# Levy Flights in the Stochastic Dynamics of Robot Swarm Gathering

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**Abstract.** We consider the problem of gathering a swarm of robots which is initially randomly dispersed over a domain in the plane. A stochastic method for the cooperative control of a swarm of mobile robots is presented. The network of mobile robots is modeled by a swarm performing a directed random walk. The swarm dynamics are governed by a system of stochastic difference equations. The motion is controlled by a robot leader, which transmits the coordinates of the gathering point to the swarm as the network cooperative control signal. We study the case where the control signal is corrupted by noise and find that the gathering process is robust to noise and efficient. The swarm dynamics display anomalous diffusion and Levy flights, where the robots move along straight lines over many time steps, followed by short random walks in the vicinity of the gathering point.

## 1 Introduction

The term Levy flight was introduced by Mandelbrot and is described in his book on fractals [6] as a sequence of jumps separated by stopovers. In plates 296 and 297 in his book, he gives the example where each stopover is a star, a galaxy or a cluster of stars or galaxies, thus showing that the global structure of matter distribution in the universe is composed of clusters separated by Levy flights. The clusters themselves can be decomposed into self-similar miniclusters, resulting in a fractal structure. Since then, other phenomena have been described as displaying Levy flights and anomalous diffusion. In the present work, we show that controlled swarm robotic motion is displaying anomalous diffusion and Levy flights.

Robots are used in many practical applications such as industrial robots in manufacturing, spacecraft and rover robots for space exploration and unmanned air vehicles (UAVs) for reconnaissance, surveillance and tactical military missions. Other possible applications include underwater missions by autonomous underwater vehicles (AUVs) such as formation control and rendezvous, search and rescue missions and exploration and mapping of unknown environments. In many applications, single robots are employed in the performance of a given task. It has been recognized for some time, however, that the use of collaborating multiple mobile robots can have significant advantages in achieving complex tasks and missions, which otherwise might not be achievable with single robots. Consequently, in recent years, there has been an interest in the

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cooperative control of networked collaborating mobile robots with distributed resources such as sensors, computing power and communications [2, 10, 8].

Consider the problem where of a group of mobile robots has been dispersed in a given area, and that it is required to gather the robots in the vicinity of a given point. For example, consider the case where a group of robots has to perform a mission in a remote area, where they have landed by parachutes. The robots are now randomly scattered in a wide area and need to be gathered into a much smaller area in the vicinity of a designated location before starting their mission. The specific task now is for the robots to collectively move towards the gathering point. We consider swarms on the order of $200$ robots dispersed in a two-dimensional domain on the order of 1 km by 1 km.

It is assumed that each autonomous robot is equipped with a compass and is capable of moving in a given azimuthal direction for a given distance. Each robot has a low level control and navigation system that can detect its location at all times and guide it from one point in the domain to the next at the right speed and orientation. It is also assumed that each autonomous robot is equipped with a collision and obstacle avoidance control system for preventing collisions with other robots and obstacles. The robots network architecture consists of a leader robot acting as a server and communicating with the other robots as clients.

The robot swarm cooperative control method is described in the next section. Each robot has a microprocessor computing device on board capable of running the robot swarm algorithm. We propose to use this paradigm algorithm as a top level discrete event controller for the cooperative control of the swarm. Each robot sends the best solution found at any given time to the leader or other central processing station through its communication channel. The leader in turn computes the global best solution and transmits the result as a control signal to the network. The Robot Swarm Optimization (RSO) is a stochastic population based method that belongs to the class of biologically inspired algorithms. It is based on the paradigm of a swarm of insects performing a collaborative task such as ants or bees foraging for food using chemical or some other type of communication, see for example [1] and [3]. The swarm intelligence method was originally developed by [4] and later described in great detail in [5]. An overview of the method as extensively applied to various function optimization problems of increasing difficulty has recently been presented by [7]. Here the PSO method is and adapted for use as a top level discrete event cooperative control method for a swarm of autonomous robots.

In the next section we develop the robot swarm algorithm with communication noise and we explain how it can be applied to solve the swarm gathering problem. In section 3, results of simulations are described for a swarm of 200 robots, gathering in a noisy environment. We show that the robots trajectories follow Levy flights and compute the probability distribution for the flights lengths.

## 2 Cooperative Control of the Robot Swarm

In developing the robot swarm cooperative control method, we incorporate physical effects or constraints in order to implement the search method by actual mobile robots such as land vehicles, autonomous underwater vehicles or autonomous unmanned aerial vehicles. The first effect imposes a limitation on the speed of the vehicle, or equivalently, a limit on the distance $\Delta X_{max}$ it can move in a given typical time step $\Delta t$. Another effect taken into account is imperfect and noisy communication between the robots. At any given time, communication with one or more robots can be attenuated or corrupted by noise. Therefore, rather than assuming that the global minimum is available to the swarm at all times as in the case of perfect communication, we introduce noise in the control signal transmitted to all members of the swarm.

The robot swarm cooperative control algorithm without any robot speed constraints and with perfect communication consists of minimizing a function of several variables:

minimize $f(X)$, where $X \in \Omega \subset R^n$ and $f : \Omega \mapsto R$

subject to the side constraints

$$X_{min} \leq X \leq X_{max}$$

using a directed random walk process described by the following system of stochastic difference equations:

$$X^i(k+1) = X^i(k) + \Delta X^i(k+1) \tag{2.1}$$

$$\Delta X^i(k+1) = w(k)X^i(k) + c_1 r_1^i(P^i(k) - X^i(k)) + c_2 r_2^i(P^g(k) - X^i(k)) \tag{2.2}$$

Here $k$ is the discrete time counter, $c_1$ and $c_2$ are real constants, $r_1^i$ and $r_2^i$ are random variables uniformly distributed between 0 and 1. The superscript index $i$ denotes robot number $i \in [1, N_R]$ where $N_R$ is the number of robots in the swarm. The location $P^i(k)$ is the best solution found by robot $i$ at time $t = k$ and $P^g(k)$ is the global minimum at time $t = k$. The factor $w(k)$ can be either constant or time dependent. If it decreases with time, the search process can usually be improved as the search approaches the global minimum and smaller steps are needed for better resolution. For example, the parameter $w(k)$ can be set to decrease from an initial value of $w_0 = 0.8$ to a final value of $w_f = 0.2$ after $N$ time steps:

$$w(k) = w_f + (w_0 - w_f)(N - k)/N \tag{2.3}$$

The system of equations (2.1-2.2) describes a directed random walk for each robot $i$ in the swarm, similar to a Brownian motion of a tracer particle in a fluid. Whereas Brownian motion is an undirected random motion, the motion of a robot in the swarm will have a velocity that will start as a random motion, but will eventually decay as the particle approaches a point $P^i(k)$ in the domain where the function reaches a local minimum and as the swarm as a whole approaches a point $P^g(k)$ of the domain where the function reaches a global minimum, that is,

$$P^i(k) = argmin\{f(X^i(k))\}$$

$$P^g(k) = argmin\{f(P^i(k))\},\ i \in [1, N_R] \tag{2.4}$$

The following initial conditions are needed in order to start the solution of the system of difference equations

$$X^i(0) = X_{min} + r^i \Delta X_{max} \tag{2.5}$$

$$\Delta X_{max} = (X_{max} - X_{min})/N_x \tag{2.6}$$

$N_x$ is a typical number of grid segments along each component of the position vector $X$. For example, if the domain consists of a two dimensional square domain of 1000 m by 1000 m, then with $N_x = 50$, we can use a typical distance segment of $\Delta X_{max} = 1000$ m/$N_x = 20$ m. If we take a typical speed of an autonomous robot as $V_c = 1$ m/s, then the typical time will be $t_c = \Delta X_{max}/V_c = 20$ s. We can now measure $X$ in units of $\Delta X_{max}$, $V$ in units of $V_c$ and $\Delta t$ in units of $t_c$. The equations will then have exactly the same form in non-dimensional variables.

Placing a limit on the magnitude of the velocity component of each robot in any given direction for a given time step, we can impose a constraint on the magnitude of the distance traveled in any time step as:

$$|\Delta X^i(k+1)| < \Delta X_{max} \tag{2.7}$$

Under these assumptions, the equations of motion of the swarm become:

$$X^i(k+1) = X^i(k) +$$

$$sign(\Delta X^i(k+1))(min[|\Delta X^i(k+1)|, \Delta X_{max}]) \tag{2.8}$$

$$\Delta X^i(k+1) = w(k)X^i(k) + c_1 r_1^i (P^i(k) - X^i(k)) +$$

$$+ c_2 r_2^i (P^g(k) - X^i(k)) \tag{2.9}$$

subject to the side constraint

$$X_{min} \leq X^i(k+1) \leq X_{max} \tag{2.10}$$

The signum function term sign($\Delta X^i(k+1)$) is added in order to keep the original direction of the motion while reducing the length of the step.

## 3 Swarm Gathering in a 2-D Domain

The cooperative control method described in the previous section is applied to the problem of gathering a swarm of robots at a given point in the plane. We consider a two dimensional domain $\Omega \subset R^2$, defined by the coordinates:

$$X_1 \in [X_{1min}, X_{1max}] = [-500, 500]$$

$$X_2 \in [X_{2min}, X_{2max}] = [-500, 500] \tag{3.1}$$

which forms a square of 1000 m by 1000 m, with the origin at the center of the square. We choose the number of grid segments as $N_x = 50$, so that the maximum distance traveled by any robot in any direction $X_1$ or $X_2$ in one time step is 20 m, which we choose as one distance unit or 1 DU. The equivalent time unit $\Delta t$ =TU= 20 s is the time it takes a robot to travel along 1 DU at a typical speed of 1 m/s.

$$\Delta X_{1max} = \Delta X_{2max} =$$

$$= (X_{1max} - X_{1min})/N_x = 20m = 1DU$$

$$|V_1|_{max} = |V_2|_{max} \leq \Delta X_{1max}/\Delta t = DU/TU = 1m/s \tag{3.2}$$

Initially, the swarm is randomly distributed in the domain $\Omega$ or in a subset domain of $\Omega$. At time $k = 0$, the control is started and the swarm is set in motion. Each robot in the swarm is programmed to minimize its distance from the gathering point , by minimizing the function:

$$f(X_1^i, X_2^i) = (X_1^i - P_1^g)^2 + (X_2^i - P_2^g)^2 \tag{3.3}$$

Here the control signal $P^g = (P_1^g, P_2^g)$ transmitted to each member of the swarm specifies the gathering point and $(X_1^i, X_2^i)$ is the location of the $i$th robot in the swarm, where $i \in [1, N_R]$. The communication signal is corrupted by additive noise $\eta$:

$$P^g = (P_1^g + \eta, P_2^g + \eta) \tag{3.4}$$

$$\eta = \mathcal{N}(0, \sigma) \tag{3.5}$$

where $\mathcal{N}(0, \sigma)$ denotes random numbers having a normal distribution with zero mean and standard deviation $\sigma$. Without loss of generality, we choose the gathering point at the origin, i.e., $(P_1^g, P_2^g) = (0, 0)$.

For the Gaussian noise $\eta$ we chose a standard deviation $\sigma = \delta \Delta X_{1max}$ with $\delta = 5$. As the noise level is increased, say with values of $\delta = 10, 20, 30$,it becomes more difficult to gather all the swarm at the origin. The other parameters appearing in the equations of motion are $c_1 = c_2 = 2$ and $w_0 = w_f = 0.8$. The results of a simulation of the gathering of the 200 robots are given in Figs. 1-6. The simulation was run for N= 80 time steps. Fig.1 shows the locations of the robots as they were spread randomly over the domain at the start of the simulation. It also shows the locations after 10 time steps, after 15 time steps and the locations of the swarm as the robots gathered in the vicinity of the origin after 80 time steps.

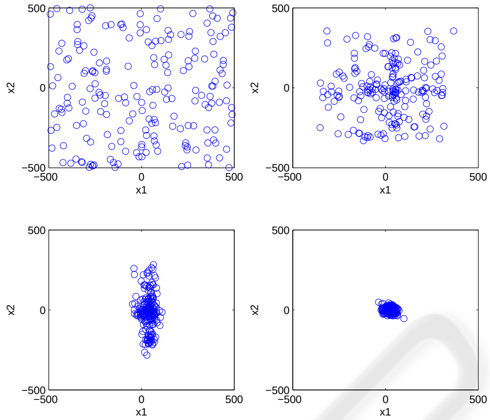


**Fig. 1.** Swarm gathering stochastic process. Top left: Initial random swarm distribution in the domain. Top right: After 10 time steps. Bottom left: After 15 time steps. Bottom right: After 80 time steps.

The trajectories of the first 50 robots in the swarm are shown in Figs. 2-4. Fig.2 displays the coordinates $X_1(t)$ as a function of time. The coordinates $X_2(t)$ as a function of time are shown in Fig.3. Most Levy flights occur at the beginning of the gathering motion, up to about 30 time steps. After that the swarm aggregates in the vicinity of the gathering point. This can also be seen in Fig.4, which displays the radial distances $r(t)$ from the origin, where $r^2(t) = X_1^2(t) + X_2^2(t)$ for the first 50 robots in the swarm.

In order to obtain the probability distribution of the Levy flights, the lengths of flights along straight lines are followed for each robot in the swarm. Then the number of flights for each given length are counted for all the robots in the swarm and put in 25 bins ordered from the shortest to the longest flights. Then a histogram is plotted showing the frequency of occurence of the various flight lengths. Such a histogram is shown in Fig. 5. The histogram does not follow a Gaussian distribution, but rather a Levy distribution, which has a very long tail and an infinite variance.

Levy flights follow power laws of the form

$$N = (L/L_0)^{\alpha} \tag{3.6}$$

which appear as straight lines with slope $\alpha$ when displayed on a log-log scale

$$logN = \alpha log(L/L_0) = \alpha logL - \alpha logL_0 \tag{3.7}$$

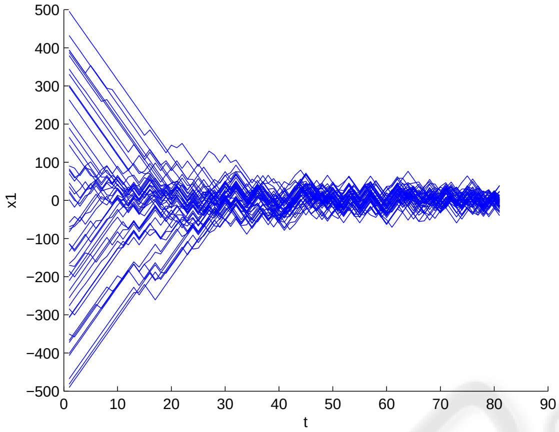


**Fig. 2.** Trajectories $X_1(t)$ of the first 50 robots in the swarm.

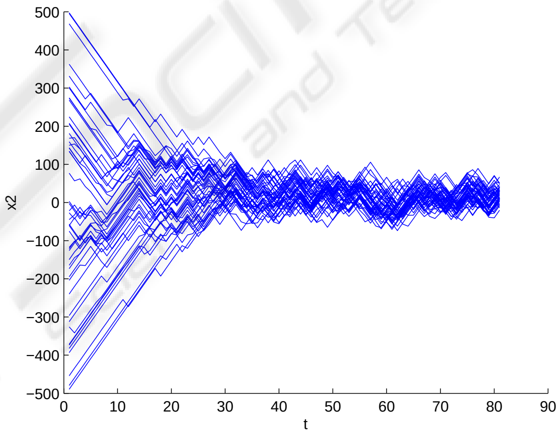


**Fig. 3.** Trajectories $X_2(t)$ of the first 50 robots in the swarm.

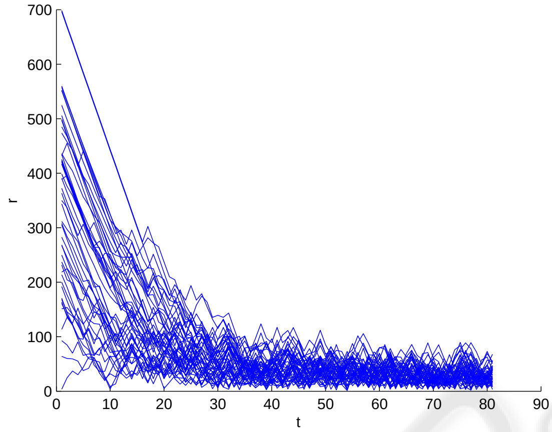


**Fig. 4.** Radial distances r(t) of the first 50 robots in the swarm.

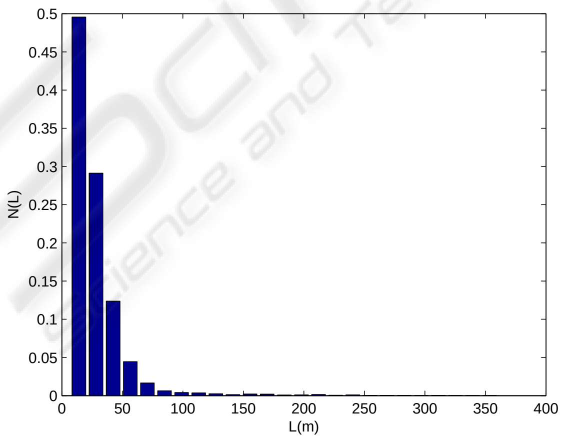


**Fig. 5.** Probability distribution of the Levy flights for the 200 robots.

where $N$ is the number of flights of length $L$ and $L_0$ is a characteristic length. For the noise level described above, a value of $\alpha = -2.446$ and a characteristic length $L_0 = 11.62$ were obtained. Such a plot is shown in Fig.6.

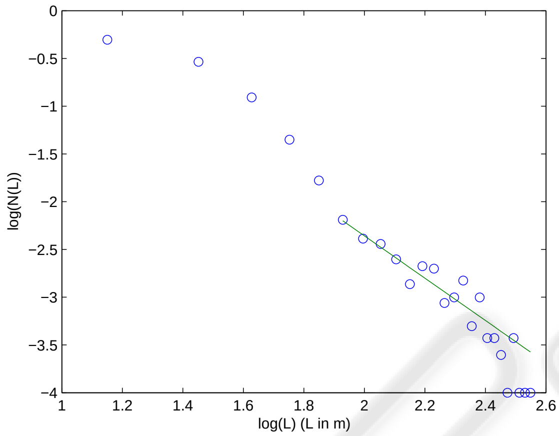


**Fig. 6.** Power law of the Levy flights for the tail of the distribution.

## 4 Conclusions

A method for the cooperative control of a group of robots based on a stochastic model of swarm motion has been developed. The network of mobile robots is modeled by a swarm moving randomly in the search domain with the global motion of the swarm directed and controlled by a central unit which can be a leader robot or a central server. The motion of each robot in the swarm is governed by a system of two stochastic difference equations. Usually, in the robot swarm method developed in this work, the best solution found collectively by the swarm serves as the control signal for the network of robots. However, in the swarm gathering problem, the problem is simpler, since the coordinates of the gathering point, which serve as the control signal, are fixed, except for additive noise that is present in the communication system. The method was used to solve the basic problem of collaborative gathering in a two-dimensional domain. It was found that the swarm can gather successfully in the vicinity of a designated point in the plane despite significant noise in the network communications. Moreover, it was found that the gathering process is efficient, in the sense that the robots trajectories exhibit anomalous diffusion, performing long distance Levy flights along straight lines, followed by local sticking random walks in a limited area of the domain in the vicinity of the gathering point.

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