A MODEL PREDICTIVE CONTROLLER BASED ON SUPPORT VECTOR REGRESSION AND GENETIC OPTIMIZATION FOR AN SP-100 SPACE NUCLEAR REACTOR

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Abstract: In this work, a model predictive control (MPC) method combined with support vector regression (SVR), is applied to the design of the thermoelectric (TE) power control in the SP-100 space reactor. The future TE power is predicted by using SVR. The objectives of the proposed model predictive controller are to minimize both the difference between the predicted TE power and the desired power, and the variation of control drum angle that adjusts the control reactivity. Also, the objectives are subject to maximum and minimum control drum angle and maximum drum angle variation speed. The genetic algorithm (GA) is used to optimize the model predictive controller. A lumped parameter simulation model of the SP-100 nuclear space reactor is used to verify the proposed controller. The results of numerical simulations to check the performance of the proposed controller show that the TE generator power level controlled by the proposed controller could track the target power level effectively, satisfying all control constraints.

1 INTRODUCTION

The SP-100 was designed to provide a realistic and reliable source of long-term power for space exploration and exploitation activities. The SP-100 system is a fast spectrum lithium-cooled reactor system with an electric power rating of 100 kW (Demuth, 2003) and its energy conversion system is based on a direct TE conversion mechanism. The control system is a key element of space reactor design to meet the mission requirements of economics, reliability, safety, survivability, and life expectancy. For a space mission with uncertain environment, rare events, and communication delays, all the control functions must be achieved through a sophisticated control system with a limited degree of human intervention from the earth.

In order to optimize the reactor power control performance, techniques for the optimal power control of nuclear reactors have been studied extensively in the past two decades (Cho and Grossman, 1983; Shtessel, 1998). But it is very difficult to design optimized controllers for nuclear systems because of variations in nuclear system parameters and modeling uncertainties, and in particular, for the long-term operation of the SP-100 reactor.

This work employs the MPC method, which has received much attention as a powerful tool for the control of industrial process systems (Kwon and Pearson, 1977; Garcia et al., 1989). The basic concept of the model predictive control is to solve an optimization problem for a finite future at the current time. Once a future input trajectory is chosen, only the first element of that trajectory is applied as the input to the plant, and the calculation is repeated at each subsequent instant. This method has many advantages over the conventional infinite horizon control because it is possible to handle input and output constraints in a systematic manner during the design and implementation of the control. In particular, it is a suitable control strategy for nonlinear time varying systems. The MPC method

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has been applied to a nuclear engineering problem (Na et al., 2003).

Also, this work incorporates the support vector machines (SVMs) that have been successfully employed to solve nonlinear regression problems (Pai and Hong, 2005; Yan et al., 2004). The SVR is used to predict the future output that is required in the optimization objective of the model predictive control. That is, at the present time the behavior of the process over a prediction horizon is considered and the process output to changes in the manipulated variable is predicted by SVMs. In this application, based on this identified reactor model that consists of the control drum angle and the TE generator power, the future TE generator power is predicted. The objective function for MPC is minimized by a GA that is widely used for optimization problems. A lumped parameter simulation model of the SP-100 space reactor is used to verify the proposed controller for a space nuclear reactor.

2 MPC CONTROLLER USING SVR

Figure 1 shows the basic concept of the model predictive control (Garcia, 1989). At first a set of present and future control moves are assumed, and the future behavior of the process outputs can be predicted over a prediction horizon L with the assumed present and future control moves. Then the optimized M present and future control moves ($M \le L$) are optimized to minimize a quadratic objective function. Although M optimized control moves are calculated, only the first control move is implemented. At the next time step, new values of the measured output are obtained, the control horizon is shifted forward by one step, and the same calculations are repeated by using updated measurements.

The purpose of taking new measurements at each time step is to compensate for unmeasured disturbances and model inaccuracies, both of which cause the measured system output to be different from the predicted one. At every time instant, model predictive control requires the on-line solution of an optimization problem to compute optimal control inputs over a fixed number of future time instants, known as the time horizon.

Also, in order to achieve fast responses and prevent excessive control effort, the associated performance index for deriving an optimal control input is represented by the following quadratic function:

$$J = \frac{1}{2}\sum_{k=1}^{L}\left[\hat{y}(t+k\,|\,t) - z(t+k)\right]^2 + \frac{1}{2}\sum_{k=1}^{M}\rho\left[\Delta u(t+k-1)\right]^2 \quad (1)$$

$$\text{subject to constraints} \begin{cases} \Delta u(t+k-1) = 0 \ \text{ for } \ k > M \\ u_{\min} \le u(t) \le u_{\max} \\ -du_{\max} \le \Delta u(t) \le du_{\max} \end{cases}$$

where the parameter ρ determines trade-off between the TE power (system output) error and control drum angle (control input) move between neighbouring time steps, and z is a setpoint (desired TE power). The estimate $\hat{y}(t+k\,|\,t)$ is an optimum k-step-ahead prediction of the system output based on data up to time t. Δu, $\Delta u(t) = u(t) - u(t-1)$, is an input move between neighbouring time steps. The parameters L and M are called the prediction horizon and the control horizon, respectively. The prediction horizon represents the limit of the instant in which it is desired for the output to follow the reference sequence. The constraint, $\Delta u(t+k-1) = 0$ for $k > M$, means that there is no variation in the control signals after a certain interval M.

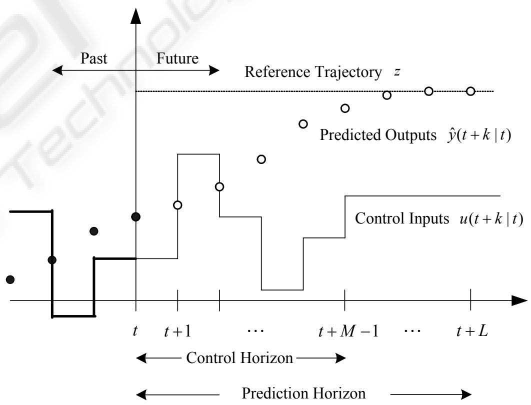


Figure 1: Basic concept of a MPC method.

In order to obtain control inputs, the predicted outputs are first calculated by function approximation using SVMs, in which inputs consist of past values of control system inputs and outputs and of future control system input signals. Along with the introduction of Vapnik's ε-insensitive loss function (Vapnik, 1995), SVMs also have been extended and widely used to solve nonlinear regression estimation problems. In SVM regression the concept is to map the input data into a high dimensional feature space and subsequently carry out the linear regression in the feature space.

Therefore, the SVM regression is used to predict the future output based on past inputs and outputs.

2.1 Output Prediction

The basic concept of the SVM regression is to map nonlinearly the original data $\mathbf{x}$ into a higher dimensional feature space. Hence, given a set of data $\{(\mathbf{x}_i, y_i)\}_{i=1}^{N}$ where $\mathbf{x}_i$ is the input vector, y_i is the actual output value and N is the total number of data patterns, the SVM regression function is

$$y = f(\mathbf{x}) = \sum_{i=1}^{N} w_i \phi_i(\mathbf{x}) = \mathbf{w}^T \boldsymbol{\varphi}(\mathbf{x}) + b, \tag{2}$$

where $\phi_i(\mathbf{x})$ is called the feature that is nonlinearly mapped from the input space $\mathbf{x}$, $\mathbf{w} = [w_1 \quad w_2 \quad \cdots \quad w_N]^T$, and $\boldsymbol{\varphi} = [\phi_1 \quad \phi_2 \quad \cdots \quad \phi_N]^T$. The parameters $\mathbf{w}$ and b are a support vector weight and a bias that are calculated by minimizing the following regularized risk function:

$$R(\mathbf{w}) = \frac{1}{2}\mathbf{w}^T\mathbf{w} + \lambda \sum_{i=1}^{N} \left| y_i - f(\mathbf{x}) \right|_{\varepsilon}, \tag{3}$$

where

$$\left| y_i - f(\mathbf{x}) \right|_{\varepsilon} = \begin{cases} 0 & \left| y_i - f(\mathbf{x}) \right| < \varepsilon \\ \left| y_i - f(\mathbf{x}) \right| - \varepsilon & \text{otherwise} \end{cases} \tag{4}$$

Here, λ and ε are user-specified parameters and $\left| y_i - f(\mathbf{x}) \right|_{\varepsilon}$ is called the ε-insensitive loss function (Vapnik, 1995). The loss equals zero if the estimated value is within an error level ε. The regularized risk function can be rewritten by the following constrained form:

$$R(\mathbf{w}, \boldsymbol{\xi}, \boldsymbol{\xi}^*) = \frac{1}{2}\mathbf{w}^T\mathbf{w} + \lambda \sum_{i=1}^{N} \left(\xi_i + \xi_i^* \right), \tag{5}$$

subject to the constraints

$$\begin{cases} y_i - \mathbf{w}^T\boldsymbol{\varphi}(\mathbf{x}) - b \le \varepsilon + \xi_i, \quad i = 1, 2, \cdots, N \\ \mathbf{w}^T\boldsymbol{\varphi}(\mathbf{x}) + b - y_i \le \varepsilon + \xi_i^*, \quad i = 1, 2, \cdots, N \\ \xi_i,\ \xi_i^* \ge 0, \quad i = 1, 2, \cdots, N \end{cases}$$

where the constant λ determines the trade-off between the flatness of $f(\mathbf{x})$ and the amount up to which deviations larger than ε are tolerated and $\boldsymbol{\xi} = [\xi_1 \quad \xi_2 \quad \cdots \quad \xi_N]^T$, $\boldsymbol{\xi}^* = [\xi_1 \quad \xi_2 \quad \cdots \quad \xi_N]^T$ are slack variables representing upper and lower constraints on the outputs of the system.

The solution to the constrained optimization problem is given by the saddle point of the Lagrange functional:

$$\begin{aligned} &\Phi\left(\mathbf{w}, b, \xi_i, \xi_i^*, \alpha_i, \alpha_i^*, \xi_i, \xi_i^*\right) = \\ &\frac{1}{2}\mathbf{w}^T\mathbf{w} + \lambda\sum_{i=1}^{N}\left(\xi_i + \xi_i^*\right) - \sum_{i=1}^{N}\alpha_i\left[\mathbf{w}^T\boldsymbol{\varphi}(\mathbf{x}_i) + b - y_i + \varepsilon + \xi_i\right] \\ &- \sum_{i=1}^{N}\alpha_i^*\left[y_i - \mathbf{w}^T\boldsymbol{\varphi}(\mathbf{x}_i) - b + \varepsilon + \xi_i^*\right] - \sum_{i=1}^{N}\left(\beta_i\xi_i + \beta_i^*\xi_i^*\right) \end{aligned} \tag{6}$$

The above equation is minimized with respect to the primal variables $\mathbf{w}$, b, ξ_i, ξ_i^*, and then maximized with respect to the nonnegative Lagrangian multipliers α_i, α_i^*, β_i, β_i^*. The minimum with respect to $\mathbf{w}$, b, ξ_i, ξ_i^* provides the following conditions:

$$\mathbf{w} = \sum_{i=1}^{N}\left(\alpha_i - \alpha_i^*\right)\boldsymbol{\varphi}(\mathbf{x}_i), \tag{7}$$

$$\sum_{i=1}^{N}\left(\alpha_i - \alpha_i^*\right) = 0,$$

$$\lambda - \alpha_i - \beta_i = 0, \quad i = 1, 2, \cdots, N,$$

$$\lambda - \alpha_i^* - \beta_i^* = 0, \quad i = 1, 2, \cdots, N.$$

The Lagrange functional can be rewritten by using the above minimum conditions as follows:

$$\begin{aligned} \Psi\left(\alpha_i, \alpha_i^*\right) = &\sum_{i=1}^{N} y_i\left(\alpha_i - \alpha_i^*\right) - \varepsilon\sum_{i=1}^{N}\left(\alpha_i + \alpha_i^*\right) \\ &- \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\alpha_i - \alpha_i^*\right)\left(\alpha_j - \alpha_j^*\right)\boldsymbol{\varphi}^T(\mathbf{x}_i)\boldsymbol{\varphi}(\mathbf{x}_j) \end{aligned} \tag{8}$$

subject to the constraints

$$\sum_{i=1}^{N}\left(\alpha_i - \alpha_i^*\right) = 0, \ 0 \le \alpha_i \le \lambda, \ 0 \le \alpha_i^* \le \lambda, \quad i = 1, \cdots, N \tag{9}$$

By solving the above equation with standard quadratic programming technique, the values of α_i, α_i^* are found out. By substituting Eq. (7) into Eq. (2), the regression function becomes

$$\begin{aligned} y = f(\mathbf{x}) &= \sum_{i=1}^{N}\left(\alpha_i - \alpha_i^*\right)\boldsymbol{\varphi}^T(\mathbf{x}_i)\boldsymbol{\varphi}(\mathbf{x}) + b \\ &= \sum_{i=1}^{N}\left(\alpha_i - \alpha_i^*\right)K(\mathbf{x}, \mathbf{x}_i) + b \end{aligned} \tag{10}$$

where $K(\mathbf{x}, \mathbf{x}_i) = \boldsymbol{\varphi}^T(\mathbf{x}_i)\boldsymbol{\varphi}(\mathbf{x})$ is called the kernel function. A number of coefficients $\alpha_i - \alpha_i^*$ are nonzero values and the corresponding training data points have approximation error equal to or larger than ε. They are called support vectors.

2.2 Objective Function Optimization by a GA

The objective function of Eq. (1) can be solved by linear matrix inequality (LMI) techniques. In this work, a GA is used to minimize the objective function with multiple objectives. The GA has been known to be effective in solving multiple objective functions and is less susceptible to getting stuck at local minima compared to conventional search methods (Goldberg, 1989).

We propose an SVM-based MPC methodology which is based on a dynamic nonlinear SVM model of the SP-100 space reactor. The optimization problem which needs to be solved online is no longer a linear problem but a complicated nonlinear problem which requires a tremendous computational effort. This calculation cannot be completed on time even by the fast computing systems [22]. Due to the peculiarity of the SVM model, conventional optimization techniques cannot be easily applied. Therefore, in this work, the online optimization problem is solved using a GA.

In the GA, the term *chromosome* is referred to as a candidate solution that minimizes a cost function. The GAs require a fitness function and the fitness function evaluates the extent to which each candidate solution is suitable for specified objectives. The GA starts with an initial population of chromosomes, which represent possible solutions of the optimization problem. The fitness function is computed for each chromosome. New generations are produced by the genetic operators, such as selection, crossover, and mutation. The algorithm stops after the maximum allowed time has elapsed.

A chromosome which is a candidate solution of the optimization problem is represented by s_g, whose elements consist of present and future control inputs and has the following structure (Sarimveis and Bafas, 2003):

$$s_g = \left[u_g(t) \quad u_g(t+1) \quad \cdots \quad u_g(t+M-1) \right], \tag{11}$$

where t indicates the current time. Assuming we have chosen the number of chromosomes G, which will constitute the initial population, the crossover probability p_c and the mutation probability p_m, the algorithm proceeds according to the following steps:

Step 1 (initial population generation): Set the number of iterations $iter = 1$. Generate an initial population consisting of a total of G chromosomes. The values are allocated randomly, but they should satisfy both input and input move constraints of Eq. (1).

Step 2 (fitness function evaluation): Evaluate the objective function of Eq. (1) for all the chosen chromosomes. Then invert the objective function values and find the total fitness of the population as follows:

$$F = \sum_{g=1}^{G} \frac{1}{J_g(t)}, \tag{12}$$

where $J_g(t)$ is the objective function value for the g-th chromosome and the inversion of $J_g(t)$ is a fitness value of the g-th chromosome. Then, calculate the normalized fitness value of each chromosome, meaning that the selection of probability p_g calculated by

$$p_g = \frac{\left(1/J_g(t)\right)}{F}, \quad g = 1, \cdots, G. \tag{13}$$

Step 3 (selection operation): Calculate the cumulative probability q_g for each chromosome using the following equation:

$$q_g = \sum_{j=1}^{g} p_j, \quad g = 1, \cdots, G. \tag{14}$$

For $g = 1, \cdots, G$, generate a random number r between 0 and 1. Select the chromosome for which $q_{g-1} \le r \le q_g$. At this point of the algorithm a new population of chromosomes has been generated. The chromosomes with high fitness value have more chance to be selected.

Step 4 (crossover operation): For each chromosome s_g, generate a random number r between 0 and 1. If r is lower than p_c, this particular chromosome will undergo the process of crossover, otherwise it will remain unchanged. Mate the selected chromosomes. The crossing point is the position indicated by a random integer number z generated between 0 and $M-1$. Two new chromosomes are produced by interchanging all the members of the parents following the crossing point. The crossover operation might produce infeasible offsprings and this situation is avoided by a simple correction mechanism for an input variable, which modifies the values of the input parameters after the cross position so that the input move constraints are satisfied.

Step 5 (mutation operation): For every member of each chromosome s_g, generate a random number r between 0 and 1. If r is lower than p_m, this particular member of the chromosome will undergo the process of mutation, otherwise it will remain unchanged. Each chromosome should satisfy both input and input move constraints of Eq. (1) after mutation.

Step 6 (repeat or stop): If the maximum allowed time has not expired, set $iter = iter + 1$ and return the algorithm to Step 2. Otherwise, stop the algorithm and select the chromosome that produced the lowest value of the objective function throughout the entire procedure.

3 APPLICATION TO THE SP-100 SPACE NUCLEAR REACTOR

The SP-100 system is a fast spectrum lithium-cooled reactor system that can generate electric power of 100 kW for space exploration and exploitation activities. The reactor system is made up of a reactor core, a primary heat transport loop, a thermoelectric generator, and a secondary heat transport loop to reject waste heat into space through radiators. The reactor core is composed of small disks of highly enriched (93%) uranium nitride fuel contained in sealed tubes. The heat generated in the reactor core is transported by liquid lithium and is circulated by electromagnetic (EM) pumps. The interface between the primary heat transport system and the energy conversion system is a set of primary heat exchangers. The energy conversion system uses the direct TE conversion mechanism. A temperature drop of about 500 K is maintained across the TE elements by the cooling effect of a second liquid lithium loop that transfers the waste heat from the converter to a heat-pipe radiator.

The model predictive controller for the power level control is subject to constraints as follows:

$\Delta u(t+j-1) = 0$ for $j > M$, $0^o \le u(t) \le 180^o$, $|\Delta u(t)| \le 1.4^o$.

The sampling interval T is 1 second. The external reactivity control uses the mechanism of the stepper motor control drum system (Shtessel, 1998).

The regression function by SVMs is solved by using one fifth of a data set shown in Fig. 2. 77 support vectors are collected at every interval (one per five data points) from the data of 1000 sampling points.

Figure 3 shows the detailed performance of the proposed model predictive controller. It is shown that the TE generator power follows its desired value very well. It was known that the proposed controller meets several constraints very well and accomplishes the fast and stable responses.

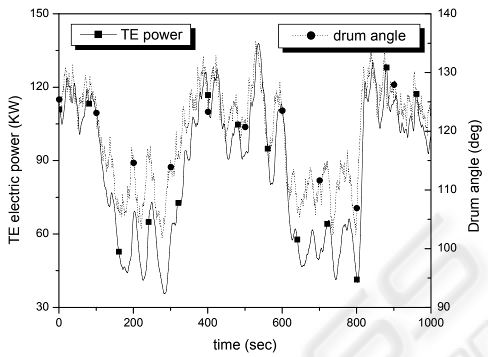
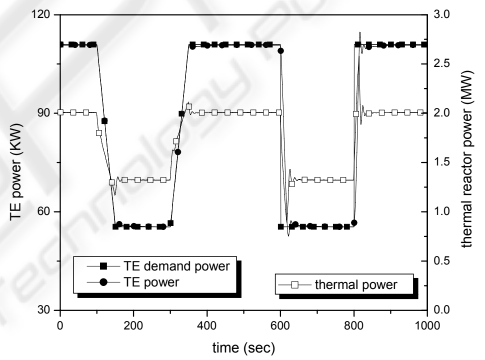
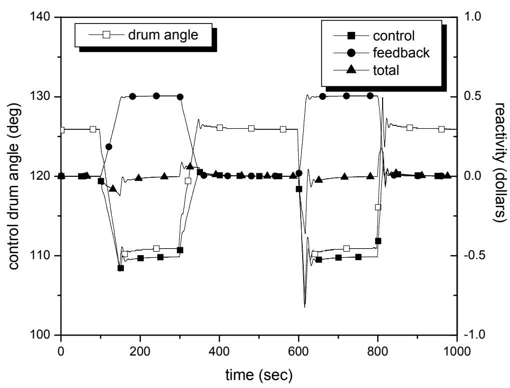


Figure 2: Training data plot.



(a) TE power and thermal reactor power



(b) control drum angle and reactivity

Figure 3: Performance of the proposed MPC controller.

In addition, a conventional proportional-integral (PI) controller was designed to compare the performance of the power level response with the proposed model predictive controller optimized by the GA (refer to Fig. 4). The PI controller has a little slower response and bigger overshoot and undershoot than the proposed MPC.

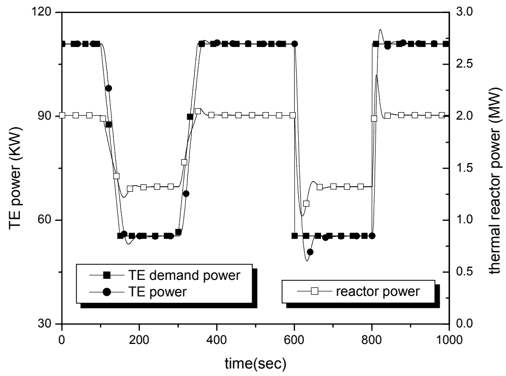


Figure 4: Performance of a PI controller.

4 CONCLUSIONS

In this work, the model predictive controller optimized by the GA and combined by SVMs was developed to control the nuclear power in the SP-100 space reactor system. The future TE power is predicted by using the SVMs and the GA was used to optimize the model predictive controller. It was determined from many numerical simulation results that the proposed controller was able to actuate the control drum to regulate the control reactivity so that the TE generator electric power followed the set point changes according to load demands. Also, the performance of the new proposed controller was proved to be more efficient than that of the conventional PI controller.

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