPP has to be run for each unit. Besides, a typical DEA LPP has a large number of redundant constraints concerning the inefficient DMUs. That results in degenerate LPPs and in some cases, multiple efficient solutions. The developed work intends to fill out a gap in current DEA software packages i.e. the lack of a piece of software capable of producing full results in classic DEA models as well as the capability of using more advanced DEA models. The software package interface as well as the models and solution algorithms were implemented in Delphi. Both basic and advanced DEA models are allowed in the package. Besides the main module that includes the DEA models, there is an additional module containing some models for decision support such as the multicriteria model called Analytic Hierarchic Process (AHP). The developed piece of software was coined as *FSDA – Free Software for Decision Analysis*.

1 INTRODUCTION

Data Envelopment Analysis (DEA) is an approach to evaluate efficiency that uses Linear Programming Problems (LPP) whose results assess the performance of Decision Making Units (DMUs).

In recent years there has been increasing interest for DEA and its corresponding LPPs. Several real

case applications have led to a need for new developments in the classic models, the CCR (Charnes et. al. 1978) and BCC (Banker et al. 1984) models, in order to include new situations. Thus several researchers have been aware to the results yielded by the models in terms of efficiency indexes, benchmarks and targets.

On the other hand, a LPP has to be solved for each DMU. So the task for efficiency evaluation can

be hard and very time consuming without an adequate specific piece of software, particularly for a large number of DMUs. Several software packages were developed so to minimize those problems. However, in several situations, the results produced by those packages either are not complete or are not consistent to the ones produced by other pieces of software for the same model. That leads to some questions about the correctness of the computing implementation of the model in those softwares.

That motivates the need for developing a reliable software package yielding complete results and covering new theoretical developments regarding DEA models. Besides the capability to cover the classic DEA models and producing complete results, it also includes advanced models that were not considered by other DEA software packages (Angulo-Meza et al., 2003a). The current version of the software package is presented in Portuguese language and is called SLAD (Software Livre de Apoio à Decisão) although a future release in English is also under development.

The SLAD was written in Delphi 7.0 due mainly to allow object Pascal coding for writing the Simplex algorithm to solve LPPs and also to its capability to deal with graphics.

The developed software package was shown to be very useful for testing new models and has been used in many papers written by the authors.

2 DATA ENVELOPMENT ANALISYS

Data Envelopment Analysis (DEA) was developed by Charnes et al. (1978) and is a methodology that uses linear programming for the comparative evaluation of DMUs efficiencies

The DEA purpose is to compare a certain number of DMUs performing similar tasks and that distinguishes themselves in the number of used inputs and the number of produced outputs. There are basically two classic DEA models: the Constant Return Scale (CRS) model also known as CCR (Charnes, Cooper and Rhodes, 1978) and the Variable Return Scale (VRS) model or BCC (Banker, Charnes and Cooper, 1984). The first model considers constant scale returns and the second one variable scale returns and does not assume proportionality among inputs and outputs.

Each k^{th} DMU, $k = 1, \dots, n$, is considered to be a production unity that uses n inputs x_{ik} , $i = 1, \dots, r$, to produce m outputs y_{jk} , $j = 1, \dots, s$. The CCR model described by (1) maximizes the ratio between a linear combination of outputs and a linear combination of inputs with the constraint that for

each DMU that ratio can not be greater than one. So for a particular DMU o , h_o is its efficiency; x_{io} and y_{jo} are its inputs and outputs and v_i and u_j are the calculated weights for the inputs and outputs.

$$\begin{aligned} \max h_o &= \frac{\sum_{j=1}^s u_j y_{jo}}{\sum_{i=1}^r v_i x_{io}} \\ \text{subject to} & \\ & \frac{\sum_{j=1}^s u_j y_{jk}}{\sum_{i=1}^r v_i x_{ik}} \leq 1, \quad k = 1, \dots, n \\ & u_j, v_i \geq 0 \quad \forall i, j \end{aligned} \tag{1}$$

After some mathematical manipulations, the model can be rewritten, yielding in a Linear Programming Problem (LPP) as shown in (2).

$$\begin{aligned} \max h_o &= \sum_{j=1}^s u_j y_{jo} \\ \text{subject to} & \\ & \sum_{i=1}^r v_i x_{io} = 1 \\ & \sum_{j=1}^s u_j y_{jk} - \sum_{i=1}^r v_i x_{ik} \leq 0, \quad k = 1, \dots, n \\ & u_j, v_i \geq 0 \quad \forall i, j \end{aligned} \tag{2}$$

As a LPP is solved for each DMU, for n DMUs n LPPs are solved, with $r + s$ decision variables. The model just presented is the basis for all other DEA models.

Besides the efficiency index, DEA models yields for each DMU the variables weights, benchmarks, and targets for the inefficient DMUs. The last two elements are determined from the values of the dual variables i.e. by solving the dual LPPs or by the use of the complement slack theorem. The set of all those results defines what we call in this paper complete results. In recent years, software packages were developed due to the great interest and the large number of applications using the DEA approach. Those programs include mostly basic models and were developed to avoid the effort of running separately LPPs without the integration provided by DEA packages software in order to get the final evaluation. On the other hand, theoretical developments have been made and are widely used.

For example, weights restrictions (Allen et al, 1997) allow the inclusion of a priori information, i.e. subjective aspects can be incorporated within DEA models that are essentially very objective. Thus the use of some advanced developments have been essential for DEA analysis and some software packages already include some advanced models.

However, in some cases different softwares running the same model yielded different results or results not following the model constraints which make them unreliable. That also happened even for the basic models. Most software packages show only efficiency indexes and targets leaving out the variable weights which might be useful in a deep assessment of the DMUs and for aid in later theoretical developments (Angulo-Meza e Lins, 2000).

Such problems motivated the authors to build a package software in order to produce complete results involving the basic models CCR and BCC (input or output oriented, Cooper et al, 2000), with open code access to check eventual problems or discrepancies among results and the capability to include models for new theoretical advanced developments.

3 THE FSDA – FREE SOFTWARE FOR DECISION ANALYSIS

A fundamental step for the development of a DEA software is the setup and choice of the algorithm to solve the LPPs associated with the problem.

The Simplex algorithm is very popular for solving LPPs, so is the interior points algorithm particularly for large scale LPPs. The FSDA uses the Simplex algorithm for the solution of linear programming problems.

The Simplex algorithm was created by Dantzig and published in 1948. Briefly, according to Dantzig (1963) a linear programming problem can be reduced to a combinatory problem in such a way that the method for searching the optimum solution is performed by testing a sequence of combinations in which the value of the objective function is incremented gradually. An optimum solution is considered to be reached after a certain number of iterations not greater than the number of constraints or the number of variables whatever is the largest.

The FSDA uses Kuenzi et al 's (Kuenzi et al, 1971) approach that includes a subroutine to avoid degenerating problems, a common problem in DEA models due to the typical structure of the involved LPPs that present a large number of redundant constraints for the inefficient DMUs. In many cases that leads to multiple optimal solutions.

Matrices and vectors are used to separate the values of the coefficients corresponding to the basic variables (variables that are part of the solution) and to the non basic variables (variables which are not part of the solution and whose values are set to zero). Moreover, two lists are kept containing those two variables which are updated at each algorithm iteration.

The mentioned procedure has two phases for evaluating the solution of a LPP. The first one searches an initial basic solution. The second starts from the initial basic solution found in part one.

Both phases have three steps. The first step determines the value of a secondary objective function that will be used in the other two steps and also a variable that will enter the base.

The second step locates the pivot element taking into consideration a possible degeneration in the corresponding LPP. If the pivot element can not be found a message is sent and the search ends.

The last step updates the matrices in order to change the set of basic and non basic variables.

There are three possible outcomes for a given LPP: an optimal solution is achieved, no solution is achieved, in those cases the LPP is said to be non limited or no initial vector is possible which means the LPP is not feasible.

The algorithm used in the FSDA software is now presented in detail:

Phase 1: Determine an initial feasible basic solution and check if that solution is the origin, otherwise search one.

Step 1: Search a variable to enter the base of the initial feasible solution and evaluate a secondary objective function.

If the current solution is feasible go to phase 2, otherwise if there is a variable to be sent into the base go to step 2, otherwise the LPP is non feasible and the program ends and a message is sent to the user.

Step 2: Locate pivot element if input go to step 3, otherwise END, LPP not feasible.

Phase 2: There is an initial basic solution go to step 1.

Step 1: Search a variable to be in the base and evaluate the secondary objective function.

If the current solution is optimum then END, otherwise if there is a variable to be in the base go to step 2, otherwise LPP is non limited.

- Step 2: Locate pivot element.
- Step 3: Update the sets of basic and non basic variables and go to step 1.

It should be stressed that in basic DEA models (CCR or BCC, whatever orientation) at least one solution can always be found with a possibility of an infinite number of optimum solutions which is quite frequent. In that case, only the first optimum solution is shown to the user.

In the case of models with restrictions to the weights, there is a possibility of appearing non feasible LPPs once the additional constraints can turn the LPP originally presenting an optimum solution to a non feasible LPP. In that cases, a message is sent and the user has the choice of changing the restriction and repeat the process.

The method for solving the LPPs is unique, what changes is the format of the LPPs, once a different LPP is performed for each DEA model. The data input have to be put in the proper format in a matrix structure depending on the used model. The data ordering process in the referred matrix in the appropriate format is the most difficult part in the software implementation.

3.1 Software Description

The FSDA(SLAD in Portuguese version) opening window is shown in Figure 1 and was developed for Windows platform with Delphi 7.0. It is capable for dealing with 150 DMUs and 20 inputs or outputs variables.

Although for other research areas 150 DMUs might be insufficient for DEA applications that value is able to deal with large scale situations since in the literature there are few applications dealing with more than a 100 DMUs. In most applications

10 variables are sufficient.

The software package allows the data input to be entered in two ways: directly inside the program using a table with the choice of the number of DMUs and variables, and by the use of a txt file having the data that are loaded to the mentioned table shown in figure 2. Figure 2 also shows the possible choices of basic models (CCR or BCC) and orientation (input or output) covering the basic and most used models.

Moreover, advanced options are also included in the software as one of the objectives of the FSDA package is to allow new DEA developments that may be chosen along with the model and its orientation. The user can also the choice of using weight restrictions, or considering data uncertainty.

The advanced option of weight restrictions leads the user to an additional window indicating the number of restrictions to be included in the chosen model.

The choice of uncertainty in some or all variables also leads the user to another window, showing the data input. A DEA analysis with uncertainties uses variation intervals for some or all variables of analysis.

Results for any model, advanced or not, are presented in an additional window as illustrated in figure 3. That window shows the efficiency indexes for all DMUs. Besides, additional options are presented to show other results: inverted frontier, that expand the results window to include the efficiency index in the inverted frontier and the composed index (standard and inverted efficiencies); the variables weights; the benchmarks of all DMUs (efficient and inefficient) and the targets for the DMUs including the slacks and the levels that the variables have to reach for turning the DMUs efficient

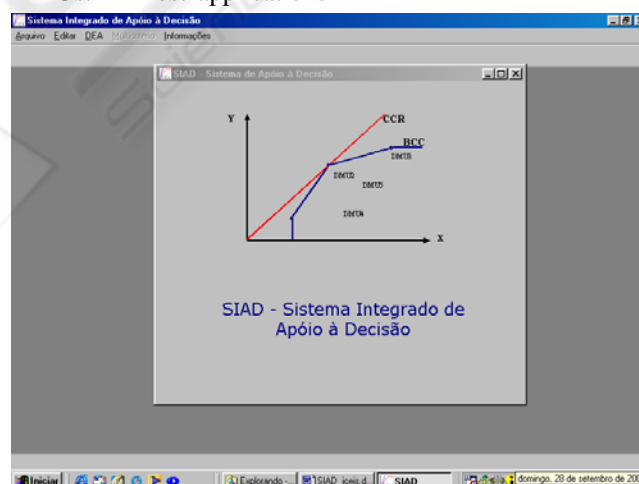


Figure 1: FSDA (SLAD) Opening Window

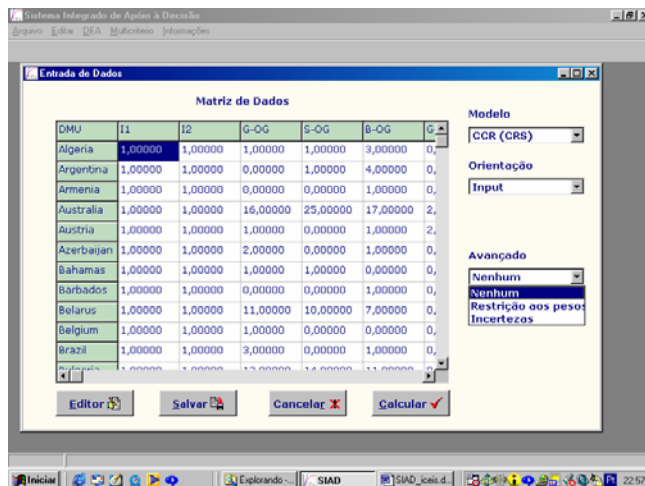


Figure 2: Options window of FSDA (SLAD).

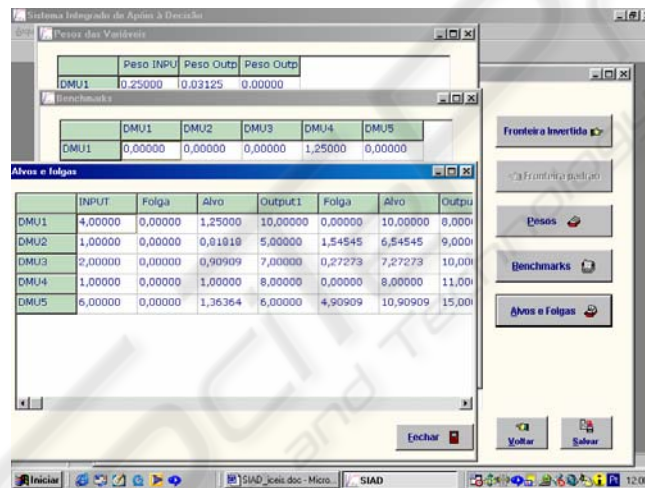


Figure 3: Results window of FSDA (SLAD).

It should be noted that the results window just shows the efficiencies of the DMUs whereas the other results are shown only when asked for by clicking the appropriate icons on the same window. That was done for the sake of easing the visualization of the results once the presentation of the complete results in one window might be difficult to interpret the results particularly for a large number of variables and DMUs.

In the data input window there is another option, not yet mentioned, that allows the use of multicriteria models. By choosing that option the window changes colour to indicate the user the substitution of the DEA approach for the multicriteria approach. As this module is under development, the only available method is the Analytic Hierarquic Process (AHP) according to

Clímaco et al. (1996), but other methods will be include in due time.

4 CONCLUSIONS

The growing interest in recent years of DEA approach for the evaluation of efficiency and the theoretical developments due to real world applications have indicated the need of a reliable software package that involve jointly a friendly user interface, yielding reliable, consistent and complete results and having the capability to work with more advanced models. There are DEA software packages but the lack of complete and reliable results in those softwares led to the development of a

software package that met the demands of the authors of this paper.

The FSDA (SLAD) was written in Delphi 7.0 and uses the Simplex algorithm to solve the LPPs required by DEA. The software includes, besides the classic models, advanced models such as DEA models with weight restrictions as well as recent models developed by the authors such as inverted frontiers and data uncertainties.

The complete results produced by the software are suitable for a deep analysis of the DMUs and could also serve as a basis for the development of new theoretical models.

The performance of the FSDA in large scale problems has been satisfactory, e. g. the time response for problems involving 80 DMUs and 7 variables was less than one second.

A great motivation of the FSDA for the authors is the possibility of including other models in the package that has been very useful for the authors in their new developments and in the testing of new models.

Finally, new models developed by the authors are under way to be considered in the FSDA such as the DEA-GSZ model (Lins et al., 2003), as well as the expansion of the multicriteria module.

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