IDENTIFICATION OF SLOWLY TIME-VARYING SYSTEMS BASED ON THE QUALITATIVE FEATURES OF TRANSIENT RESPONSE A FROZEN-TIME APPROACH

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Abstract: A method for structural and parameter identification of a slowly time-varying systems is proposed. The frozen-time method is used in this analysis. By means of this method we obtain consecutive LTI models, which are identified in consecutive discrete instants using the Qualitative System Identification (QSI) Algorithm. The proposed algorithm models the behavior of the ODE's coefficients means of polynomial functions. The algorithm models the variations of those coefficients though polynomials. An optimal model is obtained using Genetic Algorithms. The algorithm starts with a polynomial of second degree and tries to fit these polynomials, to the variations of the coefficients. If the degree of the polynomials is not enough it increases and repeats the process until achieving a good fit. The system was tested with the identification of a controlled experiment in a power systems laboratory.

1 INTRODUCTION

Practical systems are inherently time-varying, due to changes in operating conditions, drifting effects of components, on-line modeling processes, etc. One of the simplest and most tractable time-varying systems are slowly time-varying systems, whose behavior resemble linear time invariant systems over a small period of time.

Slowly time-varying systems are of great importance in both practical applications and theorical studies. Many practical systems are slowly time-varying. Enviromental condition variations are usually much slower than systems dynamics. Therefore, a dynamic system with parameters dependent on the enviroment (temperature, pressure, altitude, etc.) can often be modelled as slowly varying systems. Component aging and deteriorations are another example of slow variations of systems dynamics in operation.

One of the previous approaches for analysing slowly varying systems is the frozen-time approach introduced in the 60's (Freedman; Desoer), for stability analysis of systems with slowly time-varying parameters and used in (Le Yi) for identification and control. The main idea of the frozen-time approach can be summarized as follows: A time-varying plant is first modelled as a sequence of Linear Time-Invariant (LTI) systems, called frozen-time systems. The frozen-time system at time t represents the dynamic behavior of the plant at that frozen time. At each frozen-time, the system's identification process is carried out using the QSI software (see next section).

The resulting models of applying QSI, and the frozen-time aproach, are organized consecutively forming a matrix that describes the behavior of the coefficients in time. The behavior of the coefficients of the ODE can be modelled independently by means of a polynomial function.

Section 2 presents how Qualitative System Iidentification (QSI) works. In Section 3 we formulate the problem addressed in this paper. Section 4 explains the systems identification procedure proposed in this paper. Section 5 presents an application example. Finally, section 6 presents the conclusions of this work.

2 QUALITATIVE SYSTEM IDENTIFICATION

QSI is a qualitative and quantitative system identification algorithm and software, developed by Flores and

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(Flores05 et al; Pastor05). QSI takes as input a time-series representing the transient response of a LTI dynamic system and produces a model of the identified system.

The identification algorithm of QSI is based on the fact that the response of a LTI system can be decomposed as a sumation of exponential terms. If some of those exponentials terms are complex, in which case they are conjugate complex pairs, each pair forms a sinusoidal. We can represent the behavior of this type of systems in terms of exponential and sinusoidal components, in their response. Given that, we can make the following definitions:

$$E_{n_1}(t) = \sum_{1 \le i \le n1} C_i e^{-r_i t} \qquad (1)$$

Equation (1) represents a sum of n_1 exponential terms,

$$ES_{n_2}(t) = \sum_{1 \le i \le n2} C_i e^{-r_i t}\, sen\,(\omega_i t + \varphi) \qquad (2)$$

and Equation (2) represents a sum of n_2 decaying sinusoidal functions.

The previous definitions allow us to give a qualitative description of the behavior of the system from of the exponential and sinusoidal components. As a consequence, the response of a n-th order LTI system, could be expressed as in Equation (3).

$$y(t) = E_{n1}(t) + ES_{n2}(t) \qquad (3)$$

where $n_1 + 2n_2 = n$.

This result is evident from the definition of the Equations (1) and (2).

If the second term of the Equation (3) does not exist the response is non-oscilatory. Otherwise, it is a sinusoidal wave, where $En_1(t)$ represents its atractor, and $ESn_2(t)$ is a decaying sinusoidal component.

The algorithm separates the terms of Equation (3) to determine the structure (or qualitative form) of the system exhibiting the observed behavior. Separating the terms of the system's response is performed by a filtering process, which eliminates one component at a time, starting by the component with the highest frequency. Each time we eliminate one sinusoidal component, it is substracted from the remainder $Y^*(t)$, which initially contains the original response, with all its components. After the elimination of j sinusoidal components, the remainder is:

$$Y_j^*(t) = E_{n_1}(t) + ES_{n_2-j}(t) \qquad (4)$$

The elimination of components continues until the rest of the signal is non-oscillatory. The remainder signal, after extracting the oscillatory components, is a summation of exponential terms, which are also identified and filtered one by one. Figure (1) shows a simplified version

of the QSI algorithm (Flores05 et al). QSI determines the order of the system by adding the order of all eliminated components.

QSI(X)

$k \leftarrow 0$

$P \leftarrow 0$

$(X^*) \leftarrow TPAFilter(X, \mathrm{k}, P)$

$Modelo \leftarrow ExpFilter(X^*, \mathrm{k}, P)$

RETURN $Modelo$

Figure 1: QSI Algorithm.

There are two main functions in the QSI algorithm: TPAFilter and ExpFilter. The function TPAFilter eliminates the sinusoidal components and returns the order corresponding to those components, the remainder signal and the parameters of the eliminated sinusoidal. The function ExpFilter eliminates the exponential componentes and returns the order of the model and the parameters. At the end the remainder signal is not generally zero, it can have components of white noise with zero mean, or some other type of noise. It is considered that this noise doesn't have some statistical meaning, since all the components of the response have been eliminated at this time.

The QSI algorithm adds two units to the order of the system for each eliminated oscillatory component and one for each eliminated exponential component.

At the same time that we eliminate each component, we isolate it to determine its parameters (quantitative or parametric identification), i.e., the coefficients of the ODE that models the observed system.

QSI determines the simplest LTI system capable of exhibiting the observed behavior. Equation (5) shows the form of the ODE obtained by QSI, that models a LTI system.

$$\frac{d^n x(t)}{dt^n} + C_{n-1}\frac{d^{n-1}x(t)}{dt^{n-1}} + \cdots + C_1\frac{dx(t)}{dt} + C_0 x(t) = 0 \qquad (5)$$

3 PROBLEM FORMULATION

The observation process is carried out by means of the frozen-time method. The main assumption underlying the frozen-time approach is the existence of two time-scales. During the systems identification period, the transient dynamics time-scale is faster than the variation of the componentes of the observed system. That means, seen from the time-scale of the dynamic transients, the systems' components remain constant;

on the other hand, from the time-scale in which the system's components change, transients are instantaneous. The frozen time approach allows us to view the system, and therefore to perform its identification, as time-invariant, during the transients. On each the frozen-time instants the system's transient response is captured and it is processed by QSI to obtain a LTI model. This process repeats for several consecutive moments, producing a set of differential equations that describes the behavior of the system for each instant of those frozen-times.

These ODEs describe the changes the system undergoes through time. These changes are perceived as variations in the coefficients of the ODEs that describe the system in the different instants of time. With this set of ODEs we can form a matrix of coefficients that will allow us to observe their trends (see Table 1).

Table 1: Structure of coefficients Matrix.

FT	Coefficients of QSI			
1	$C_{n_1} = 1$	$C_{(n-1)_1}$	$\ldots$	C_{0_1}
$\vdots$	$\vdots$	$\vdots$	$\ldots$	$\vdots$
k	$C_{n_k} = 1$	$C_{(n-1)_k}$	$\ldots$	C_{0_k}

The first column of Table 1 represents the frozen-time instants and the remainder columns the variation of the coefficients though time.

One way to characterize slowly time-varying systems is by an ODE, where the coefficients are functions of time. Thus, linear time-varying systems are charaterized by Equation (6).

$$C_n(t)\frac{d^n x}{dt^n} + C_{n-1}(t)\frac{d^{n-1}x}{dt^{n-1}} + C_1(t)\frac{dx}{dt} + C_0(t)\,x = 0 \tag{6}$$

where

$$C_i(t) = \sum_{j=0}^{D_i} p_{i,j} t^j \tag{7}$$

and D_i is the highest degree of the polynomial that can model the variations of the i-th coefficient. In other words, $a_i(t)$ is a polynomial of degree D_i.

The problem we are addressing in this paper can be stated as: given a sequence of observations of the transient behavior of the system at times $t_1, \ldots, t_k$, determine a model of the system that includes the variation of the parameters.

4 THE SYSTEMS IDENTIFICATION PROCEDURE

Building models using QSI and the frozen-time approach involves three basic elements; data, set of models, and functions that fit the time-varying coefficients. The data set are the time series captured from transient responses observed at each frozen instant. The excitation signal with those that QSI can work are: impulse, step and sinusoidal functions. The set of models is obtained from processing this set of time series through QSI. The polynomials approach (see Equation (7)) are determined according with the algorithm of the Figure 2.

```
QSITimeVarying(Data)
    [k, N] ← size(Data)
    do
        M[i] ← QSI(Data[i])
    until   i = k
    do
        TVmodel ← GeneticAlg(M)
        r ← valida(TVmodel)
    until   r ≥ 0.9
    RETURN TVmodel
```

Figure 2: Time-varying system identification algorithm.

This algorithm works on data organized in a $k \times N$ matrix where k is the number of frozen-instants and N represents the size of the time series that captured the dynamics of each transient response for each frozen instant. Table 2 shows the organization of the data.

Table 2: Input data for QSITV.

Instant	System's Response		
1	$x_{(1,1)}$	$\ldots$	$x_{(1,N)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	$x_{(k,1)}$	$\ldots$	$x_{(k,N)}$

QSI identifies the models for each frozen instant, recording the obtained models with the structure of the Table 1. The columns of this Table describe the behavior in time of each coefficient of the characteristic equation. This set of models is processed through a genetic algorithm to determine the function that best describes the behavior of the coefficients, i.e., we are identifying the functions

$C_{n-1}(t), \ldots, C_0(t)$ of Equation (6).

We estimate the functions that approach these variations by means of genetic algorithms (Pastor). The scheme of the Genetic Algorithm used was: The Breeder Genetic Algorithm (MuhDirk93), this optimization technique offers following advantages: it can maintain several potentials solutions in paralel, it has a better chance of getting a global optima, and its computational complexity is $O(n)$, i.e., the complexity is linear. This method performs an optimization process in such a way that it adjusts the behavior of the coefficients to polynomial functions. The first approach of the functions is made with second order polynomials, if the approach does not satisfy the criterion of a correlation coefficient $r > 0.90$ then the procces repeats

increasing the degree of the polynomial until the criteria is met.

The validation procces is performed in the following way; we evaluate the functions $C_{n-1}(t)$,...,$C_0(t)$ with $t = \{t_1, t_2, \ldots, t_k\}$. With these evaluations we obtain a vector of results for each C_i. For each one of these vectors we compute the correlation coefficient, r_i, with their corresponding column in Table 1. Now we define cr as the correlation coefficient average, and we compute it as the average of the $r_i's$.

The output of the algorithm is a Table with the coeficients that best fits the observed data.

The aplication of this algorithm allow us to obtain the coefficients of Equation (6). The next section presents one application case of this methodology; the problem is the identification of atransmision line experiment.

5 RESULTS

In order to illustrate this algorithm, we use a laboratory experiment representing a transmission line. In this experiment we simulate the aging of the line by increasing its resistance.

This experiment was performed in a power systems laboratory. The experiment consists in capturing the transient effect in a transmission line during the disconnection of the load, see Figure 3(a).

The disconnection of the load is equivalent to apply an inverted step excitation.

The equipment used was an experimental console LabVolt with an AC source of 20 volts; for capturing the transient data we used the acquisition card of National Instruments NI PCI 5112, (100 MHz, 100 MS/s 8-Bit Digitizer). The model used in this test is the π model of the transmission line, this single-phase transmission line is shown in Figure 3(b). The values for the elements of this model were; $Vs = 20v$, $C_1 = 1.017\mu F$, $C_2 = 0.967\mu F$, $L = 29.65mH$, and R varies as shown in Equation (8).

$$R(t) = 0.0415t + 0.386 \tag{8}$$

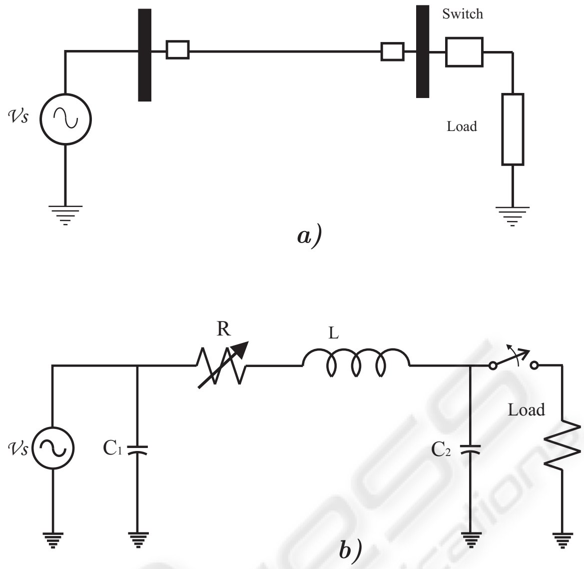

Figure 3: a) Single-phase transmission line, b) Equivalent circuit.

We used these laboratory devices to simulate a transmission line exhibiting the effects of aging. Every two seconds R was adjusted (simulated by a variable resistor), the transmission line was powered, and the load disconnected for four cycles (approx 70 msec). The disconnection transient effect was recorded. This experiment takes 21 seconds approximately, during which the transient response corresponding to each desconexion of the load is captured. During the experiments, the transient was recorded by measuring the voltage in C_2.

During data acquisition in an experiment, data are not generally in good shape to be processed, and therefore it becomes necessary to pretreat them to eliminate noise and other components that can affect the identification process. The frequency of the noise is generally bigger than the modes of the system. In the carried out experiments, the typical range of frequencies of the system was between 100 and 300 Hz, while the noise range was above the 900 Hz. If the noise overlaps the frequencies of the system, the identification process will see it as a characteristic of the response; this situation cannot be avoided, since there is no way to distinguish between componentes in the same frequency range, where some are genuine componentes and others are noise components. Figure 4 shows the acquired signal and the detail shows the result of the filtering process.

Once the captured signals were filtered, we use the algorithm shown in Figure 2 to process the signal. Table 3 shows the matrix of coefficients produced by QSI.

The π model of a transmission line expressed as an

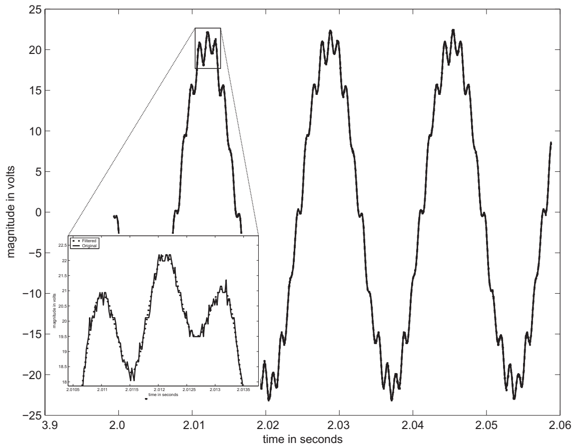

Figure 4: Acquired signal and detail of filtering process.

Table 3: Coefficients of a transmission line experiment.

t	$C_2(t)$	$C_1(t)$	$C_0(t)$
2	1	11.33780349	32147873.02
4	1	11.63152897	31834739.03
6	1	11.92525445	32076947.89
8	1	12.5127054	31982739.73
10	1	12.80643088	32087805.38
12	1	13.9813328	31982739.78
14	1	15.15623471	31955757.71
16	1	17.50603855	32018490.09
18	1	22.79309717	32087809.44
20	1	35.71701824	32018492.74

ODE is shown in equation (9)

$$L(t)\,C(t)\,\frac{d^2 v_{c_2}}{dt^2} + R(t)\,C(t)\,\frac{dv_{c_2}}{dt} + v_{c_2} = v_s \quad (9)$$

therefore

$$C_2(t) = \frac{L(t)\,C(t)}{L(t)\,C(t)} = 1 \quad (10)$$

$$C_1(t) = \frac{R(t)\,C(t)}{L(t)\,C(t)} = \frac{R(t)}{L(t)} \quad (11)$$

$$C_0(t) = \frac{1}{L(t)\,C(t)} \quad (12)$$

The genetic algorithm must determine the functions for $R(t)$, $L(t)$ and $C(t)$ in such a way that Equation (13) is minimized.

$$dif(t) = \left(\frac{R(t)}{L(t)} - C_1(t)\right) + \left(\frac{1}{L(t)\,C(t)} - C_0(t)\right) \quad (13)$$

Following the procedure described in section 4, the genetic algorithm first tests fitness with second degree polynomials. As the approach provided by the second degree polynomials is not enough to give a good fit to the data the algorithm tests with a polynomials of different degree, the degree

is increased until the fitness criteria is met. This process is carried out repeatedly until achieving a good fit.

The polynomials that best describe the behavior of the data of Table 3 are the following:

$$\begin{aligned} R(t) &= 0.000474t^4 - 0.006177t^3 + \cdots \\ &\quad +0.026557t^2 - 0.02843t + 0.268339 \end{aligned} \quad (14)$$

$$\begin{aligned} L(t) &= -0.0000001t^4 - 0.0000003t^3 + \cdots \\ &\quad +0.0000002t^2 - 0.0000574t + 0.0232831 \end{aligned} \quad (15)$$

$$\begin{aligned} C(t) &= -0.000015x10^{-6}t^4 + 0.0004x10^{-6}t^3 - \\ &\quad -0.003x10^{-6}t^2 + 0.01x10^{-6}t + 1.33x10^{-6} \end{aligned} \quad (16)$$

As we can observe in the functions given by Equations (15) and (16), the coefficients of the terms from the first to the fourth order are small compared to the independent term.

That is to say, these terms do not contribute significantly in the evaluation of their respective functions. In practical terms we can assume that those functions are constant.

Figure 5 shows the signal observed in a frozen-instant, k, and their corresponding simulated signal after the identification process. We compute the coefficients using Equations (14), (15), and (16) evaluated at instant k.

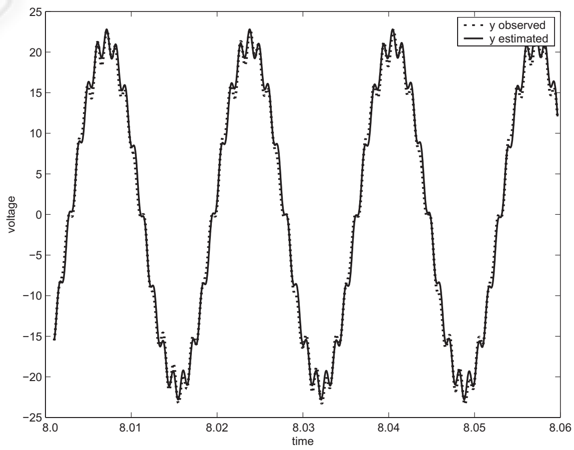

Figure 5: Comparing between observed response and estimated model response.

The correlation coefficient between the model estimated and the expected model is $r = 0.9988$. i.e., the model that we estimate reproduces the dynamics of the system appropriately.

6 CONCLUSIONS

In this paper, a new identification algorithm for slowly time-varying systems has been proposed. The algorithm is based on the QSI and frozen-time approaches. We pretreated the signal using a low-pass filter to eliminate the inherent noise to the laboratory measurements. The example of aplication of this method was satisfactory, since the model can reproduce the dynamics of the system with great accuracy. The algorithm has been extensively tested with synthetic examples (i.e. simulations) using matlab and simulink, and also with real laboratory measurements. The validation tests of the estimated models are acceptable, with correlation coefficients very near to unity.

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REFERENCES

C. A. Desoer. ”Slowly varying discrete systems $x_{i+1} = A_i x_i$,” Electronics Letters 6, pp. 339-340, 1970.

J.E. Dennis Jr. and Schnabel Robert B. ”Numerical Methods for Uncostrained Optimization and Nonlinear Equations”. Classics in Applied Mathematics. SIAM. 2000.

Flores Juan and Pastor Nelio. ”Qualitative and Quantitative Systems Identification for Linear Time Invariant Dynamic Systems”, Workshop on Qualitative Reasoning 2002 Sitges Barcelona España. Junio 2002.

Pohlheim Hartmut. “”Evolutionary Algorithms: Principles, Methods and Algorithms”” , document part of the Genetic and Evolutionary Algorithm Toolbox for use with MatLab. http://www.geatbx.com. November 2001.

Le Yi Wang. ”Identification and Control of Slowly Varying Systems: Recent Advances”; Proceedings of the 34th Conferece on decision & control, New Orleans, LA December 1995.

Lenart Ljung. ”System Identification: Theory for the user”, 2nd Edition Upper Saddle River, NJ: Prentice Hall 1999.

M. Freedman and G. Zames. ”Logarithmic variation criteria for stability of systems with time-varying gains”, SIAM J. Control vol. 6,N$^\circ$ 3. pp. 487- 507. 1968.

Mahmood Samavat and Ali Jabar Rashidie. ”A new Algorithm for Analisys and Identification of Time Varying Systems”, Proceedings of the American Control Conference, Seattle Washington, June 1995

Mühlenbein H. Schlierkamp-Voosen D., ”Predictive Models for the Breeder Genetic Algorithm: I. Continuos Parameter Optimization”, Evolutionary Computation 1(1):25-49, 1993.

Pastor G. Nelio. ”Identificación de Sistemas Usando Algoritmos Genéticos”, Masters Thesis. Facultad de Ingeniería Eléctrica, Universidad Michoacana de San Nicolás de Hidalgo, Morelia, México. 2000.

Pastor G. Nelio. ”Identificación de Sistemas Dinámicos Variantes e Invariantes en el Tiempo Basada en las Características Cualitativas de la Respuesta Transitoria”, Doctoral Thesis. Facultad de Ingeniería Eléctrica, Universidad Michoacana de San Nicolás de Hidalgo, Morelia, México. 2005.

Flores R. Juan and Pastor G. Nelio. ”Time-Invariant Dynamic Systems Identification Based on the Qualitative Features of the Response”. Engineering Applications of Artificial Intelligence Volume 18, Issue 6, Pages 719-729. doi:10.1016/j.engappai.2005.01.003 September 2005.

Smith Steven W. ”The Scientist and Engineer’s Guide to Digital Signal Processing”, Second Edition, California Technical Publishing, San Diego California 1999.