ON THE FORCE/POSTURE CONTROL OF A CONSTRAINED SUBMARINE ROBOT

Ernesto Olguín-Díaz and Vicente Parra-Vega
Robotics and Advanced Manufacturing, CINVESTAV-IPN, Saltillo, México

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Abstract: An advanced, yet simple, controller for submarine robots is proposed to achieve simultaneously tracking of time-varying contact force and posture, without any knowledge of its dynamic parameters. Structural properties of the submarine robot dynamic are used to design passivity-based controllers. A representative simulation study provides additional insight into the closed-loop dynamic properties for regulation and tracking cases. A succinct discussion about inherent properties of the open-loop and closed-loop systems finishes this work.

1 INTRODUCTION

In the last decade, we have witness a surprising leap on scientific knowledge and technological achievements for submarine robots, from a simple torpedo to modern UAV. Those vehicles are at the same time tantamount scientific and technological challenges to robotics, control and mechatronic researchers. First efforts for free motion submarine robots were focused for motion control, with sensing capabilities, similar to the case of fix-base robots in our labs. Of course, stable contact for submarine robots is a more complex problem in comparison to the typical force control problem of robot manipulators fixed to ground because not only due complementary complex dynamics such as buoyancy and added masses, but to de fact that the vehicle reference frame is not longer inertial.

However, some tasks requires more challenging goal of establishing stable contact, which leads us to study the problem of simultaneous force and pose (position and orientation) control of submarine robots. When the submarine robot achieves contact, like pushing itself against a wall or polishing a sunken surface vessel or manipulating tools on submarine pipe lines, forces are presented, and little is known about these contact forces, let alone exploit them either for design or control. The main general reason that this problem remains rather open is that we really know so little about, on one hand, in how to model and control properly a fully immersed vehicle constrained by rigid object; and on the other hand, the submarine force control technology lies behind system requirements, such as very fast sampling rates, even when the bandwidth of the submarine robot is very low.

Despite brilliant control schemes for free motion submarine robots in the past few years, in particular those of (Yoerger, 1985; Smallwood, 2001; Smallwood, 2004) those schemes does not guarantee formally convergence of tracking errors, let alone simultaneous convergence of posture and contact force tracking errors, there are several results that suggest empirically that a simple PD control structure behaves as stiffness control for submarine robots. So far the simultaneous convergence of time-varying contact forces and posture remains an open problem for the PD-like controller (notice that for submarine robots the role of model-free controllers are very important because it is very hard to known exactly the dynamic model and its dynamic parameters).

Recently, some efforts have focused on how to obtain simple control structures to control the time-varying pose of the UAV under the assumption that the relative velocities are low. For force control, virtually none complete control system has been published. However, to move forward more complex force controllers, we believe that more understanding of the structural properties of submarine robots in stable contact to rigid objects are required.

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To understand more about this problem, we show that the open-loop rigid dynamics of submarine robots subject to holonomic constraints (rigid contact) exhibits similar properties of fixed-base robots. Then almost any force/position control scheme with the proper adequation can be used. In this paper we have chosen the orthogonalization principle (Parra-Vega and Arimoto, 1996) to propose a simple controller with advanced tracking stability properties.

1.1 Contribution

Despite the complexity of the problem, in this paper, the performance of a robot submarine carrying out a very simple contact tasks under a novel orthogonalization-based model-free controller is analyzed and discussed, with emphasize on structural properties. The closed-loop shows robust tracking of time-varying contact force and posture, without any knowledge of submarine dynamics. The proposed controller stands itself as a new controller which guarantees simultaneous tracking of submarine robot with a model-free decentralized control structure.

1.2 Organization

Section 2 introduces the nonlinear differential algebraic equations of the robot submarine dynamics in error coordinates. Section 3 presents the control design. Simulation data is presented for a Testbed robot of IFREMER in 4, and finally section 5 discusses some structural properties useful for further understand robot submarine in contact tasks. Conclusions are stated in Section 6.

2 THE MODEL

2.1 A Free Floating Submarine

The model of a submarine can be obtained via Kirchhoff formulation (Fossen, 1994), the inclusion of hydrodynamic effects such as added mass, friction and buoyancy and the account of external forces/torques like contact effects (Olguín-Díaz, 1999). The model is then expressed by the next set of equations:

$$M_v\dot{\nu} + C_v(\nu)\nu + D_v(\cdot)\nu + g_v(q) \; = \; u + F_c^{(v)} \qquad (1)$$

$$\nu \; = \; J_\nu(q)\dot{q} \qquad (2)$$

From this set, (1) is called the dynamic equation while (2) is called the kinematic equation. Both had been used long widely to express the dynamical behavior of free moving object in the 3D space.

The generalized coordinates vector $q \in \mathbb{R}^6$ is given on one hand by the 3 Cartesian positions x, y, z of the origin of the submarine frame (Σ_v) with respect to a inertial frame (Σ_0) fixed in some place on the surface of the Earth; and on the other hand by any set of attitude parameters that represent the rotation of the vehicle's frame with respect to the inertial one. Most common sets of attitude representation such a Euler angles, in particular roll-pitch-yaw (ϕ, θ, ψ), use only 3 variables (which is the minimal number of orientation variables). Then, for a submarine vehicle, the generalized coordinates $q = (x_v, y_v, z_v, \phi_v, \theta_v, \psi_v)$ represents its 6 degrees of freedom.

The vehicle's velocity $\nu \in \mathbb{R}^6$ is the vector representing both linear and angular velocity of the submarine expressed in its own vehicle's frame. This vector is then defined as $\nu = (\mathbf{v}_v^{(v)^T}, \omega_v^{(v)^T})^T$. The relationship between this vector and the generalized coordinates is given by the kinematic equation. The linear operator $J_\nu(q) \in \mathbb{R}^{6\times 6}$ in (2) is built by the concatenation of two transformations. The first is defined as:

$$J_q(q) = \begin{bmatrix} I & 0 \\ 0 & J_\theta(\phi_v, \theta_v, \psi_v) \end{bmatrix} \in \mathbb{R}^{6\times 6}$$

Where $J_\theta \in \mathbb{R}^{3\times 3}$ is an operator that converts time derivatives of attitude (orientation) parameters in angular velocity: $\omega_v^{(0)} = J_\theta(\theta_v)\dot{\theta}_v$, with $\theta_v = (\phi_v, \theta_v, \psi_v)^T$ defined as the orientation parameter vector. The operator $J_q(q)$ maps the first time derivative of the generalized coordinates to linear and angular velocities of the submarine vehicle, both expressed in the inertial frame Σ_0.

The second operator transforms a 6 dimension tensor from the inertial frame to vehicle's frame by $J_R(q) = diag\{R_0^v, R_0^v\} \in \mathbb{R}^{6\times 6}$. Thus the *Jacobian* of the vehicle used in the kinematic equation is a linear operator defined as:

$$J_\nu(q) = J_R^T(q) J_q(q)$$

In the dynamic equation (1), matrices $M_v, C_v(\nu), D_v(\cdot) \in \mathbb{R}^{6\times 6}$ are respectively Inertia matrix, Coriolis matrix and Damping matrix. M_v includes the terms of classical inertia plus the hydrodynamic coefficients of the added mass effect (due to the amount of extra energy needed to displace the surrounding water when the submarine is moving). This Inertia matrix is constant, definite positive and symmetric only when the submarine is complete immersed and the relative water incidence velocity is small (Fossen, 1994). This condition is met for most common activities of a ROV, for example during sampling recollection in not shallow waters. The Coriolis vector $C_v(\nu)\nu$ represents the Coriolis forces, gyroscopic terms of the classical inertia effects plus the quadratic velocity terms induced by the added mass. The Coriolis matrix in this representation does not depend on the position but only on the velocity, in contrast to the same expression for a Robot Manipulator (Spong M.W., 1989). However it fulfills the classic relationship for Lagrangian systems: $\dot{M}_v - 2C_v(\nu) = Q; Q + Q^T = 0$, and is indeed skew symmetric. The Damping matrix represents all the hydrodynamic effects of energy dissipation. For that reason it is a strictly positive definite matrix, $D_v(q,\nu,t) > 0$ (Fossen, 1994). Its arguments are commonly the vehicle's orientation ϕ_v, θ_v, ψ_v, the generalized velocity ν, and the velocity of the surrounding water. The diagonal components represents the drag forces while the off-diagonal components represent the lift forces.

Finally, vectors $g_v(q), u, F_c^{(v)} \in \mathbb{R}^6$ are all force wrenches (force-torque vector) in the vehicle's frame. They represent respectively: gravity, input control and the contact force. Gravity vector includes buoyancy effects and it does not depend on velocity but on the orientation (attitude) of the submarine with respect to the inertial frame. The contact force wrench is the one applied by the environment *to* the submarine. The input control are the forces/torques induced by the submarine thrusters in the vehicle frame.

The dynamic model (1)-(2) can be rearranged by replacing (2) and its time derivative into (1). The result is one single equation model:

$$M_q(q)\ddot{q} + C_q(q,\dot{q})\dot{q} + D_q(\cdot)\dot{q} + g_q(q) = u_q + \tau_c \quad (3)$$

which has the form of any Lagrangian system. Its components fulfills all properties of such systems i.e. definite positiveness of inertia and damping matrices, skew symmetry of Coriolis matrix and appropriate bound of all components (Sagatun, 1991).

The control input in this equation is obtained by a linear transformation of the real input using the linear operator given by the kinematic equation:

$$u_q = J_\nu^T(q)u. \quad (4)$$

This last equation becomes important because it relates the nominal control signal u_q to be designed in the general coordinates space with the real physical input u that shall be implemented in the real vessel. The contact effect is also obtained by the same transformation. However it can be expressed directly from the contact wrench in the inertial frame (Σ_0) by the next relationship

$$\tau_c = J_\nu^T(q)F_c^{(v)} = J_q^T(q)F_c^{(0)} \quad (5)$$

where the contact force $F_c^{(0)}$ is the one expressed in the inertial frame. By simplicity it will be noted as F_c from this point further. The relationship with the one expressed in the vehicle's frame is given by $F_c = J_R^T(q)F_c^{(v)}$. This wrench represents the contact forces/torques exerted by the environment to the submarine as if measured in a non moving (inrtial) frame. These forces/torques are given by the normal force of an holonomic constraint when in contact and the friction due to the same contact. For simplicity in this work, tangential friction is not considered.

2.2 Contact Force Due to a Holonomic Constraint

A holonomic constraint can be expressed as a function of the generalized coordinates of the submarine as

$$\varphi(q) = 0 \quad (6)$$

with $\varphi(q) \in \mathbb{R}^r$. The number r stand for the number of contact points between the submarine and the motionless rigid object. Stable contact appears only when the robot submarine does not deattach from the object modeled by $\varphi(q) = 0$. In this case, the time derivatives of this constraint are zero

$$\frac{d^n\varphi(q)}{dt^n} = 0; \ \forall\, n \geq 1 \quad (7)$$

For $n = 1$ the last constraint becomes:

$$\dot{\varphi}(q) = J_\varphi(q)\dot{q} = 0 \quad (8)$$

where $J_\varphi(q) = \frac{\partial\varphi(q)}{\partial q} \in \mathbb{R}^{r\times n}$ is the constraint jacobian. Last equation means that velocities of the submarine in the directions of constraint jacobian are restricted to be zero. This directions are then normal to the constraint surface $\varphi(q)$ at the contact point. As a consequence, the normal component of the contact force has exactly the same direction as those defined by $J_\varphi(q)$. Then the contact force wrench can be expressed as

$$F_c = J_{\varphi+}^T(q)\lambda \quad (9)$$

where $J_{\varphi+}(q) \triangleq \frac{J_\varphi}{\|J_\varphi\|}$ is a normalized version of the constraint jacobian; and λ is the magnitude of the normal contact force at the origin of vehicle frame: $\lambda = \|F_c\|$.

The free moving model expressed by (1)-(2), when in contact with the holonomic constraint can be rewritten as:

$$\begin{aligned} M_v\dot{\nu} + h_v(q,\nu,t) &= u + J_R^T(q)J_{\varphi+}^T(q)\lambda && (10)\\ \nu &= J_\nu(q)\dot{q} && (11)\\ \varphi(q) &= 0 && (12) \end{aligned}$$

where $h_v(q,\nu,t) = C_v(\nu)\nu + D_v(q,\nu,t)\nu + g_v(q)$.

Equivalently, the model (3) is also expressed as

$$M_q(q)\ddot{q}+h_q(q,\dot{q},t) \;=\; u_q+J_{\varphi}^T(q)\lambda \tag{13}$$

$$\varphi(q) \;=\; 0 \tag{14}$$

with $h_q(q,\dot{q},t)=C_q(q,\dot{q})\dot{q}+D_q(q,\dot{q},t)\dot{q}+g_q(q)$ and $J_{\bar{\varphi}}(q)=J_{\varphi+}(q)J_q(q)$. Equations (13)-(14) are a set of Differential Algebraic Equations index 2 (DAE-2). To solve them numerically, a DAE solver is required. This last representation has the same structure and properties as those reported in (Parra-Vega, 2001).

2.3 Numerical Considerations

To compute the value of λ, the constrained Lagrangian that fulfils the constrained movement, can be calculated using the second derivative of (7): $\ddot{\varphi}(q)=0$. Then using the dynamic equation (either of them) and after some algebra its expression becomes either:

$$\lambda \;=\; \left[J_{\varphi}J_{\nu}^{-1}M_{\nu}^{-1}J_R^TJ_{\varphi+}^T\right]^{-1}\Big(J_{\varphi}J_{\nu}^{-1}M_{\nu}^{-1}\left(h_{\nu}-u\right) - \frac{d}{dt}\left(J_{\varphi}J_{\nu}^{-1}\right)\nu\Big) \tag{15}$$

$$= \left[J_{\varphi}M_q^{-1}J_{\bar{\varphi}}^T\right]^{-1}\Big(J_{\varphi}M_q^{-1}\left(h_q-u_q\right)-\dot{J}_{\varphi}\dot{q}\Big) \tag{16}$$

The set of equations (10)-(11)-(15) or the set (13)-(16) describes the constrained motion of the submarine when in contact to infinitely rigid surface described by (6). Numerical solutions of these sets can be obtained by simulation, however the numerical solution, using a DAE solver, can take too much effort to converge due to the fact that these sets of equation represent a highly stiff system. In order to minimize this numerical drawback, the holonomic constraint has been treated as a numerically compliant surface which dynamic is represented by

$$\ddot{\varphi}(q)+D\dot{\varphi}(q)+P\varphi(q)=0 \tag{17}$$

This is known in the force control literature of robot manipulators as constrained stabilization method, which bounds the nonlinear numerical error of integration of the backward integration differentiation formula. With a appropriate choice of parameters $P\gg 1$ and $D\gg 1$, the solution of $\varphi(q,t)\to 0$ is bounded. This dynamic is chosen to be fast enough to allow the numerical method to work properly. In this way, it is allowed very small deviation on the computation of λ, typically in the order of -10^6 or less, which may produce, according to some experimental comparison, less than 0.001% numerical error. Then, the value of the normal contact force magnitude becomes:

$$\lambda \;=\; \left[J_{\varphi}J_{\nu}^{-1}M_{\nu}^{-1}J_R^TJ_{\varphi+}^T\right]^{-1}\Big(J_{\varphi}J_{\nu}^{-1}M_{\nu}^{-1}\left(h_{\nu}-u\right) -\frac{d}{dt}\left(J_{\varphi}J_{\nu}^{-1}\right)\nu-DJ_{\varphi}J_{\nu}^{-1}\nu-P\varphi(q)\Big) \tag{18}$$

$$= \left[J_{\varphi}M_q^{-1}J_{\bar{\varphi}}^T\right]^{-1}\Big(J_{\varphi}M_q^{-1}\left(h_q-u_q\right)-\dot{J}_{\varphi}\dot{q} -DJ_{\varphi}\dot{q}-P\varphi(q)\Big) \tag{19}$$

It can be pointed out that equation (19) compute the contact force magnitude using the generalized coordinates variables only. From this equation, desired force can be easily obtained. Also note that the preliminary inverse operator is only expressed by the inertia matrix inverse and the two constraint jacobians. The first is the gradient of the contact surface, while the second is the normalized gradient with a simple kinematic variation given by the operator J_q.

3 CONTROL DESIGN

The introduction of a so called Orthogonalization Principle has been a key in solving, in a wide sense, the force control problem of a robot manipulators. This physical-based principle states that the projection (dot product) of contact forces and joint generalized velocities is zero. Then passivity arises from torque input to generalized velocities. To preserve passivity, then, the closed-loop system must satisfy the passivity inequality for controlled generalized error velocities. This is true for robot manipulators with fixed frame, and here we prove that this also happens with robot whose reference frame is not inertial (fixed).

3.1 Orthogonalization Principle and Linear Parametrization

Similar to (Liu et al., 1997), an orthogonal projection of $J_{\varphi}(q)$ arises onto the tangent space at the contact point. This tangent space can be calculated by the definition of the following operator

$$Q(q)\stackrel{\triangle}{=}I_n-J_{\varphi}^T(q)J_{\varphi}(q)\in\mathbb{R}^{n\times n} \tag{20}$$

where I_n is an identity matrix of proper size. Notice that $rank\{Q(q)\}=n-r$ since $rank\{J_{\varphi}(q)\}=r$. Also notice that $Q\dot{q}=\dot{q}$ due to the definition of Q and (8). Therefore, and according to the Orthogonalization Principle the integral of $(\tau,\dot{q})$ is upper bounded by $-\mathcal{H}(t_0)$, for $\mathcal{H}(t)=K+P$, and passivity arise for fully immersed submarines without inertial frame. Therefore, passivity-based control is a powerful option to control such systems.

On the other hand it is known that equation (13) can be linearly parameterized as follows

$$M_q(q)\ddot{q}+C_q(q,\dot{q})\dot{q}+D_q(\cdot)\dot{q}+g_q(q)=Y(q,\dot{q},\ddot{q})\Theta \tag{21}$$

where the regressor $Y(q,\dot{q},\ddot{q}) \in \mathbb{R}^{n\times p}$ is composed of known nonlinear functions and $\Theta \in \mathbb{R}^p$ by p unknown but constant parameters.

3.2 Change of Coordinates

A variation Y_r of regressor Y can be designed such that

$$\begin{aligned} Y_r(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Theta \ = \ & \\ M_q(q)\ddot{q}_r+[C_q(q,\dot{q})+D_q(\cdot)]\,\dot{q}_r+g_q(q) \end{aligned} \tag{22}$$

where $(\dot{q}_r,\ddot{q}_r)$ are some time functions *yet to be defined*. Then the open loop equation (13) can be written by adding and subtracting (22) as

$$\begin{aligned} M_q(q)\dot{s} \ &= \ -[C_q(q,\dot{q})+D_q(\cdot)]\,s \\ &\quad -Y_r(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Theta+J_\varphi^T(q)\lambda+u_q \end{aligned} \tag{23}$$

where $s \triangleq \dot{q}-\dot{q}_r$ is called the extended error. The problem of control design for the open loop equation (23) is to find u_q such that exponential convergence arises when $Y_r\Theta$ is not available.

3.3 Orthogonal Nominal Reference

Consider that $q_d(t)$ and $\lambda_d(t)$ are the desired trajectories functions for position and contact force. Consider as well, the tracking errors defined as $\tilde{q} \triangleq q(t)-q_d(t)$ for position error while $\tilde{\lambda} \triangleq \lambda(t)-\lambda_d(t)$ stand for the force contact error. Consider also the next set of definitions:

$$S_p \ \triangleq \ \dot{\tilde{q}}+\sigma\tilde{q} \tag{24}$$

$$S_{dp} \ \triangleq \ S_p(t_0)e^{-\alpha(t-t_0)} \tag{25}$$

$$S_{qp} \ \triangleq \ S_p-S_{dp} \tag{26}$$

$$S_F \ \triangleq \ \int_{t_0}^{t}\tilde{\lambda}dt \tag{27}$$

$$S_{dF} \ \triangleq \ S_F(t_0)e^{-\eta(t-t0)} \tag{28}$$

$$S_{qF} \ \triangleq \ S_F-S_{dF} \tag{29}$$

with $\alpha>0$, $\eta>0$. Finally, consider the following reference $\dot{q}_r$ function:

$$\begin{aligned} \dot{q}_r \ &= \ Q\left(\dot{q}_d-\sigma\tilde{q}+S_{dp}-\gamma_1\int_{t_0}^{t} sgn\{S_{qp}(t)\}dt\right) \\ &\quad +\beta J_\varphi^T\left(S_F-S_{dF}+\gamma_2\int_{t_0}^{t} sgn\{S_{qF}(t)\}dt\right) \end{aligned} \tag{30}$$

where the parameters β, σ, γ_1 and γ_2 are constant matrices of appropriate dimensions; and $sgn(y)$ stands for the entrywise signum function of vector y,

Using (30) into s, it arises

$$s=Q(q)S_{vp}-\beta J_\varphi^T(q)S_{vF} \tag{31}$$

where the orthogonal extended position and force manifolds S_{vp} and S_{vF}, respectively, are given by

$$S_{vp} \ = \ S_{qp}+\gamma_1\int sgn(S_{qp}(\varsigma))d\varsigma \tag{32}$$

$$S_{vF} \ = \ S_{qF}+\gamma_2\int sgn(S_{qF}(\varsigma))d\varsigma \tag{33}$$

Notice that although the time derivative of $\dot{q}_r$ is discontinuous, it will not be used in the controller. In the next subsection we show that if $\dot{S}_{vp}$, $\dot{S}_{vF}$ are bounded, and for large enough γ_1,γ_2, then simultaneous tracking of contact force and posture is achieve.

3.4 Model-Free Second Order Sliding Mode Control

Consider the following nominal continuous control law:

$$\begin{aligned} u_q \ &= \ -K_d\left(Q(q)s_{vq}-\beta J_\varphi^T(q)s_{vF}\right) \\ &\quad +J_\varphi^T(q)\left(-\lambda^d+S_F\right) \\ &\quad +\gamma_2 J_\varphi^T(q)\left(\tanh(\mu S_{qF})+\eta\int \mathrm{sgn}(S_{qF})\right) \end{aligned} \tag{34}$$

with $\mu>0$ and $K_d=K_d^T>0,\in\mathbb{R}^{n\times n}$. This nominal control, designed in the q-space can be mapped to the original coordinates model, expressed by the set (1)-(2), using the next relationship

$$u=J_\nu^{-T}(q)u_q \tag{35}$$

thus, the physical controller u is implemented in terms of a key inverse mapping J_ν^{-T}.

3.4.1 Closed-loop System

The open loop system (23) under the continuous model-free second order sliding mode control (34) yields to

$$\begin{aligned} M_q\dot{s} \ &= \ -[C_q+D_q+K_d]\,s-Y_r\Theta \\ &\quad +J_\varphi^T\left(\tilde{\lambda}+\eta S_F\right)+\gamma_2 J_\varphi^T Z+\tau^* \\ &\quad +\gamma_2 J_\varphi^T\left(sgn(S_{qF})+\eta\int sgn(S_{qF})\right) \end{aligned} \tag{36}$$

where $Z = tanh(\mu S_{qF}) - sgn(S_{qF})$ and $\tau^* \equiv 0$ is the virtual control input for the passivity analysis.

3.4.2 Stability Analysis

Theorem 1 *Consider a constrained submarine (13)-(16) under the continuous model-free second order sliding mode control (34). The Underwater system yields a second order sliding mode regime with local exponential convergence for the position, and force tracking errors.*

Proof. A passivity analysis $\langle S, \tau^* \rangle$ indicates that the following candidate Lyapunov function V qualifies as a Lyapunov function

$$V = \frac{1}{2}(s^T M_q s + \beta S_{vF}^T S_{vF}) \tag{37}$$

for a scalar $\beta > 0$. The time derivative of the Lyapunov candidate equation immediately leads to

$$\begin{aligned} \dot{V} &= -s^T(K_d + D_q)s - \beta\eta S_{vF}^T S_{vF} - s^T Y_r \Theta \\ &\quad + s^T \gamma_2 J_\varphi^T Z \\ &\leq -s^T K_d s - \beta\eta S_{vF}^T S_{vF} + \|s\| \|Y_r \Theta\| \\ &\quad + \|s\| \|\gamma_2\| \|J_{\bar{\varphi}}\| \|Z\| \\ &\leq -s^T K_d s - \beta\eta S_{vF}^T S_{vF} + \|s\| \|\varepsilon\| \end{aligned} \tag{38}$$

where it has been used the skew symmetric property of $\dot{M} - 2C(q, \dot{q})$, the boundedness of both the feedback gains and submarine dynamic equation (there exists upper bounds for $M, C(q, \dot{q}), g(q), \dot{q}_r, \ddot{q}_r$), the smoothness of $\varphi(q)$ (so there exists upper bounds for J_φ and $Q(q)$), and finally the boundedness of Z. All these arguments establish the existence of the functional ε. Then, if K_d, β and η are large enough such that s converges into a neighborhood defined by ε centered in the equilibrium $s = 0$, namely

$$s \to \varepsilon \; as \; t \to \infty \tag{39}$$

This result stands for local stability of s provided that the state is near the desired trajectories for any initial condition. This boundedness in the $\mathcal{L}_\infty$ sense, leads to the existence of the constants $\varepsilon_3 > 0$ and $\varepsilon_4 > 0$ such that

$$|\dot{S}_{vp}| < \varepsilon_3, \tag{40}$$

$$|\dot{S}_{vF}| < \varepsilon_4 \tag{41}$$

An sketch of the local convergence of S_{vp} is as follows[1]. Locally, in the $n - r$ dimensional image of Q, we have that $S_{qp}^* = QS_{qp} \in \mathbb{R}^n$. Consider now that under an abuse of notation that $S_{qp} = S_{qp}^*$, such that for small initial conditions, if we multiply the derivative of S_{qp} in (32) by S_{qp}^T, we obtain

$$\begin{aligned} S_{qp}^T \dot{S}_{qp} &= -\gamma_1 |S_{qp}| + S_{qp}^T \dot{S}_{vp} \\ &\leq -\gamma_1 |S_{qp}| + |S_{qp}| |\dot{S}_{vp}| \end{aligned} \tag{42}$$

Substituting (40) into (42) yields

$$S_{qp}^T \dot{S}_{qp} \leq -(\gamma_1 - \varepsilon_3)|S_{qp}| \tag{43}$$

where γ_1 must be chosen such that $\gamma_1 > \varepsilon_3$. The equation (43) is precisely the condition for the existence of a sliding mode at $S_{qp}(t) = 0$. The sliding mode is established in a time $t \leq |S_{qp}(t_0)|/(\gamma_1 - \varepsilon_3)$, and according to the definition of S_{qp} (below (32)), $S_{qp}(t_0) = 0$, which simply means that $S_{qp}(t) = 0$ for all time. We see that if we multiply the derivative of (33) by S_{vf}^T, we obtain

$$\begin{aligned} S_{qF}^T \dot{S}_{qF} &= -\gamma_2 |S_{qF}| + S_{qF}^T \dot{S}_{vF} \\ &\leq -\gamma_2 |S_{qF}| + |S_{qF}| |\dot{S}_{vF}| \end{aligned} \tag{44}$$

substituting (41) into (44) yields

$$S_{qf}^T \dot{S}_{qf} \leq -(\gamma_2 - \varepsilon_4)|S_{qf}| \tag{45}$$

where γ_2 must be chosen such that $\gamma_2 > \varepsilon_4$. The equation (45) is precisely the condition for the existence of a sliding mode at $S_{qF}(t) = 0$. The sliding mode is established in a time $t \leq |S_{qF}(t_0)|/(\gamma_2 - \varepsilon_4)$ and, according to (33), $S_{qF}(t_0) = 0$, which simply means that $S_{qf}(t) = 0$ for all time, which simply implies that $\lambda \to \lambda_d$ exponentially fast.

4 SIMULATION RESULTS

Force control of submarine robots is in its infancy, therefore simulations study must be kept rather simple to enter into the intricacies of the numerical solutions. Later on, the complexity of the simulation study will be be increased such that underwater disturbances can include unmodelled submarine dynamics, such as lateral marine current with high Reynolds number, to mention one. Then, the simulation study has been made on a simplified platform of a real submarine. Data has been obtained from the Vortex system of IFREMER (IFREMER, 1992). Simulator presents only vertical planar results (only in the x-z plane), so the generalized coordinates for this case of study are:

$$q = \begin{pmatrix} x_v \\ z_v \\ \theta_v \end{pmatrix} \tag{46}$$

The vehicle velocities are

$$v = \begin{pmatrix} u_v \\ w_v \\ q_v \end{pmatrix} \tag{47}$$

[1]The strict analysis follows Liu, *et. al.*

where u_v and w_v are linear velocities (surge and heave) and q_v is the angular velocity in the x-z plane. The holonomic constraint is given by a vertical surface located at 2 meters from the origin. This is expressed as:

$$\varphi(q) \equiv x - 2 \tag{48}$$

Initial conditions were calculated to be at the contact surface with no force. Simulations with simple PD were also performed as a comparison tool. The model-free control parameters are as follows:

α	β	γ_1	γ_2
4	0.025	0.0025	10^{-3}
K_d	σ	η	μ
$200\hat{M}_v$	5	1000	1

where $\hat{M}_v$ is made by the diagonal values of the constant Inertia matrix when expressed in the vehicle's frame. For de PD the gains were defined as $K_p = 100\hat{M}_v$ and $K_d = 20\hat{M}_v$. The numerical dynamic induced in the constraint surface were performed with $P = 9x10^6$ and $D = 36x10^3$.

4.1 Set-Point Control Case

The task is very simple. It consist on a change of deep, from 1m to 1.5m, and orientation from 5 degrees to 0, while the submarine remains in contact with a constant contact force of 170N.

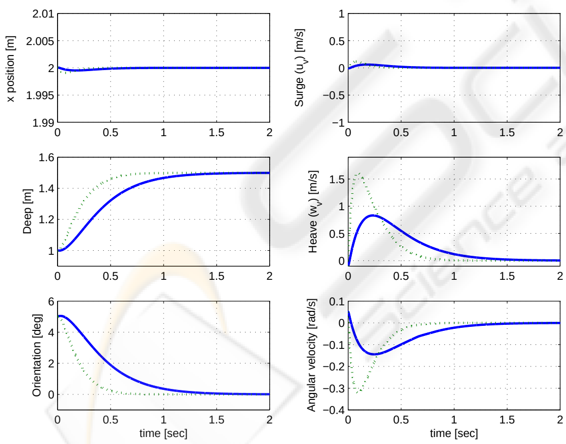

Figure 1: Generalized coordinates q and vehicle velocities ν, in vehicle's frame (continuous line for model-free second order sliding mode control, dotted line for PD control).

Figures 1 and 2 shows respectively position, velocity and force inputs. The difference in settling time has been explicitly computed in order to visualize the qualitative differences. In any case, this time interval can be modified with appropriate tuning of gain parameters. Notice that there is some transient and

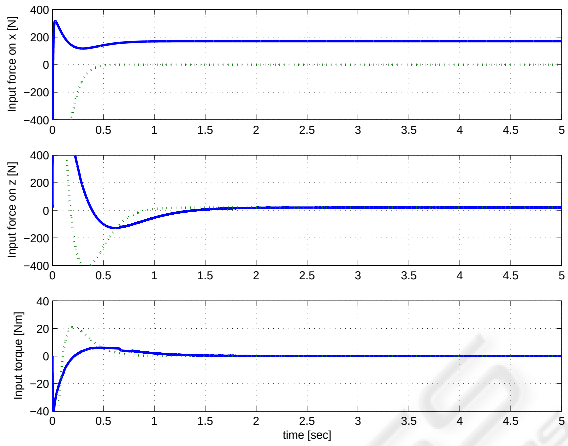

Figure 2: Inputs controlled forces u, in vehicle's frame (continuous line for model-free second order sliding mode control, dotted line for PD control).

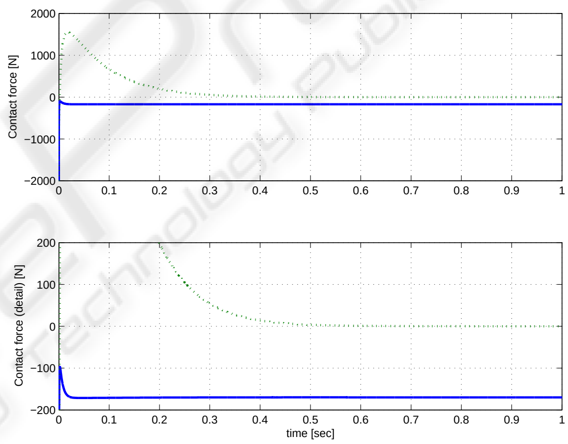

Figure 3: Contact force magnitude λ (continuous line for model-free second order sliding mode control, dotted line for PD control).

variability in the position of the contact point in the the direction normal to that force (the x coordinate). This is more visible in the velocity curve where velocity in the x direction should not occur if the constraint is infinitely rigid. The same transients are present in the force graphic where a noticeable difference between simple PD and model-free second order sliding mode control is present. The big difference is that although PD can regulate with some practical relative accuracy it is not capable to control force references, either constant nor variable. Figure 3 shows the transient of the contact force magnitude λ (only the 1st second is shown). The second window shows with more detail the transient in this contact force due to the numerical modification. In this graphic is is clear that no force is induced with the PD scheme. In the

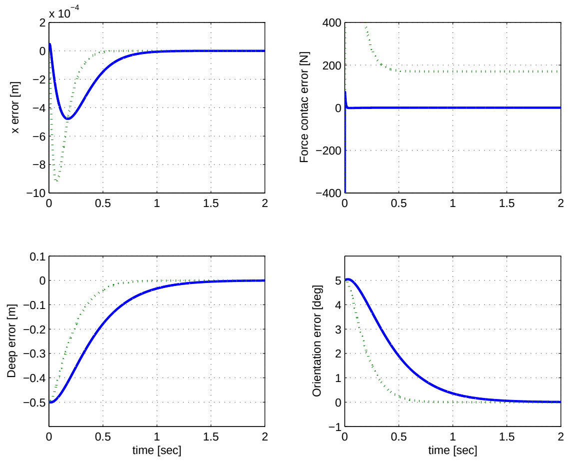

Figure 4: Set-point control errors: $\tilde{q} = q(t) - q^d$ and $\tilde{\lambda} = \lambda(t) - \lambda^d$ (continuous line for model-free second order sliding mode control, dotted line for PD control).

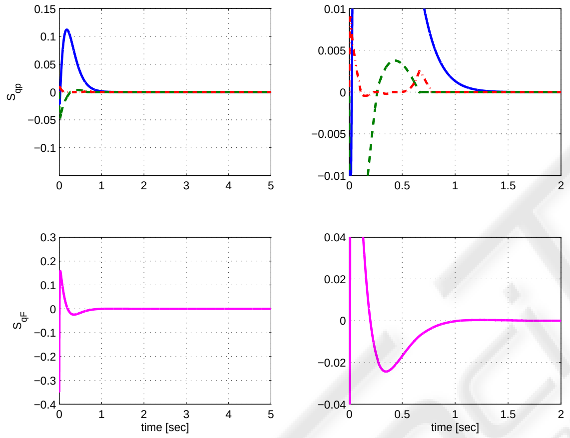

Figure 5: Control variables S_{qp} and S_{qF} for the model-free second order sliding mode control.

case of the model-free 2nd order sliding mode, force set-point control is achieved very fast indeed. Figure 4 shows the set-point control error for position (3 curves) and force. Is again evident the substantial difference in schemes for force control. Figure 5 shows the convergence of the extended position and force manifolds. They do converge to zero and induce there after the sliding mode dynamics.

4.2 Tracking Control Case

This task differs from the set-point control case in that the deep is a periodic desired trajectory center at 1 meter deep with 1 meter amplitude (pick to pick) and a 5 sec period. Contact desired force remains constant at 170N.

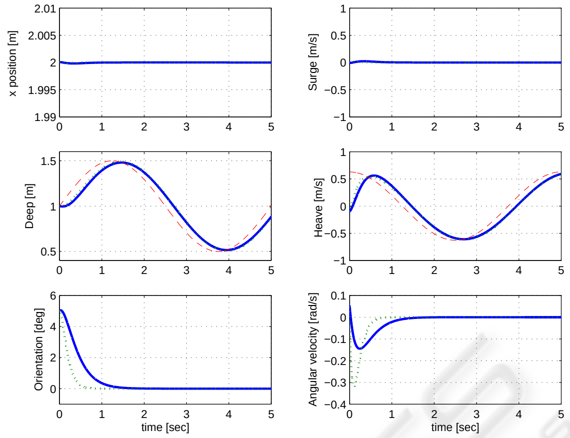

Figure 6: Generalized coordinates q and vehicle velocities ν, in vehicle's frame for the tracking case (continuous line for model-free second order sliding mode control, dotted line for PD control, dashed line for the tracking reference).

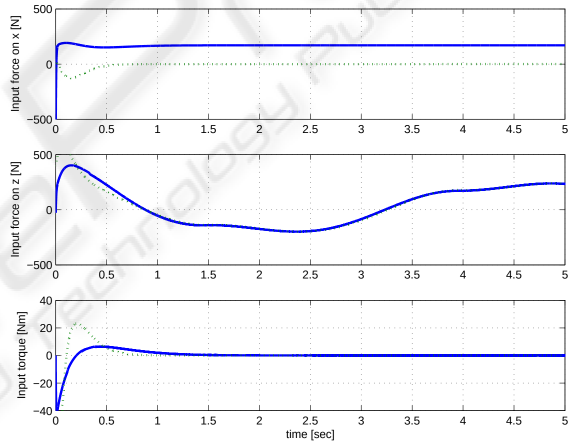

Figure 7: Inputs controlled forces u, in vehicle's frame (continuous line for model-free second order sliding mode control, dotted line for PD control).

As for the set-point control case, the position error and velocities in the constrained direction remain very small as can be see in figure 6. Also in this figure, the deep response has a lag from the tracking reference. As for the input control forces u the steady state resemble to those of the set-point control. For the deep coordinate, the control input tends to to be the same after some relative small time. Again, the PD does not provide any force in the constraint direction. The last can be verified in figure 8 where, for the PD case, there is no contact force although the position of the contact point is indeed in contact. For the model-free 2nd order sliding mode control, the contact force has the same shape as for the set-point control case. The same transient, due to the numerical consideration for

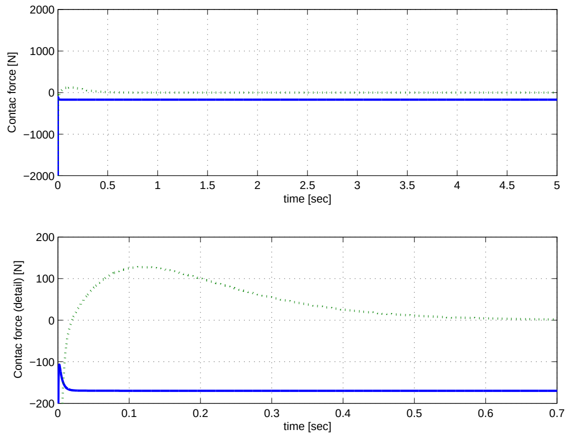

Figure 8: Contact force magnitude λ (continuous line for model-free second order sliding mode control, dotted line for PD control).

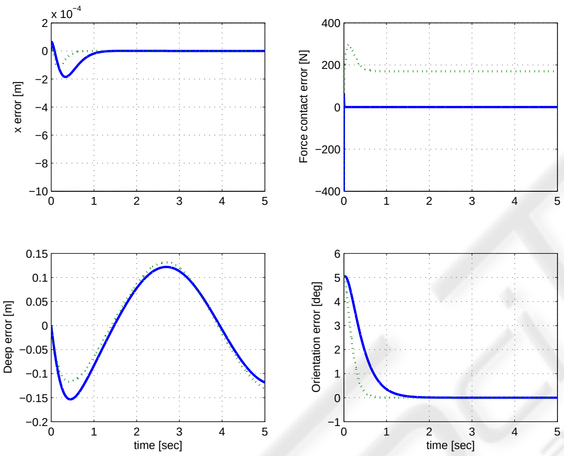

Figure 9: Set-point control errors: $\tilde{q} = q(t) - q^d$ and $\tilde{\lambda} = \lambda(t) - \lambda^d$ (continuous line for model-free second order sliding mode control, dotted line for PD control).

the almost rigid constraint surface, is present. The tracking errors for the x position and force has been reduced for the PD case certainly due to numerical initial conditions. In the tracking of deep, the model-free 2nd order sliding mode has apparently the same performance as the simpler PD controller. As it has been pointed out by other authors, for this kind of system, the PD controller remains as the best and simpler controller for position tracking. However, it does not track any force reference, which is the essence of the model-free 2nd order sliding mode control. The last figure 10 shows the convergence of the extended position and force manifolds to zero and induce there after the sliding mode dynamics.

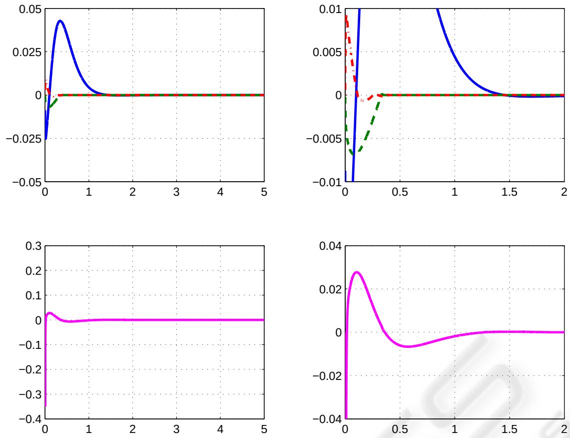

Figure 10: Control variables S_{qp} and S_{qF} for the model-free second order sliding mode control.

5 SOME DISCUSSIONS ON STRUCTURAL PROPERTIES

In the beginning of this study, we were expecting an involved control structure for submarine robots, in comparison to fixed-base robots, with additional control terms to compensate for hydrodynamic and buoyancy forces. However, surprisingly, no additional terms were required! Due to judicious treatment of the mathematical model, and careful mapping of velocities through jacobians in vehicle frame and generalized frame, it suffices only proper mapping of the gradient of contact forces. This results open the door to study similar force control structures developed for fixed-base robots, and so far, the constrained submarine robot control problem seems now at reach.

5.1 Properties of the Dynamics

As it as pointed out in Section 2, the model of a submarine robot can be expressed in either a *self space* (where the inertia matrix is constant for some conditions that in practice are not difficult to met) or in a generalized coordinate space (in which the inertial matrix is no longer constant but the model is expressed by only one equation, likewise the kinematic lagrangian chains). Both representations has all known properties of lagrangian systems such skew symmetry for the Coriolis/Inertia matrix, boundless of all the components, including buoyancy, and passivity preserved properties for the hydrodynamic added effects. The equivalences of these representation of the same model can be found by means of the kinematic equation. The last one is a linear operator that maps generalized coordinates time derivative with a generalized physical velocity. This relationship is specially important for the angular velocity of a free moving object due to the fact that time derivative of angular representations (such a roll-pitch-yaw) do not stand for the angular velocity. However, there is always a correspondence between these vectors. For external forces this mapping is indeed important. It relates a physical force/torque wrench to the generalized coordinates $q = (x_v, y_v, z_v, \phi_v, \theta_v, \psi_v)$ whose last 3 components does not represent a unique physical space. In this work, such mapping is given by J_q and appears in the contact force mapping by the normalized operator $J_{\bar{\varphi}}$. The operator J_q can be seen as the difference of a Analytical Jacobian and the Geometric Jacobian of a robot manipulator.

5.2 The Controller

Notice that the controller exhibits a PD structure plus a nonlinear I-tame control action, with nonlinear time-varying feedback gains for each orthogonal subspace. It is in fact a decentralized PID-like, coined as ”Sliding PD” controller. It is indeed surprising that similar control structures can be implemented seemingly for a robot in the surface or below water, with similar stability properties, simultaneous tracking of contact force and posture. Of course, this is possible under a proper mapping of jacobians, key to implement this controller.

5.3 Friction at the Contact Point

When friction at the contact point arises, which is expected for submarine tasks wherein the contact object is expected to exhibits a rough surface, with high friction coefficients, a tangential friction model should be added in the right hand side. Particular care must be employed to map the velocities. Complex friction compensators can be designed, similar to the case of force control for robot manipulators, therefore details are omitted.

6 CONCLUSIONS

Although a PD controller, recently validated by (Smallwood, 2004) has a much simpler structure and very good performance for underwater vehicles in position control schemes, it does not deal with the task of contact force control.

The desired contact force can be calculated via equation (19) directly in the generalized frame in which it occurs.

Structural properties of the open-loop dynamics of submarine robots are key to design passivity-based controllers. In this paper, an advanced, yet simple, controller is proposed to achieve simultaneously tracking of time-varying contact force and posture, without any knowledge of his dynamic parameters. This is a significant feature of this controller, since in particular for submarine robots the dynamics are very difficult to compute, let alone its parameters.

A simulation study provides additional insight into the closed-loop dynamic properties for set-point control and tracking case.

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