DESIGN OF MAX-PLUS CONTROL LAWS TO MEET TEMPORAL CONSTRAINTS IN TIMED EVENT GRAPHS

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Abstract: The aim of the work presented is the control of timed event graph in order to meet tight temporal constraints. The temporal constraint represents the maximal duration of a chemical or thermal treatment, for instance. We formulate the problem in terms of control of linear Max-Plus models. A method for the synthesis of a control law ensuring the meeting of constraints is first described for a single input single constraint. Then, the single input multi constraint problem is tackled and finally, the method is extended to the multi inputs, multi constraints problem. The proposed method is illustrated on an example.

1 INTRODUCTION

We consider in the sequel a class of deterministic controlled processes subject to strict time constraints. Such time critical systems are frequent in the industry, for instance in the case of a thermal or chemical treatment, in the car industry for the rubber parts, in the semiconductor industry and also in the food industry. Of course the question is to validate some temporal conditions (see for instance (Berthomieu and al., 1991), (Ghezzi and al., 1991), (Bonhomme and al., 2001), (Cofer and Garg, 1995)). In the present contribution, we formulate this problem in terms of a control problem, assuming that some inputs of the process can be controlled (it is generally the case). We propose a method to solve the inverse problem, synthesizing a control law so that the temporal constraint is validated. We use the formalism of timed event graph, and their algebraic models which are linear over dioids (Baccelli and al., 1992).

The timed event graph behavior is modeled with Max-Plus equations, and the temporal constraints meeting problem is represented with inequalities, also in the so-called Max-Plus algebra. The control approach that we propose is quite different from that considered within the so-called supervisory control framework ((Holloway and al., 1997), (Moody and al., 1996)). Here the time is explicitly taken into account. Timed event graphs and dioids formalism has been used by ((Lahaye et al., 2004), (Atto et al., 2006)) to treat slightly different timed constraint problems. In (Lahaye et al., 2004), the question is formulated as a model matching problem and the temporal constraint appeared as an additional requirement. (Atto et al., 2006) are interested to particular temporal constraints and they suppose that the places subjected to these constraints are initial marking null. Our work also differs from the existing literature on the control of (timed) discrete event systems, since the control laws we consider may involve some delays.

In the present paper, we propose a method for the synthesis of control law that permits to meet a given set of time constraints. The resulting control law itself is finally defined a Max-Plus linear difference equation, involving a finite number of delays. Such an equation corresponds to feedback that is also a timed event graph. A first approach of control for timed event graph under strict temporal constraints was presented in (Amari and al., 2005). This approach has been developed in the Min-Plus algebra, under the hypothesis that all temporizations of the considered graph are integers. In this present contribution, this condition is not required, we

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consider a timed event graph with temporizations that may be real numbers.

The paper is organized as follows. In Section 2, some backgrounds are recalled, featuring some notations concerning the Max-Plus semiring, the timed event graphs and their Max-Plus linear models, the concept of a state equation. The problem, of finding a causal control verifying critical time constraints, is formulated in Section 3. We propose in Section 4 a procedure for the control synthesis, considering first the case of a single input system with a single temporal constraint. Two conditions are proposed, which are sufficient for ensuring the existence of a solution. A simpler condition, which is satisfied in many practical cases, and is simpler to check, is also provided. Then we extend the method to the case of many different constraints. The multivariable case is examined in Section 5 and Section 6 is devoted to illustrative example. Finally Section 7 is devoted to the conclusion.

2 BACKGROUNDS

2.1 Max-Plus Algebra

A monoid is a set, say D, endowed with an internal law, noted $\oplus$, which is associative and has a neutral element, denoted ε, $\forall a \in D, a \oplus \varepsilon = \varepsilon \oplus a = a.$ A semiring is a commutative monoid endowed with a second internal law, denoted $\otimes$, which is associative, distributive with respect to the first law $\oplus$, has a neutral element, denoted e, and admits ε as absorbing element: $\forall a \in D, a \otimes \varepsilon = \varepsilon \otimes a = \varepsilon.$

A dioid is a semiring with an idempotent addition: $\forall a \in D, a \oplus a = a.$ The dioid is called commutative if the second law $\otimes$ is commutative.

We shall consider in the sequel the so-called Max-Plus algebra that is $(\mathbb{R} \cup \{-\infty\} \cup \{+\infty\}, \max, +)$. The Max–Plus algebra, denoted $\overline{\mathbb{R}}_{\max}$ is a commutative dioid, the law $\oplus$ is the operation max, having the neutral element $\varepsilon = -\infty$ and the second law $\otimes$ is the usual addition, with neutral element $e = 0$.

If $n \in \mathbb{N}$ and $v, w \in \overline{\mathbb{R}}^n_{\max}$, we denote $v \oplus w$ the vector with components $v_i \oplus w_i = \max(v_i, w_i)$ for $i = 1$ to n. If no confusion can arise, when $p, q \in \mathbb{N}$, $A \in \overline{\mathbb{R}}^{p\times n}_{\max}$ and $B \in \overline{\mathbb{R}}^{n\times q}_{\max}$ are given matrices, $A \otimes B$ (or just $A.B$) will denote the matrix multiplication in $\overline{\mathbb{R}}_{\max}$, defined by the following expression: $(A.B)_{ij} = \overset{n}{\underset{k=1}{\oplus}} (A_{ik} \otimes B_{kj}) = \max_k (A_{ik} + B_{kj}).$

The Kleene star of a square matrix $M \in \overline{\mathbb{R}}^{n\times n}_{\max}$, denoted M^* is defined by $M^* = \underset{i\in\mathbb{N}}{\oplus} M^i$, where M^0 equals the unit matrix, which entries equal e on the diagonal, and ε elsewhere. Let us recall that $v \in \overline{\mathbb{R}}^n_{\max}$ then $x = M^*.v$ is the maximal solution of both the inequality, $x \geq M.x \oplus v$, and the equality, $x = M.x \oplus v$, (Baccelli and al., 1992).

2.2 Timed Event Graphs, Linear Max-Plus Models

An event graph is an ordinary Petri net where each place has exactly one upstream transition and one downstream transition. A timed event graph is obtained by associating delays to the places or to transitions of an event graph. In our case, the delays are associated to places. We note *n* the number of transitions having at least one upstream place, and m stands for the number of source transitions, noted *tu*, having no upstream place. The unique place relying t_j to t_i is denoted p_{ij}, if any, the corresponding delay is denoted τ_{ij} and its marking is denoted m_{ij}.

A transition t_j is controllable, if there exist a path, denoted α, from transition tu to transition t_j. This path is a sequence of transitions and places, of the form $(tu, p_{uk_1}, t_{k_1}, p_{k_1k_2}, t_{k_2}, ..., t_j)$. We denote m_α the sum of marking along the path α, $m_\alpha = \underset{p_{kl}\in\alpha}{\otimes} m_{kl} = \sum_{p_{kl}\in\alpha} m_{kl}$.

To represent the dynamic of the timed event graph in Max-Plus algebra, we associate to each transition a firing time for the k^{th} occurrence. We note $u_s(k)$, for source transitions tu_s and $\theta_i(k)$ for other transitions t_i.

Example:

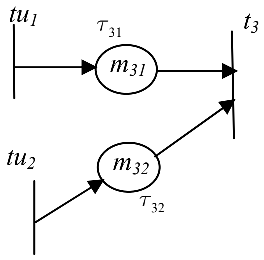

Figure 1: Example.

The timed event graph for the Figure 1, leads to the following equation:

$$\theta_3(k) = \max(\tau_{31} + u_1(k - m_{31}), \tau_{32} + u_2(k - m_{32})),$$

which, in Max-Plus algebra appears to be linear:

$$\theta_3(k) = \tau_{31} \otimes u_1(k - m_{31}) \oplus \tau_{32} \otimes u_2(k - m_{32}). \qquad \square$$

In general, the dynamical behavior of a timed event graph can be expressed by means of a linear equation in Max-Plus algebra, as follow,

$$\theta(k) = \underset{m \geq 0}{\oplus} (A_m.\theta(k-m) \oplus B_m.u(k-m)), \qquad (1)$$

where the components of the vector $\theta(k)$ are the firing times of the n transitions t_i, the components of $u(k)$ is the firing dates of the source transitions tu. The matrix A_m that belongs to dioid $\overline{\mathbb{R}}_{\max}^{n \times n}$, is a matrix which entry $A_{m,ij}$ equals to τ_{ij}, the delay associated to place p_{ij}, if this place exists and the associated delay is τ_{ij}, and ε else. Similarly, the entries of matrices $B_m \in \overline{\mathbb{R}}_{\max}^{n \times m}$ correspond to the delays of the places following source transitions.
Equation (1) is implicit in general. It is worth replacing it by the following explicit equation,

$$\theta(k) = \underset{m > 0}{\oplus} (A_0^*.A_m.\theta(k-m) \oplus A_0^*.B_m.u(k-m)), \qquad (2)$$

where A_0^* is the Kleene star of A_0, defined in the previous section. (See (Baccelli and al., 1992)).
Analogously to the case of usual linear systems, the explicit equation 2 can be brought in state space form. In order to obtain a state space model, one first expands all the places with marking $m > 1$ into m places with marking equal to 1. Hence one adds $(m-1)$ intermediate transitions. One has then the resulting extended state vector $x(k)$, which belongs to $\overline{\mathbb{R}}_{\max}^{N}$, with $N = n + n'$ and n' is the number of added intermediate transitions.
The dynamic behavior of the expanded timed event graph is then described by an equation of the form $x(k) = \hat{A}_0.x(k) \oplus \hat{A}_1.x(k-1) \oplus \hat{B}.u(k)$,
which can be rewritten into the following explicit form, where $A = \hat{A}_0^*.\hat{A}_1$ and $B = \hat{A}_0^*.\hat{B}$,

$$x(k) = A.x(k-1) \oplus B.u(k). \qquad (3)$$

All these formulations permit to point out that the behaviour of a controlled timed event graph is deterministic, depending on the input $u(k)$ and on some initial conditions. This dependence can be explicated, and we shall make use of the following formulation:

$$x(k) = A^{\varphi}.x(k-\varphi) \oplus \left[\underset{k'=0}{\overset{\varphi-1}{\oplus}} A^{k'}.B.u(k-k') \right], \qquad (4)$$

which holds true, for each integer $\varphi \geq 1$.
In the following, we shall assume that the input $u(k)$ is actually a control, which can be arbitrarily assigned. For instance in a production process, the input corresponds to the authorization of performing a certain operation. Typically the beginning of a task performed by a robot, for instance, is subject to such a control input.

3 PROBLEM OF TEMPORAL CONSTRAINTS

Strict time constraints are frequent in flexible manufacturing system (Amari and al., 2004) and semiconductor manufacturing (Kim and al., 2003). One can for instance consider the example of a production process with a furnace for realizing a thermal treatment. The duration of any treatment in the furnace is fixed, or defined by a time interval. One wants to control the system to respect this constraint.
The definition of a timed event graph already takes into account a delay on each place that corresponds to a minimal holding time. The maximal duration appears as an additional constraint that the system should meet. Rather than a verification problem, we formulate the question as a control problem.

3.1 Temporal Constraint

In general, the sojourn time of the tokens in place p_{ij} can be higher or equal to τ_{ij}. In our case, one imposes a maximum sojourn time, noted τ_{ij}^{max}.
Hence p_{ij} is a place subject to a strict time constraint, an interval of time $\left[\tau_{ij}, \tau_{ij}^{max}\right]$ is associated to the place p_{ij}. See Figure 2.

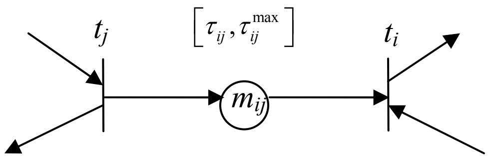

Figure 2: Temporal constraint.

This additional temporal constraint is expressed through the following inequality:

$$x_i(k) \leq \tau_{ij}^{\max}.x_j(k - m_{ij}), \qquad (5)$$

where the product is over $\overline{\mathbb{R}}_{\max}$.

3.2 Causal Feedback

We consider a process modelled by (3), subject to the additional constraint (5). We want to determine a control $u(k)$ that satisfies the constraint (5) for $k > 0$. We shall a priori research this control in the form of a well posed causal feedback of the form, $u(k) = F.x(k-1)$, with $k > 1$.

Remark 1:

Let us note that a static control law $u(k) = G.x(k)$ could lead to implicit loops. For example, given a system with state equation $x(k) = u(k)$ ($x(k) = A.x(k-1) \oplus B.u(k)$, with $A = \varepsilon$ and $B = e$), the control law equation would be $u(k) = x(k)$. On one hand, a badly posed feedback may appear using $u(k) = G.x(k)$, and the other hand a feedback of the form $u(k) = F.x(k-1)$ is always well posed, leading to the closed loop $x(k) = (A \oplus B.F).x(k-1)$. Furthermore, given a system $x(k) = A.x(k-1) \oplus B.u(k)$, one can always build a well posed feedback of the form $u(k) = G.x(k)$, with $F = (G.(B.G)^*.A)$. Therefore, it is not restrictive to suppose that the feedback is of the form $u(k) = F.x(k-1)$, with $F = (G.(B.G)^*.A)$.

4 SINGLE CONTROL

4.1 Single Constraint

We have in this case a timed event graph modelled by the linear Max-Plus equation (3) and subject to a single temporal constraint (5). We propose a method for the synthesis of a control law solving the problem of temporal constraint, provided that the following additional hypothesis is satisfied.
We suppose that the transition t_j is controllable, i.e. there exists a path α from tu to t_j. One note m_α the cumulated markings along this path.
The j[th] component $x_j(k)$ satisfies (6):

$$(A^{m_\alpha}.B)_j.u(k - m_\alpha) \leq x_j(k) \qquad (6)$$

Inequality (6) translate the relation between tu and t_j.
According to equation (4), the i[th] component of the vector $x(k)$ is given by the following explicit expression:

$$x_i(k) = \overset{N}{\underset{r=1}{\oplus}}(A^\varphi)_{ir}.x_r(k-\varphi) \oplus \left[\overset{\varphi-1}{\underset{k'=0}{\oplus}}(A^{k'}.B)_i.u(k-k')\right] \quad (7)$$

for every integer $\varphi \geq 1$.
Taking (7) into account, it appears that the constraint (5) is satisfied if the two following inequalities hold,

$$\overset{N}{\underset{r=1}{\oplus}}(A^\varphi)_{ir}.x_r(k-\varphi) \leq \tau_{ij}^{\max}.x_j(k-m_{ij}), \qquad (8)$$

$$\overset{\varphi-1}{\underset{k'=0}{\oplus}}(A^{k'}.B)_i.u(k-k') \leq \tau_{ij}^{\max}.x_j(k-m_{ij}). \qquad (9)$$

Further, taking (6) into account, we have

$$\overset{N}{\underset{r=1}{\oplus}}(A^\varphi)_{ir}.x_r(k-\varphi) \leq \tau_{ij}^{\max}.(A^{m_\alpha}.B)_j.u(k-m_\alpha-m_{ij}) \quad (10)$$

$$\overset{\varphi-1}{\underset{k'=0}{\oplus}}(A^{k'}.B)_i.u(k-k') \leq \tau_{ij}^{\max}.(A^{m_\alpha}.B)_j.u(k-m_\alpha-m_{ij}) \quad (11)$$

If inequalities (10) and (11) hold, then (8) and (9) are satisfied. Condition (10) holds true for all controls:

$$u(k) \geq \bigoplus_{r=1}^{N}((A^{\varphi})_{ir} - (A^{m_\alpha}.B)_j - \tau_{ij}^{\max}).x_r(k-1),$$

with $(\varphi = m_{ij} + m_\alpha + 1)$. If condition (12) holds for all $(A^{\varphi})_{ir} \neq \varepsilon$, therefore the former expression define suitable causal control laws *u(k)*.

$$(A^{\varphi})_{ir} \geq \tau_{ij}^{\max}.(A^{m_\alpha}.B)_j, \quad (12)$$

with $r = 1$ to N. Inequality (11) is satisfied if the inequalities (13) and (14) are respected.

$$(A^{k'}.B)_i \leq \tau_{ij}^{\max}.(A^{m_\alpha}.B)_j, \quad (13)$$

$$u(k-k') \leq u(k - m_\alpha - m_{ij}), \quad (14)$$

with $k' = 0$ to $\varphi - 1$.

Inequality (13) is a condition which depends on temporizations of the considered timed event graph. As the function $u(k)$ is non decreasing, then inequality (14) is checked if and only if the following inequality is satisfied:

for $k' = 0$ to $\varphi - 1$, $k' \leq m_\alpha + m_{ij}$.

This last inequality is true for $(m_\alpha = m_{ij} = 0)$.

The conditions (12), (13) and the hypothesis $(m_\alpha = m_{ij} = 0)$ are sufficient to ensure the existence of feedbacks which guarantee the meeting of the temporal constraint.

Theorem 1:

The control laws defined by the inequality:

$$u(k) \geq \bigoplus_{r=1}^{N}\left[(A_{ir} - B_j - \tau_{ij}^{\max}).x_r(k-1)\right],$$

guarantee the meeting of the temporal constraint (5) if the hypothesis $(m_\alpha = m_{ij} = 0)$ holds and if conditions (12) and (13) are satisfied.

Proof:

Previously, we saw that the two inequalities (10) and (11) imply the temporal constraint (5). As specified in Theorem 1, the condition (12) is satisfied, and then feedback given as in the theorem ensures the respect of the condition (10). We have the hypothesis $(m_\alpha = m_{ij} = 0)$ and $\varphi = 1$, then the inequality (11) is written as follows:

$$B_i.u(k) \leq \tau_{ij}^{\max}.B_j.u(k)$$

This last inequality is equivalent to (13) which is checked by hypothesis. □

4.2 Multiple Constraints

We consider now the case of a timed event graph, having one source transition which is a control, but Z places are constrained, noted p_z, for $z = 1$ to Z. For each constrained place p_z, let m_z, τ_z and $\tau_z^{\max}$ respectively denote the initial marking, the minimal and maximal delays. Further, let t_z and $t_{z'}$ respectively denote the input and output transitions of the place, $x_z(k)$ and $x_{z'}(k)$ denote the corresponding firing dates, and m_{α_z} denote the cumulated marking along a path α_z going from the source transition tu to t_z. These added temporal constraints are expressed by the inequalities:

$$x_{z'}(k) \leq \tau_z^{\max}.x_z(k - m_z) \quad (15)$$

for $z = 1$ to Z.

We denote $u_z(k)$ the control law calculated as in the previous section to satisfy the z[th] temporal constraint. The following Theorem defines a causal feedback which ensures the respect of all Z temporal constraints.

Theorem 2:

The equation $u(k) = \bigoplus_{z=1}^{Z} u_z(k)$,

with $u_z(k) = \bigoplus_{r=1}^{N}(A_{z'r} - B_z - \tau_z^{\max}).x_r(k-1)$,

defines a causal control which ensures the meeting of all the temporal constraints (15), if the following conditions are satisfied: for *z=1* to *Z*,

$$m_z = m_{\alpha_z} = 0,$$

$$B_{z'} \leq \tau_z^{\max}.B_z,$$

and $A_{z'r} \geq \tau_z^{\max}.B_z$, for *r=1* to *N*.

Proof:

The conditions quoted in this theorem are sufficient so that the feedback $u_z(k)$ satisfies the z[th] temporal constraint. The following inequality:

$$\overset{Z}{\underset{z=1}{\oplus}} u_z(k) \geq u_z(k),$$

is true for *z=1* to *Z*.
It is finally clear that $u(k) = \oplus_{z=1}^{Z} u_z(k)$ validates all the *Z* temporal constraints. □

5 MULTIVARIABLE CONTROL

In this section, a considered timed event graph contains *m* source transitions, with $m \geq 1$. Its dynamical behaviour is represented by a linear Max-Plus system (3). The control law is a vector of *m* components. Firstly, we suppose that p_{ij} is the single place subjected to an additional temporal constraint (5). We calculate a vector $u(k) \in \overline{\mathbb{R}}_{\max}^{m}$, with $m \geq 1$, which is a control law that must satisfy the constraint (5). The components of $u(k)$ are noted $u_s(k)$, for $s = 1$ to m. We note by m_{α_s}, the cumulated markings along a path α_s from tu_s to t_j. We suppose that $m_{\alpha_s} = m_{ij} = 0$, i.e. the initial marking of the place p_{ij} and along equal zero the path α_s.
This hypothesis is translated by the following inequality:

$$B_{js}.u_s(k) \leq x_j(k). \tag{16}$$

Theorem 3:

The meeting of the temporal constraint (5) is guaranteed if:
(a) There exits s such that:

$$u_s(k) \geq \overset{N}{\underset{r=1}{\oplus}} \left[(A_{ir} - B_{js} - \tau_{ij}^{\max}).x_r(k-1) \right],$$

and $u_l(k) = \varepsilon = -\infty$ for $l \neq s$, and
(b) The both following sets of conditions are satisfied:

(i) $A_{ir} \geq \tau_{ij}^{\max}.B_{js}$ for *r=1* to *N* and $A_{ir} \neq \varepsilon$.

(ii) $B_{is} \leq \tau_{ij}^{\max}.B_{js}$ for *s=1* to *m*.

Proof.

Applying (3), the *i*[th] component of $x(k)$ is

$$x_i(k) = \left[\overset{N}{\underset{r=1}{\oplus}} A_{ir}.x_r(k-1) \right] \oplus \left[\overset{m}{\underset{s=1}{\oplus}} B_{is}.u_s(k) \right]. \tag{17}$$

Taking (17) into account, it appears that constraint (5) (with $m_{ij} = 0$) is satisfied if both following conditions hold,

$$\overset{N}{\underset{r=1}{\oplus}} A_{ir}.x_r(k-1) \leq \tau_{ij}^{\max}.x_j(k)$$

and

$$\overset{m}{\underset{s=1}{\oplus}} B_{is}.u_s(k) \leq \tau_{ij}^{\max}.x_j(k).$$

Futher, taking (16) into account, these conditions become

$$\overset{N}{\underset{r=1}{\oplus}} A_{ir}.x_r(k-1) \leq \tau_{ij}^{\max}.B_{js}.u_s(k)$$

and

$$\overset{m}{\underset{s=1}{\oplus}} B_{is}.u_s(k) \leq \tau_{ij}^{\max}.B_{js}.u_s(k).$$

Conditions *(i)* and *(ii)* being verified, and the control law satisfying the inequality of the Theorem 3, one can check that the constraint (5) is satisfied. □

Corollary:

Let a timed event graph with *m* source transitions ($m \geq 1$) and *Z* additional temporal constraints (15). The causal control law which guarantees the respect of the *Z* constraints is defined by:

$$u(k) = \overset{Z}{\underset{z=1}{\oplus}} u^{s_z}(k),$$

where $u^{s_z}(k)$ is the control law, calculated by Theorem 3, to check the z[th] constraint.

Proof:

A control law $u^{s_z}(k)$, validates the z[th] constraint, if conditions *(i)* and *(ii)* of Theorem 3 are satisfied. Thus, we have, for $z = 1$ to *Z*, $u^{s_z}(k) \leq \overset{Z}{\underset{z=1}{\oplus}} u^{s_z}(k)$.
According to Theorem 3, it is clear that the control law $u(k) = \oplus_{z=1}^{Z} u^{s_z}(k)$ guarantees the respect of all Z temporal constraints. □

6 EXAMPLE

Consider the timed event graph of Figure 3. This graph contains two source transitions modelling respectively, control $u_1(k)$ and control $u_2(k)$, $(m=2)$.

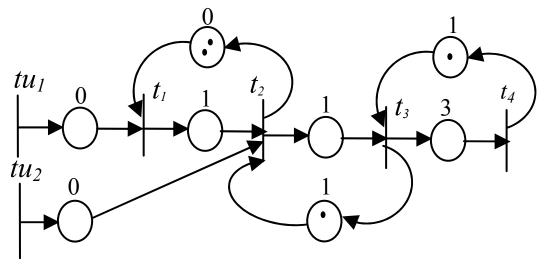

Figure 3: Timed event graph.

Two additional temporal constraints are added to this graph, and are expressed respectively by the following inequalities:

$$x_2(k) \leq 1.x_1(k),$$
$$x_3(k) \leq 1.x_2(k).$$

The problem consists in calculating a control vector,

$$u(k) = \begin{pmatrix} u_1(k) \\ u_2(k) \end{pmatrix},$$

which satisfies these both constraints.

By applying our approach, the previous graph has been transformed into the graph Figure 4, with $m^{\max} = 1$. To do so, place p_{12} marked to 2 has been split into two places marked to 1 and the intermediate transition t_5 is added.

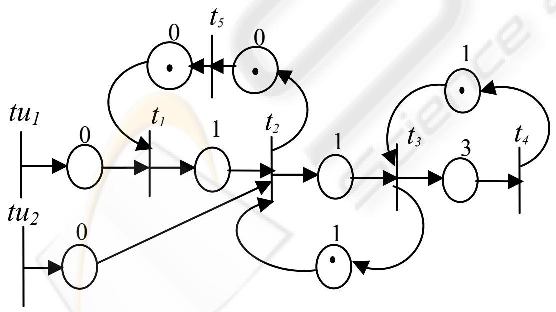

Figure 4: Extended equivalent graph.

The state equation associated with this new timed event graph is:

$$x(k) = \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & e \\ \varepsilon & \varepsilon & 1 & \varepsilon & 1 \\ \varepsilon & \varepsilon & 2 & 1 & 2 \\ \varepsilon & \varepsilon & 5 & 4 & 5 \\ \varepsilon & e & 1 & \varepsilon & \varepsilon \end{pmatrix} .x(k-1) \oplus \begin{pmatrix} e & \varepsilon \\ 1 & e \\ 2 & 1 \\ 5 & 4 \\ \varepsilon & e \end{pmatrix} .u(k),$$

where the components of $x(k)$ are the firing times of the transitions t_1, t_2, t_3, t_4 and t_5, and the vector $u(k)$ is the control law. We shall then apply Corollary to calculate a control $u(k)$ which guarantees the meeting of both temporal constraints. This example, it is enough to find, for each temporal constraint, only one component of the vector $u(k)$ to guarantee the meeting of this constraint.

Firstly, by applying of Theorem 3, we determine a component of the vector $u(k)$, which satisfies the first constraint. We have $\tau_{ij}^{\max} = \tau_{21}^{\max} = 1$ and the initial marking of the place p_{21} is $m_{21} = 0$. It exist a path α_1 from transition tu_1 to transition t_1 and its initial marking is $m_{\alpha_1} = 0$. We can check that one has $\tau_{ij}^{\max} + B_{js} = \tau_{21}^{\max} + B_{11} = 1$, and $A_{ir} = A_{2r} = (\varepsilon \;\; \varepsilon \;\; 1 \;\; \varepsilon \;\; 1)$, hence the condition *(i)* of Theorem 3 holds. Similarly, we check that $B_{is} = B_{21} = 1$, so that the condition *(ii)* of Theorem 3 holds too. Thus, the component of $u(k)$ which guarantees the meeting of the first constraint is

$$u_1(k) = \bigoplus_{r=1}^{5} \left[(A_{2r} - 1).x_r(k-1) \right] = x_3(k-1) \oplus x_5(k-1).$$

Secondly, we determine also by Theorem 3 a component of the vector $u(k)$, which satisfies the second constraint. In this case, we have, $\tau_{ij}^{\max} = \tau_{32}^{\max} = 1$ and the initial marking of the place p_{32} is $m_{32} = 0$. It exist a path α_2 from transition tu_2 to transition t_2 and its initial marking is $m_{\alpha_2} = 0$. We can check that one has $\tau_{ij}^{\max} + B_{js} = \tau_{32}^{\max} + B_{22} = 1$, and $A_{ir} = A_{3r} = (\varepsilon \;\; \varepsilon \;\; 2 \;\; 1 \;\; 2)$, hence the condition *(i)* of Theorem 3 holds. We check also that $B_{is} = B_{32} = 1$, so that the condition *(ii)* of Theorem 3 holds too. Thus, the component of $u(k)$ which guarantees the respect of the second constraint is

$$u_2(k) = \bigoplus_{r=1}^{5} \left[(A_{3r} - 1).x_r(k-1) \right]$$
$$= 1.x_3(k-1) \oplus x_4(k-1) \oplus 1.x_5(k-1).$$

Finally, according to Corollary, the control law which guarantees the meeting of both temporal constraints is given by the following vector:

$$u(k)=\begin{pmatrix} x_3(k-1)\oplus x_5(k-1) \\ 1.x_3(k-1)\oplus x_4(k-1)\oplus 1.x_5(k-1) \end{pmatrix}.$$

After comparison between the terms of each component of the vector, the control law is simplified to:

$$u(k)=\begin{pmatrix} x_3(k-1) \\ x_4(k-1) \end{pmatrix}.$$

This feedback can be interpreted by two places of control connected to the timed event graph to guarantee the respect of the temporal constraints. The controlled graph is given in Figure 5.

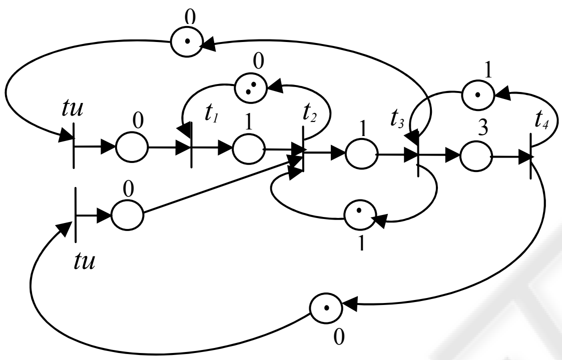

Figure 5: Controlled timed event graph.

Remark 2:

The same example was treated with the method developed in Min-Plus algebra (Amari and al., 2005). It is worth noting that the synthesis of the Max-Plus control is easier than that in the Min-Plus algebra. In this case, there are not necessary compute of power for the matrix.

7 CONCLUSION

We have developed a method for control synthesis of timed event graphs subject to strict temporal constraints. A generalization for timed event graphs with multivariable control has been proposed in this paper. This method is illustrated on an example. The conditions (12) and (13) are shown here to be sufficient conditions, we are investigating actually the existence of necessary and sufficient conditions for the synthesis of control laws which ensure the meeting of the temporal constraints. We will continue the comparison of this method with those developed in (Lahaye et al., 2004) and (Atto and al., 2006). We hope to apply this method for real systems, notably for the verification and validation of automated systems as well as telecommunication processes and real-time software.

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