

# CONSIDERATIONS FOR SELECTING FUNCTIONS AND TERMINALS IN GENETIC PROGRAMMING FOR FAULT-DETECTION IN EMBEDDED SYSTEMS

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**Abstract:** The article describes the terminals and functions used by genetic programming to discover specific parameters for fault-detection in embedded control systems design. Choice of different functions and terminals affects the convergence speed. The state of embedded controller is mapped into a space of valid/invalid points and genetic programming is used to divide the space into hypercubes that can be used to trivially recognize faults during system operation. The fault-detection logic operates by monitoring the input and output variables of the embedded controller. It is based on acquired and built-in knowledge about the normal behaviour in order to detect abnormalities. The fault-detection problem is approached by the use of monitoring cells, which implement the system supervising logic.

## 1 INTRODUCTION

Embedded control systems are present in all modern technological products with a strong position even in safety critical environments. Highly dependable programmable electronic systems for safety critical embedded control and regulation applications are mandatory. Due to the complexity of the control systems faults are an unavoidable fact. A discipline that handles faults is called “fault management”. One possibility to implement the *fault-detection* (a fault-management technique) is to use a “monitoring cell” (MC), which monitors the validity of inputs and outputs and (possibly) the internal states of the control process. As a result, the fault-status is reported to a higher fault management layer.

Run-time faults must be recognized by the fault-detection mechanism of the MC. Such faults are the most difficult to discover because they are a consequence of totally unpredictable events or chain of events. One way to detect run-time faults is to observe whether the system behaves *normally* at all times. The system is observed during normal operation by recording all inputs, outputs and internal states which are believed to affect its future behavior. Then a machine learning technique can be used to “learn” the normal behaviour from collected data. Genetic algorithms (GA) are the most easily applied evolutionary

paradigm to solve this problem, yet the article sets focus on genetic programming (GP) because GP’s evolved programs closely resemble the fault-detecting functions of MCs and are therefore of special interest.

Section 2 describes the concepts behind the monitoring cell. Next the importance of selecting appropriate functions and terminals for GP is emphasized. In section 3 evolutionary computing (EC) is used to discover fault-detecting functions for monitoring cells. For GP various functions and terminals are discussed.

## 2 EVALUATION OF THE MONITORED SIGNALS

The monitoring cell effectively performs some evaluation function. A proper evaluation function can be constructed in different ways. As a basic evaluation, the integrity of each individual signal is verified. The information on its basic properties is acquired from the system specifications, technical documentation or similar sources. This way at least information on the data ranges – valid and invalid values of different signals – is extracted.

A more thorough validation of the system should be performed to make sure that outputs are consistent with the inputs. For this, the properties of transforma-

tion function of the system must be known. To learn the behaviour of the control function two general approaches are possible: to derive it analytically from the available explicit knowledge about the process, or to learn it by observation. In the latter case, the monitored control system is observed during an interval of valid operation to produce a learning set  $L$  consisting of  $n$  instances  $s$ , each instance representing separate input signals to the MC. This instance is actually a point in a  $D$ -dimensional space, where  $D$  is the count of signals in instance  $s$ .

The learning dataset  $L$  is a set of instances of (co-related) signals and MC's task is to determine whether a previously unseen instance  $s$  belongs to the space of known valid instances or not (a classification/clustering problem). The MC's limited processing abilities mostly do not support the use of complex rules for that purpose. The monitoring function can be created off-line and a sequence of simple comparisons, which is sufficient to determine if a point lies within a faulty region or not, can then be executed by the MC during system operation.

Assuming the learning set includes all relevant signals then the simplest general clustering solution is a *hypercube* – a generalization of the cube within  $D$  dimensions. The smallest hypercube that includes *all* learning instances from  $L$  is actually the space  $S$  of all possible states of the control system. Beside  $L$ 's (valid) learning instances,  $S$  contains plenty of other points, most of which are descriptions of faulty states of the control system. Learning algorithm has to group (cluster) the valid points into smaller hypercubes with mostly valid points. The learning process can result either in (1) one, (2) too-many, or (3) few hypercubes. If (1) then no optimization was performed and all instances are always proclaimed valid. This is possible if  $L \sim S$ . If (2) the hypercubes are too small and fail to group related instances. This is a non-general partitioning of space  $S$  over-fitting the learning set. Such hypercubes fail to classify correctly most of the valid points *not* included in  $L$ . This is mainly because no relations between instances exist or the collected dataset does not include all relevant signals for fault-detection. Few hypercubes (3) partition the space  $S$  into valid/invalid regions.

In general this is a multi-objective optimization problem with conflicting goals – many small hypercubes have low classification error yet they fail to generalize and require more (scarce) computing resources. Depending on hardware MC implementation, the MC can perform only a *limited* number of point-in-hypercube tests. Therefore certain hypercubes will include noise (instances of the invalid type).

## 2.1 Establishing the Monitoring Function

From the learning set  $L$  the monitoring evaluation function  $E$  must be constructed using a combination of optimization, unsupervised learning and classification techniques. Final solution must be verifiable in order to be allowed for use in an embedded control system.

All machine learning techniques are able to extract information from the learning data. Most of them (e.g. K-means or minimal spanning tree clustering algorithms, support vector algorithms, neural networks), however, tend to produce overly complex solutions unsuitable for MC implementation. The hypercubes paradigm suits the MC well and discovery of hypercubes can be trusted to any evolutionary computation technique. Although several evolutionary techniques (e.g. genetic algorithms or evolution strategies (Banzhaf et al., 1998)) can be used, the focus here is on GP as it is the only evolutionary paradigm capable of evolving a complete fault-detection *logic*.

## 3 USING EC TO DISCOVER FAULT-DETECTION RULES

The basic difference between GA and GP is that GAs directly produce solutions according to some predefined structure, while GP produces a self-structured computer program that effectively calculates a desired output for particular inputs. GP's program is therefore similar in concept to the MC's monitoring function.

In order to use EC to discover the monitoring cell's evaluation function, a description of controller's behaviour is needed. Normal operation of the controller is described by the empirical learning set  $L$ . Genetic programming could be used as a symbolic regression tool to discover the symbolic expression that satisfies the data points in the learning set. However, the hidden symbolic expression is probably a complex structure consisting of numerous mathematical operations and functions (e.g. trigonometric functions) and is probably too complex to be implemented in MC.

For monitoring cells the partitioning of the space of all possible signals into two disjoint spaces  $S^-$  and  $S^+$  is required – space  $S^+$  includes all points (instances) from the learning set and thus describes the “normal” states of the controller;  $S^-$  includes no such points and therefore represents a space of *possible* faults (even if system produces signals belonging to  $S^-$ , the system itself is not necessarily experiencing a fault).

The example used throughout this section is based on some unknown calculation  $C$  using a single scalar

input  $x$  to produce a scalar output  $y = C(x)$ . The behavior of the example system using function  $C$  was recorded to obtain a learning set  $L$  with 316  $(x, y)$  points displayed in Fig. 1. The coordinates in  $L$  are limited:  $-10 \leq x \leq 10$  and  $-32 < y < 775$ . The relationship between  $x$  and  $y$  coordinates is unknown, yet human can easily draw a clustering region (marked with grey). This same region can be mostly covered by three inclined dashed hypercubes  $S'_0, S'_1$  and  $S'_2$ . The approximation using three upright hypercubes  $S_0, S_1$  and  $S_2$  covers not only  $L$  but also many unspecified points in  $S$ . The remaining space  $S^-$  is free of any recorded valid points. A general MC has only enough processing power to test whether a point belongs to an upright hypercube.

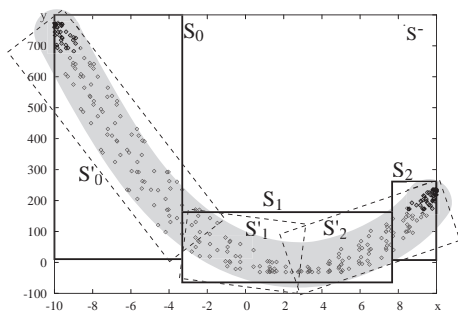


Figure 1: Dependency of  $y$  on  $x$  in learning set  $L$ . Clustering done by a human is marked with grey.

By using  $n$  upright hypercubes  $S_i$  the monitoring cell can perform only a very rough detection of faults. The  $S_0$ , for example, covers a large area designated as invalid by a human expert. The optimization task is to group available valid points into one or more hypercubes while keeping the error (count of invalid points inside the hypercubes) at a low level. Problem is that it is impossible to determine if a hypercube includes any invalid points because there are no invalid points in  $L$  to verify this assertion.

It is possible, however, to invert the problem and look for a hypercube  $H_i$  which contains *no* valid points:  $H_i \cap L = \emptyset$ . This inverted task results in a hypercube  $H_i \subseteq S^-$ , contrary to the original optimization task of producing a hypercube  $S_i \subseteq S^+$ .

The machine learning algorithm must therefore divide the space  $S$  into two mutually exclusive hyper-spaces  $S^-$  and  $S^+$  in a way which maximizes the hyper-space  $S^-$  that consists of a finite number of hypercubes  $H_n$ . For the GP to evolve a computer program, which calculates the  $n$  hypercubes  $H_i$  needed by the MC, a function set and terminal set are needed which produce  $2nD$  coordinates (each hypercube is uniquely defined by two opposite corners, each corner point with  $D$  scalar coordinates, respectively).

A similar approach to hypercubes is a matrix ap-

proach – each dimension ( $d_i, i = 1..D$ ) is divided into  $I_i$  intervals and the (hypercube) cells  $M_k$  inside the matrix represent clustering regions. A cell that contains at least one valid sample represents a valid hypercube. This approach is simpler than the hypercube approach, but can have more generalization difficulties.

### 3.1 Optimization Using Genetic Algorithm

GA is the simplest evolutionary optimization approach for hypercube sizing and positioning. Suppose the MC hardware is able to execute enough instructions to test the inclusion of a point in three hypercubes, each hypercube being defined by two opposite corner points. For three hypercubes 6 points in the 2-D space  $S$  are needed and GA has to produce 6  $(x, y)$  pairs within the specified range ( $S = [-10, 10] \times [-31, 775]$ ). Genetic algorithm typically operates with fixed-size bit strings, which are first transformed into  $x, y$  pairs and then into hypercubes. Fitness function simply returns the score  $|H_{1+2+3}|/(err + 1)$ , where the  $|H_{1+2+3}|$  operator returns the size of the hypercube  $H_{1+2+3}$ , and  $err$  is the number of valid samples therein.

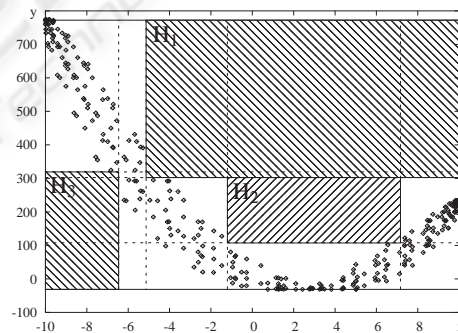


Figure 2:  $H_1, H_2$  and  $H_3$  produced by OpenBeagle GA using fixed-length bit strings (12 bits/coordinate,  $P=50$ ,  $G=1000$ ).

Fig. 2 shows the results obtained from GA using the OpenBeagle library (Gagne and Parizeau, 2006). This same picture also shows the matrix of 20 cells, which are designated by a dashed lines, and their partitioning of the search space  $S$  according to intervals as determined by three hypercubes.

### 3.2 GP: Basic Scalar Functions and Terminals

To use GP several prerequisites must be met, including the closure and sufficiency property of the func-

tions and terminals (Koza, 1992). The most straightforward and general GP approach is to use the basic scalar function set and terminal set to produce hypercube coordinates (+, -, \*, /). Basic terminal set of  $T = 0, 1$  can be extended with basic problem-specific constants ( $x_{min}, x_{max} \dots$ ) and ephemeral (Koza, 1992) constants. This approach suffers from the following problems:

- Ignorance of the “smaller” search space  $S$ ; by using the basic functions and terminals the search takes place in the range of underlying scalar coordinate data-type rather than inside the bounded search space  $S$ .
- Produces only one value.
- Has low convergence speed.

Advantages of this approach are:

- Is most generally applicable and is not limited by any assumptions about the problem.
- Is supported by most available GP libraries and therefore easy to implement – this is the basic kit in all GP implementations.

The first problem is the search through the definition space of the data-type rather than through problem space  $S$ . This is because GP is searching for a computer program that calculates values in the space of *all* numbers, not in the range of *valid* numbers as defined by  $S$ . For example, to calculate any  $x$  coordinate for the  $H_1$  hypercube from Fig.1, the program is not limited to the  $[-10, 10]$  range; GP actually generates expressions for the full data-range of the data type (e.g. float). A solution is to use an artificial mapping of calculated values into valid intervals. This action improves both the convergence speed and the (genetic) redundancy (important for the discovery of robust solutions), but is problematic to implement as GP is unlikely to produce numbers in, for example, *float* range with linear distribution. . .

Next problem of basic function set and terminal set is that only one number is produced. This is obviously unacceptable as at least  $2 \cdot D$  coordinates are needed to define one hypercube; this issue can be solved in one of the following ways:

**Multiple computer programs** can be used to produce respective coordinates; advantageous is the general applicability and simplicity of implementation, problematic are the high demand for processing resources and sensitiveness to ordering of programs – change in ordering affects the partitioning of the search space  $S$ .

**Special function** can be introduced to the function set, which “records” interim values in a pre-defined buffer until enough values are collected or the program has finished; advantage is that this function is called in a self-adaptive manner (Schwefel, 1987),

on the down side the crossover operator is more lethal and destructive.

The special *rec* function approach is illustrated in Fig.3 showing a random computer program created in the initialization phase by employing the function set  $F = \{+, -, *, /, \sin, \cos, \text{rec}\}$  and the terminal set  $T = \{1, -10, 10, -32, 775, 20, 807\}$ , where instruction *rec* records the argument and passes it on unchanged, and  $T$  consists of constants describing the search space  $S$  from Fig. 1 ( $1, x_{min}, x_{max}, y_{min}, y_{max}, \Delta x$  and  $\Delta y$ ). The example program represents the expression  $-10 \cdot 1 + \sin(775)$  and has a result of  $-9.18$ . The five *rec* instructions, however, recorded interim values  $-10, 1, -10, 775$  and  $\sin(775)$ . The interpretation of these is left to the individual’s evaluation function (for example, consecutive values can be treated as coordinates of the hypercube corners).

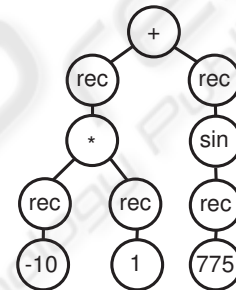


Figure 3: Example program using the extended basic function set  $F$  and terminal set  $T$  of problem-specific constants.

The main problem of the basic approach is its slow convergence speed. One possible improvement is the use of *automatically defined functions* (ADFs) as defined by (Koza, 1992), but this does not resolve the problem. The convergence speed can be improved at the expense of general applicability of functions and terminals and this is what the next approach does.

### 3.3 GP Functions for Hypercube Manipulation

Hypercube functions and terminals enable the GP to manipulate D-dimensional hypercubes. Functions do not handle scalar values as is the case with the basic function set; rather they operate with and exchange special objects that represent hypercubes.

A D-dimensional hypercube is uniquely defined by at-least two opposite corner points ( $c_1$  and  $c_2$ ) in a D-dimensional hyper-space  $S$  (hyper-space  $S$  is actually the smallest hypercube encompassing all learning points in  $L$ ). The function set and terminal set must satisfy both the closure and the sufficiency criteria. Additionally the instructions have to be highly efficient in making the hypercube transformations.

A hypercube  $H_i$  is a sub-space of the search space  $S$  ( $H_i \subseteq S$ ) and main idea is either to shrink and re-position  $S$  into  $H_i$  or “grow” the empty space  $\emptyset$  to become the final hypercube  $H_i$ . Input to the computer program can be any valid *random* hypercube  $S'$  ( $S' \subseteq S$ ).

Basic hypercube transformation functions and terminals have mathematical origins and include (but are not limited to):

$union(A, B)$  function calculates the smallest hypercube that includes both hypercubes  $A$  and  $B$ .  $union$  is efficient and very simple to implement function with results that quickly fill the whole search space.

$intersection(A, B)$  function returns the intersection of hypercubes  $A$  and  $B$ . Common results is a small hypercube or even a hyperpoint – latter may be useful as an argument for other functions as it has zero size and a fixed position in space.

$scale_k(A)$  function resizes the hypercube’s edges by a constant pre-defined factor  $k$ . For different factors different functions must be defined.

$move_i(A, B)$  function must be defined for each dimension. It moves the hypercube  $A$  in the  $i$ -th dimension by the length of hypercube  $B$  in this dimension:  $A_i = A_i + \Delta B_i$ . Optionally a negative variant is possible, which can be invoked if special criteria are met, e.g. if  $|B| > |A|$  the  $A_i = A_i - \Delta B_i$  rule applies. This set of functions allows for fine positioning of hypercube  $A$ .

*comparison* functions, e.g.  $<, \leq, = \dots$ , can be based on the size of the compared hypercubes. Furthermore, logical operators, e.g. *if*, can be declared to give GP even more freedom in the search for more robust solutions. They all return one of the hypercube arguments; boolean cast is a simple test of the non-emptiness of the hypercube.

( $x$ ) terminal is a hypercube with size 0, positioned at a random hyperpoint ( $x$ )  $\in L$ .

$S'$  terminal returns a random hypercube inside the search space  $S$ . Of course each hypercube instance is initialized differently every time it is used by a newly created computer program, similarly to an ephemeral constant. This allows great variability in hypercubes offered by the terminal set.

$S$  terminal returns  $S$  – the search space hypercube.

All these functions and terminals are universal – they can be applied in any combination. One possible *hypercube* GP setup is:

$$\begin{aligned} F &= \{union, intersection, scale_2, scale_{1/2}\}, \\ T &= \{S', S\}. \end{aligned} \quad (1)$$

GP equipped with (1) is able to find solutions similar to those found by GA, but more processing time

was needed to discover solutions of comparable quality. This is probably due to a fact that *union* and *intersection* are binary functions and will produce a perfect result only if *both* of its arguments are perfect. This severe problem is addressed next.

### 3.4 GP: Extended Hypercube Functions

The motivation is to create more robust versions of the *union* and *intersection* operations. The perfect hypercube is (1) correctly positioned, and (2) has perfect dimensions. If, for example, a binary *union* function is to be made more robust, it should no longer be dependable on correct position and size of *both* arguments. Instead, a more relaxed transformation can be introduced, which is based on complete correctness of the first argument and for example, only on correct size of the second argument. A call to  $union'(A, B)$  will produce same result as  $union'(A, C)$ , as long  $|B| = |C|$ .

The *inflate* operation inflates the hypercube  $A$  to “include”  $B$  as if  $B$  were positioned right next to  $A$  and then  $union(A, B')$  were called. This is depicted in Fig. 4, where the original  $A$  and  $B$  hypercubes are drawn with thick lines. The  $inflate(A, B)$  operation aligns the  $b_1$  corner of  $B$  with the  $a_2$  of  $A$ .

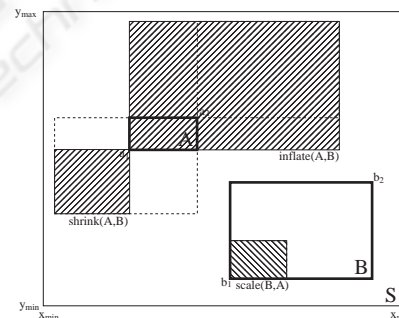


Figure 4: Functions *inflate*, *shrink* and *scale* create hatched hypercubes.

A similar operation is *shrink*, which aligns the  $B$ ’s  $b_2$  corner with  $A$ ’s  $a_2$  and returns a hypercube between  $b'_1$  and  $a_1$  (Fig. 4). The chosen two implementations are not commutative because the result is always aligned according to the first hypercube:  $inflate(A, B) \neq inflate(B, A)$ . However, the size of the resulting hypercubes is commutative:  $|inflate(A, B)| = |inflate(B, A)|$  and  $|shrink(A, B)| = |shrink(B, A)|$ .

Both functions can produce results outside of the search space  $S$ . Such interim hypercubes may be needed to produce final results within the *whole*  $S$ . The final hypercube can be easily clipped to fit inside  $S$ .

Next a powerful function that scales a hypercube can be defined. Function  $scale(X, Y)$  scales (inflates or shrinks) the hypercube  $X$ ; it effectively calls the  $scale_k(X)$ , where  $k$  is calculated using:

$$k = \begin{cases} \sqrt[D]{\frac{|Y|}{|X|}} & \text{if } |X| \neq 0 \\ 1 & \text{otherwise.} \end{cases}$$

Example in Fig. 4 shows the function  $scale(B, A)$ , which shrinks the hypercube  $B$  to approximately 40% of its original size. The  $scale$  instruction is much more flexible than the static  $scale_k$  because it allows GP to find a “perfect” scaling factor. The described three functions accompanied by the two terminals  $S$  and  $S'$  make the *improved* GP hypercube manipulating setup:

$$\begin{aligned} F &= \{inflate, shrink, scale\}, \\ T &= \{S', S\}. \end{aligned} \quad (2)$$

Table 1 shows the summary for 10 independent runs of each method for the problem from Fig. 1. Higher values are considered better although the overfitting limit is unknown. The GA used setup as described in section 3.1, and GP was equipped with both basic hypercube function set (1) and improved function set (2). Column 1 gives the average size of the discovered hyperspace of invalid points  $S^-$  and column 2 the biggest  $S^-$  found by the respective method. GP results come very close to GA’s (probably overfitted) scores. Use of basic scalar functions and terminals produced statistically inferior results when compared to GA and GP using (1) or (2) and is not included in this table.

Table 1: Summarized results for GA and GP equipped with hypercube and improved hypercube manipulating functions and terminals.

Method	Average( $ S^- $ )	Max( $ S^- $ )
GA	9703	10051
GP using (1)	9251	9780
GP using (2)	9585	9808

## 4 CONCLUSION

The aim was to discover a general assortment of GP functions and terminals in order to build programs for fault-detection. The first step, presented here, was to use GP as a machine learning algorithm to solve a typical problem in embedded system design; next is the discovery of other GP functions and terminals which would allow GP to discover ideal adaptive run-time fault detection logic.

In many cases the analytical methods for space partitioning and clustering are more appropriate than

evolutionary computation. For monitoring cells in embedded control system design, however, EC is quite suitable. The described GP functions and terminals are able to define simplistic fault-detection rules. Although the GAs are easier to implement and have best convergence properties, GP is a promising option as it’s structure is self-evolving. The GA on the contrary has a limited fixed solution structure.

The only advantage for using the basic scalar function and terminal set with GP lies in the simplicity of use as they are available in all GP libraries. However, they are not efficient in producing hypercubes directly. A better option would be to calculate optimal partitioning of problem’s dimensions into intervals, what effectively divides the search space into a matrix of hypercubes. The hypercube manipulating functions are a better choice: the improved set (2) has better convergence properties than the basic set (1). Another way to improve GP performance is to search for partial solutions first by dividing the original  $D$ -dimensional problem into a set of easier  $2-D$  problems.

The hypercubes are a simple and efficient strategy for fault-detection. However, ambiguous points exist at hypercube boundaries and corners; such points are close to the cluster of valid points. This also limits the GP in discovering effective adaptive run-time rules for fault detection as currently all results have to be approved by a human. Further handling of detected faults is out of scope of this paper.

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