NONPARAMETRIC STATISTICAL LEVEL SET SNAKE BASED ON THE MINIMIZATION OF THE STOCHASTIC COMPLEXITY

P. Martin[1,2], Ph. Réfrégier[1], F. Galland[1], F. Guérault[2]
[1] *Physics and Image Processing group, Fresnel Institute, UMR CNRSTIC 6133*
Université Paul Césanne Aix-Marseille III, EGIM 13397 Marseille France.
[2] *Simag Développement, 2 allée Sacoman 13016 Marseille France.*

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Abstract: In this paper, we focus on the segmentation of objects not necessarily simply connected using level set snakes and we present a nonparametric statistical approach based on the minimization of the stochastic complexity (Minimum Description Length principle). This approach allows one to get a criterion to optimize with no free parameter to be tuned by the user. We thus propose to estimate the probability law of the gray levels of the object and the background of the image with a step function whose order is automatically determinated. We show that coupling the probability law estimation and the segmentation steps leads to good results on various types of images. We illustrate the robustness of the proposed nonparametric statistical snake on different examples and we show on synthetic images that the segmentation results are equivalent to those obtained with a parametric statistical technique, although the technique is non parametric and without *ad hoc* parameter in the optimized criterion.

1 INTRODUCTION

An important goal of computational vision and image processing is to automatically recover the shape of objects from various types of images. The first snakes (Kass et al., 1988) were driven by the minimization of a function in order to move them towards desired features, usually edges. They are well adapted to a certain class of problems, but they can fail in the presence of strong noise although several improvements and reformulations have been proposed to overcome these limitations. An other strategy consists in considering not only the edges, but also the inner and the outer regions defined by the active contour (Leclerc, 1989; Ronfard, 1994). Different statistical region-based formulations have been proposed in (Figueiredo and Leitão, 1992; Storvik, 1994) and some of their advantages have been illustrated in (Jain et al., 1996; Germain and Réfrégier, 1996).

Concerning the region approaches, the contour is deformed in order to minimize a criterion which is the sum of two terms : the external energy which is based on the gray levels of the image and on a statistical model, and the internal energy which allows one to regularize the contour. It has been shown (Figueiredo et al., 2000; Ruch and Réfrégier, 2001; Martin et al., 2004) that one can determine the external energy for a given probability density function (pdf) model and the complexity of the contour model with the minimum stochastic complexity (Rissanen, 1989) approach.

Nevertheless, even if the pdf models (which belong to the exponential familly) proposed in (Chesnaud et al., 1999; Ruch and Réfrégier, 2001; Martin et al., 2004) allow one to deal with many aplications (radar images, low photon flux, ...), the user may not know the physical origin of the image. More important, some pdf can not be easily described by a parametric pdf model. To overcome these limitations, the authors propose in (Kim et al., 2002) a nonparametric statistical approach by estimating the pdf with the Parzen window (Parzen, 1962) method. However, the width σ_P of the Gaussian kernel needs to be tuned in this approach. If σ_P is too small, the histogram estimation could be noisy and if σ_P is too large, it could be too smooth.

We thus propose to simultaneously segment the image with a level set snake and estimate the pdf of the object and the background with a step function whose order is automatically determined.

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2 MINIMUM STOCHASTIC COMPLEXITY APPROACH

2.1 General Model

Our purpose is to segment an image $\mathbf{s} = \{s(x,y)|(x,y) \in [1,N_x]\times[1,N_y]\}$ with $N_x \times N_y$ pixels, assumed to have been quantized in Q levels such as $\mathbf{s} \in [1,Q]$ with $Q \in \mathbb{N}$ (for example, $Q = 256$) and composed of two regions Ω_t and Ω_b (not necessarily simply connected) denominated the target and the background regions. The target's gray levels $\mathbf{t}$ (with N_t pixels) and the background gray levels $\mathbf{b}$ (with N_b pixels) are assumed to be spatially uncorrelated and independently distributed (bold font symbols will denote vectors). Furthermore, their respective pdf will be noted P^t and P^b.

With statistical region-based snakes, the criterion which has to be optmized in order to obtain the final contour Γ can be determined by minimizing the stochastic complexity of the image (Minimum Description Length principle) (Figueiredo et al., 2000; Ruch and Réfrégier, 2001; Martin et al., 2004). In that case, one has to determine the sum of the number of bits needed for the description of the data with an appropriate dictionary and of the number of bits needed to describe the dictionary (Rissanen, 1989). For snake segmentation, considering the image $\mathbf{s}$ and a contour Γ, the stochastic complexity is the sum of 3 terms

$$\begin{aligned}\Delta(\mathbf{s},\Gamma,P^t,P^b) = \;& \Delta_S(\mathbf{s},\Gamma,P^t,P^b) \\ & +\Delta_P(\Gamma,P^t,P^b) + \Delta_C(\Gamma),\end{aligned} \tag{1}$$

where $\Delta_S(\mathbf{s},\Gamma,P^t,P^b)$ is the number of bits needed fog the description of the gray levels of the image $\mathbf{s}$ with a given contour Γ and a given dictionary for coding the gray levels, $\Delta_P(\Gamma,P^t,P^b)$ is the number of bits needed to code the dictionary and $\Delta_C(\Gamma)$ is the number of bits needed to describe the contour Γ. In the following, the number of bits will be measured in nats, or in other words, the natural logarithm will be considered ($1bit = log(2)nats$).

2.2 Determination of the Stochastic Complexity and Estimation of the PDF

Our goal is to estimate the contour Γ assuming the pdf of the object and the background gray levels are nuisance parameters. We thus propose to demonstrate that a simple pdf estimation technique can lead to efficient results for the estimation of Γ. Let us consider the estimation of the pdf in each region Ω_t and Ω_b with an irregular step function $\widehat{P}_q^u(s)$ ($u = t,b$) of order q defined as follow

$$P_q^u(s) = \sum_{j=1}^{q} p_u(j) R_j(s), \tag{2}$$

with $u = \{t,b\}$, $R_j(s) = 1$ if $s \in [a_j, a_{j+1}[$ and $R_j(s) = 0$ otherwise where $a_j \in [1,Q]$, $a_j > a_i$ if $j > i$, $a_{q+1} = Q+1$ and $a_1 = 1$. The step function is irregular since the different steps are not necessarily of equal length.

The Maximum Likelihood estimated of $p_u(j)$ is $\widehat{p}_u(j) = \frac{N_u(j)}{b_j Nu}$ where $b_j = a_{j+1} - a_j$, N_u is the number of pixels in Ω_u and $N_u(j)$ is the number of pixels in Ω_u such as $s \in [a_j, a_{j+1}[$. One can show that, for $\widehat{P}_q^u = \sum_{j=1}^{q} \widehat{p}_u(j) R_j(s)$ and Γ given, the number of nats needed to code the gray levels of $\mathbf{s}$ is given by

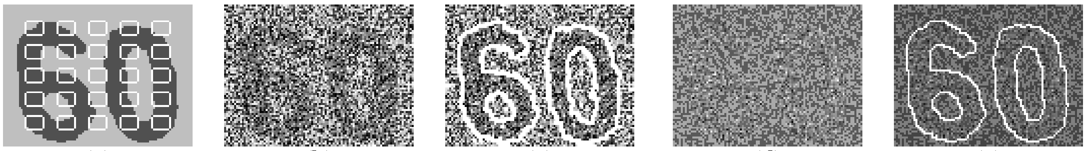
$$\Delta_S(\mathbf{s},\Gamma,\widehat{P}_q^t,\widehat{P}_q^b) = -\sum_{u\in\{t,b\}} \sum_{j=1}^{q} N_u(j) log\left[\frac{N_u(j)}{b_j N_u}\right]. \tag{3}$$

The number of nats required to code the dictionary for the gray levels can be approximated by

$$\Delta_P(\Gamma,\widehat{P}_q^t,\widehat{P}_q^b) = -\sum_{u}\sum_{j=1}^{q} log\left(\frac{N_u(j)}{N_u}\right) + \sum_{j=1}^{q} log(1+b_j). \tag{4}$$

In the following the contour will be described with a level set snake which allows one to segment objects not necessarily simply connected. It has been shown (Martin et al., 2004) that in that case, the number of nats needed to code the contour can be approximated by $\Delta_C^{LS}(\Gamma) = log(8)|\Gamma|$, where $|\Gamma|$ is the length in pixel units of the contour. These expresions of $\Delta_S(\mathbf{s},\Gamma,\widehat{P}_q^t,\widehat{P}_q^b)$, $\Delta_P(\Gamma,\widehat{P}_q^t,\widehat{P}_q^b)$ and of $\Delta_C^{LS}(\Gamma)$ allow one to obtain $\Delta(\mathbf{s},\Gamma,P_q^t,P_q^b)$ (see eq. 1) which is the free parameter criterion to minimize in order to get the level set snake convergence.

The contour Γ, the order q of the step function, the values of a_j and $\widehat{p}_u(j)$ are estimated by minimizing the stochastic complexity $\Delta(\mathbf{s},\Gamma,P^t,P^b)$. More precisely, a first segmentation (i.e. estimation of Γ) is performed with $q = Q$ which corresponds to $a_j = j$. Then, the adjacent steps $R_j(s)$ and $R_{j+1}(s)$ whose fusion leads to the largest decrease of the stochastic complexity criterion are merged and Γ is again deformed to optimize the stochastic complexity criterion. The optimal estimation of q and of the a_j correspond to the values which minimize the stochastic complexity. This criterion thus does not include any free parameter.



(a) (b) (c) (d) (e)

Figure 1: (a) Synthetic image without noise (72×96 pixels) with initial contour. (b) Noisy version of image (a), pdf of the background and object gray levels are shown in Fig. 2. (d) Noisy version of image (a), gray levels of the object region are distributed with a Gaussian pdf and gray levels of the background have a bi-valuated pdf with the same mean and variance values as those of the object region. (c), (e) Final contour respectively obtained on image (b) and (d) with the proposed approach.

3 EXPERIMENTAL RESULTS

We show in Fig.1c a result of segmentation on an image quantized with $Q = 256$ levels. The noisy image in Fig.1b has been generated with a step function of order $q = 4$ for the pdf models of the object and the background. One can see the histograms in Fig.2a and Fig. 2b in solid line. The minimum value of the stochastic complexity is obtained for $q = 4$ (Fig.2a and Fig.2b in dotted line) which corresponds to the true value. In Fig.1e, we show the segmentation result on an image with a Gaussian noise on the object region and a bi-valuated noise on the background region chosen in order to get the same mean and variance values in both regions.

An example of application of this approach to a RGB image is shown in Fig.3a. The final contour Γ is obtained on the saturation component S in HSV representation. One can see in Fig.3c the final contour on a real SAR image corrupted with speckle noise. In Fig. 4, one can see a segmentation result on a textured image. In order to get homogeneous regions, the image $\mathbf{s}$ has been preprocessed to obtain the new image $\mathbf{f}$ defined by $f(x, y) = F * s(x, y)$ where F is the Roberts filter (Jain, 1989) defined with 3×3 pixel neighborhoods and $*$ is the convolution operator.

4 ROBUSTNESS OF THE PROPOSED TECHNIQUE

Let us analyse in this section the efficiency of the proposed nonparametric approach pdf estimation based on irregular step functions. For that purpose, we compare the results obtained either with this approach or with a parametric modelization of the actual pdf of the background and object gray levels which will be distributed with a pdf which belongs to the exponential family (Martin et al., 2004).

The results shown in Fig.5 demonstrate that the proposed approach leads to segmentation results equivalent to those obtained with a parametric model for the pdf which is adapted to the noise which describe the gray level fluctuations. On the other hand, if an inadapted parametric model is used for the pdf, one can see that poor results can be obtained. It has been shown (Goudail et al., 2003) for polygonal (Ruch and Réfrégier, 2001) and level set snakes (Martin et al., 2004) that the Bhattacharyya distance (Cover and Thomas, 1991), defined by $\mathcal{B} = -\log \int \sqrt{P^t(z)P^b(z)}dz$, is an appropriate measure of contrast between the target and the background.

We have thus determined the average number of misclassified pixels (ANMP) obtained as a function of $\mathcal{B}$ on the images described in Fig.5 for the previous discussed parametric and nonparametric approaches. The number of misclassified pixels is determined from the final contour by counting the pixels which belong to the true target shape but lie outside the contour, and those which belong to the true background but lie inside the contour. Both approaches lead to equivalent ANMP (with a precision lower than 1.7%) when $\mathcal{B} \geq 0.29$. The parametric statistical approach leads to lower ANMP than the proposed nonparametric statistical technique only when $\mathcal{B} < 0.29$.

5 CONCLUSION

We have proposed a nonparametric statistical level set snake based on the minimization of the stochastic complexity. We have demonstrated that this approach leads to good segmentation results when the pdf of the object and the background are approximated by a step function whose values and order are estimated. This approach leads to minimize a criterion without free parameter to be tuned by the user. We have seen that this technique provides good results on SAR, video (color) and textured images. Moreover, we have shown that the segmentation results of the proposed approach are closed to those obtained with a parametric statistical approach but with a better robustness.

There exists different perspectives to this work. It will be interesting to generalize this technique to a multi region approach and to take into account possible spatial correlations.

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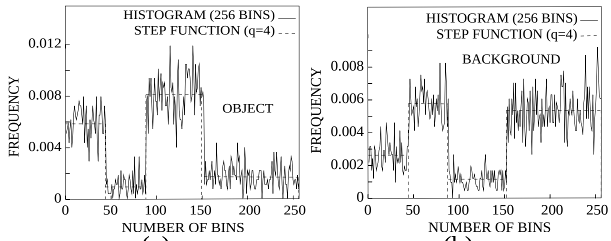
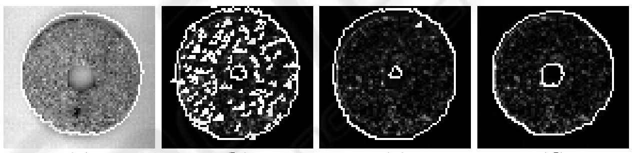


(a) (b)

Figure 2: (a), (b) Histograms (solid line) and estimated pdf (dotted line) of the object and the background of Fig.1b.

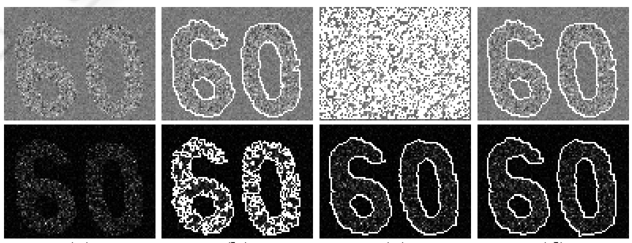
(a) (b) (c)

Figure 3: (a) RGB image acquired with a CCD camera (231×228 pixels) with the initial contour. (b) Segmentation result obtained on the saturation component S in HSV representation with the proposed approach. (c) Segmentation result obtained on a SAR image (97×127 pixels) of an agricultural area obtained by the ERS-1 satellite (distributed by the ESA and provided by the CNES). A multi-initialization analogous to the one of Fig.3a has been used.



(a) (b) (c) (d)

Figure 4: Segmentation of an image (73×69 pixels) acquired with a CCD camera. (a) Segmentation result with a Gaussian model for the pdf. Results of the segmentation on a preprocessed version of (a) obtained with different models for the pdf : (b) Gaussian, (c) Gamma, (d) the proposed approach. A multi-initialization has been used.



(a) (b) (c) (d)

Figure 5: (a) Synthetic images (72×96 pixels) perturbated with Gaussian (first line) and Gamma (second line) noises with $\mathcal{B} = 0.29$. Segmentation results obtained with a Gaussian and Gamma models for the pdf (Martin et al., 2004) and the proposed approach are shown respectively in (b), (c) and (d). The initial contour is the one of Fig.1.