ROBUST CONTROL OF HYSTERETIC BASE-ISOLATED STRUCTURES UNDER SEISMIC DISTURBANCES

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Abstract: The main objective of applying robust active control to base-isolated structures is to protect them in the event of an earthquake. Taking advantage of discontinuous control theory, a static discontinuous active control is developed using as a feedback only the measure of the velocity at the base. Moreover, due to that in many engineering applications, accelerometers are the only devices that provide information available for feedback, our velocity feedback controller could be easily extended by using just acceleration information through a filter. The main contributions of this paper are: (a) a static velocity feedback controller design, and (b) a dynamic acceleration feedback controller design, for seismic attenuation of structures. Robustness performance is analyzed by means of numerical experiments using the 1940 *El Centro* earthquake.

1 INTRODUCTION

Base isolation has been widely considered as an effective technology to protect flexible structures up to eight storeys high against earthquakes. The conceptual objective of the isolator is to produce a dynamic decoupling of the structure from its foundation so that the structure ideally behaves like a rigid body with reduced inter-story drifts, as demanded by safety, and reduced absolute accelerations as related to comfort requirements. Although the response quantities of a fixed-base building are reduced substantially through base isolation, the base displacement may be excessive, particularly during near-field ground motions (Yang and Agrawal, 2002). Applications of hybrid control systems consisting of active or semi-active systems installed in parallel to base-isolation bearings have the capability to reduce response quantities of base-isolated structures more significantly than passive dampers (Ramallo et al., 2002; Yang and Agrawal, 2002).

In this paper, two versions of a decentralized robust active control are developed and applied to a base-isolated structure. The first controller uses the velocity at the base of the structure as feedback information, and it is analyzed via Lyapunov stability techniques as proposed in (Luo et al., 2001). Due to the fact that, in civil engineering applications, accelerometers are the most practically available sensors for feedback control, the second controller is an extension of the first one where just acceleration information is used. Performance of the proposed controllers, for seismic attenuation, are evaluated by numerical simulations using the 1940 El Centro earthquake (California, United States).

This paper is structured as follows. Section 2 describes the problem formulation. The solution to the problem statement using just velocity measurements is described in Section 3, meanwhile the solution employing only acceleration information is stated in Section 4. Numerical simulations to analyze the performance of both proposed controllers are presented in Section 5. Finally, on Section 6 final comments are stated.

2 PROBLEM STATEMENT

Consider a basic forced vibration system governed by:

$$m\ddot{x} + c\dot{x} + \Phi(x,t) = f(t) + u(t), \tag{1}$$

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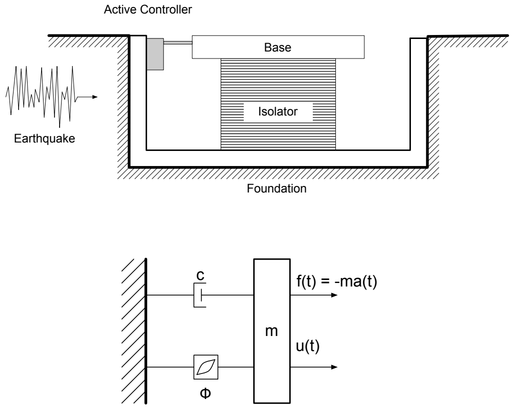

Figure 1: Building structure with hybrid control system (up) and physical model (down).

where m is the mass; c is the damping coefficient; Φ is the restoring force characterizing the hysteretic behavior of the isolator material, which is usually made with inelastic rubber bearings; $f(t)$ is the unknown excitation force; and $u(t)$ is the control force supplied by an appropriate actuator.

In structure systems, $f(t) = -m\ddot{x}_g(t)$ is the excitation force, where $\ddot{x}_g(t)$ is the earthquake ground acceleration. The restoring force Φ can be represented by the Bouc-Wen model (Ikhouane et al., 2005) in the following form:

$$\Phi(x,t) = \alpha K x(t) + (1-\alpha) D K z(t) \tag{2}$$

$$\dot{z} = D^{-1}\left(A\dot{x} - \beta|\dot{x}||\dot{z}|^{n-1}z - \lambda\dot{x}|z|^n\right) \tag{3}$$

where $\Phi(x,t)$ can be considered as the superposition of an elastic component $\alpha K x$ and a hysteretic component $(1-\alpha)DKz(t)$, in which the yield constant displacement is $D > 0$ and $\alpha \in [0,1]$ is the post- to pre-yielding stiffness ratio. $n \geq 1$ is a scalar that governs the smoothness of the transition from elastic to plastic response and $K > 0$. The hysteretic part in (2) involves an internal dynamic (3) which is unmeasurable, and thus inaccessible for seismic control design. A schematic description of the base-isolated system structure and its physical model are displayed in Fig. 1.
The following assumptions are stated for system (1)-(3):

Assumption 1 *The acceleration disturbance* $f(t) = -m\ddot{x}_g$ *is unknown but bounded;* i.e., *there exists a known constant* F *such that* $|f(t)| \leq F, \quad \forall t \geq 0.$

Assumption 2 *In the event of an earthquake, it is assumed that* $z(0) = 0$ *in equation (1) and that the structure is at rest;* i.e., $x(0) = \dot{x}(0) = 0$.

Assumption 3 *There exists a known upper bound on the internal dynamic variable* $z(t)$, i.e., $|z(t)| \leq \bar{\rho}_z, \quad \forall t \geq 0$.

Assumption 1 is standard in control of hysteretic systems or base-isolated structures (Ikhouane et al., 2005). Assumption 2 has a physical meaning because it is assumed that the structure is at rest when the earthquake strikes it. The upper bound in $z(t)$ expressed in Assumption 3 is computable, independently on the boundedness of $x(t)$ by invoking Theorem 1 in (Ikhouane et al., 2005).
Control objective: Our objective is to design a robust controller for system (1) such that, under earthquake attack, the trajectories of the closed-loop remain bounded.

To this end, the theorems in the following sections satisfy this control objective.

3 SEISMIC ATTENUATION USING ONLY VELOCITY FEEDBACK

Theorem 1 *Consider the nonlinear system (1)-(3) subject to Assumptions 1-3. Then, the following control law*

$$u = -\rho sgn(\dot{x}_0) \tag{4}$$

solves the control objective if

$$\rho \geq (1-\alpha)DK\bar{\rho}_z + F. \tag{5}$$

Proof. The closed-loop system (1)-(3) and (4) yields

$$\begin{aligned} m_0\ddot{x}_0 + c_0\dot{x}_0 + k_0x_0 + \Phi(x_0,t) &= -m_0\ddot{x}_g - \rho\mathrm{sgn}(\dot{x}_0) \\ m_0\ddot{x}_0 + c_0\dot{x}_0 + (k_0+\alpha K)x_0 &= -\rho\mathrm{sgn}(\dot{x}_0) + \Delta(z,t) \end{aligned} \tag{6}$$

where

$$\Delta(z,t) = -m_0\ddot{x}_g - (1-\alpha)Dkz.$$

Then

$$\begin{aligned} |\Delta(z,t)| &\leq |f(t)| + |(1-\alpha)DKz| \\ &\leq F + (1-\alpha)DK|z| \\ &\leq F + (1-\alpha)DK\bar{\rho}_z = \rho_1. \end{aligned}$$

Given the Lyapunov function

$$V(x_0,\dot{x}_0) = \frac{k_0+\alpha K}{2}x_0^2 + \frac{m_0}{2}\dot{x}_0^2,$$

its time derivative along the trajectories of the closed-loop system (1)-(3) and (4) yields

$$\begin{aligned}\dot{V}(x_0,\dot{x}_0) &= (k_0+\alpha K)x_0\dot{x}_0 + m_0\dot{x}_0\ddot{x}_0 \\ &= \dot{x}_0\left[(k_0+\alpha K)x_0 + m_0\ddot{x}_0\right] \\ &= \dot{x}_0\left[-c_0\dot{x}_0 - \rho\mathrm{sgn}(\dot{x}_0) + \Delta(z,t)\right] \\ &= -c_0\dot{x}_0^2 - \rho\dot{x}_0\mathrm{sgn}(\dot{x}_0) + \dot{x}_0\Delta(z,t) \\ &= -c_0\dot{x}_0^2 - \rho|\dot{x}_0| + \dot{x}_0\Delta(z,t) \\ &\le -c_0\dot{x}_0^2 - \rho|\dot{x}_0| + |\dot{x}_0|\rho_1 \\ &= -c_0\dot{x}_0^2 + (\rho_1-\rho)|\dot{x}_0|.\end{aligned}$$

The choice of $\rho \ge \rho_1$ makes $\dot{V}$ negative semidefinite, as we wanted to show. ■

Remark 1 (on solution of non-smooth systems) *The closed-loop system (6) has a non-smooth right-hand side, the signum function. Solutions to this non-smooth class of systems in the sense of Filippov has been widely studied (Wu et al., 1998). It is worth noting that non-smooth dynamic systems appear naturally and frequently in many mechanical systems (Wu et al., 1998). Due to the fact that classical solution theories to ordinary differential equations require vector fields to be at least Lipschitz continuous, main difficulties with non-smooth systems are that these systems fail the Lipschitz-continuous requirement. However, if (a) the vector field is measurable and essentially bounded; (b) the solution of the system is absolutely continuous; and (c) the Lyapunov function V is continuous and positive definite and its time derivative $\dot{V}$ along the trajectories of the closed-loop system is continuous and negative semi-definite, then the system under consideration has a solution in the sense of Filippov and it is stable in the sense of Lyapunov (Wu et al., 1998). This is exactly our case.*

Remark 2 *The signum function in the control law in Theorem 1 –common in sliding mode control theory– produces chattering (Utkin, 1982; Edwards and Spurgeon, 1998). One way to avoid chattering is to replace the signum function by a smooth sigmoid-like function such as*

$$\nu_\delta(s) = \frac{s}{|s|+\delta},$$

where δ is a sufficiently small positive scalar (Edwards and Spurgeon, 1998).

Consequently, the following Corollary is stated:

Corollary 1 *Consider the nonlinear system (1)-(3) subject to Assumptions 1-3. Then, the following control law*

$$u = -\rho\frac{\dot{x}_0}{|\dot{x}_0|+\delta} \tag{7}$$

solves the control objective if

$$\rho \ge (1-\alpha)DK\bar{\rho}_z + F$$

and δ is a sufficiently small positive scalar.

Proof. The time derivative of the Lyapunov function

$$V(x_0,\dot{x}_0) = \frac{k_0+\alpha K}{2}x_0^2 + \frac{m_0}{2}\dot{x}_0^2,$$

along the trajectories of the closed-loop system (1)-(3) and (7) yields

$$\begin{aligned}\dot{V} &= (k_0+\alpha K)x_0\dot{x}_0 + m_0\dot{x}_0\ddot{x}_0 \\ &= \dot{x}_0\left[(k_0+\alpha K)x_0 + m_0\ddot{x}_0\right] \\ &= \dot{x}_0\left[-c_0\dot{x}_0 - \rho\frac{\dot{x}_0}{|\dot{x}_0|+\delta} + \Delta(z,t)\right] \\ &= -c_0\dot{x}_0^2 - \rho\dot{x}_0\frac{\dot{x}_0}{|\dot{x}_0|+\delta} + \dot{x}_0\Delta(z,t) \\ &= -c_0\dot{x}_0^2 - \rho\frac{\dot{x}_0^2}{|\dot{x}_0|+\delta} + \dot{x}_0\Delta(z,t) \\ &\le -c_0\dot{x}_0^2 + \rho_1|\dot{x}_0| - \rho\frac{\dot{x}_0^2}{|\dot{x}_0|+\delta} \\ &= -c_0\dot{x}_0^2 - (\rho-\rho_1)|\dot{x}_0| + \rho\left(|\dot{x}_0| - \frac{\dot{x}_0^2}{|\dot{x}_0|+\delta}\right).\end{aligned}$$

The objective of guaranteeing global boundedness of solutions is equivalently expressed as rendering $\dot{V}$ negative outside a compact region. The choice of $\rho \ge \rho_1$ and considering that

$$\lim_{\delta\to 0}\rho\left(|\dot{x}_0| - \frac{\dot{x}_0^2}{|\dot{x}_0|+\delta}\right) = 0$$

guarantees the existence of a small compact region $D \subset \mathbb{R}^2$ (depending on δ) such that $\dot{V}$ is negative outside this set. This implies that all the closed-loop trajectories remain bounded, as we wanted to show. ■

4 SEISMIC ATTENUATION USING ONLY ACCELERATION FEEDBACK

Motivated by the fact that in many civil engineering applications accelerometers are the only devices that provide information available for feedback, Theorem 2 (below) presents a control law based on equation (4) where only acceleration information is required.

Theorem 2 *Consider the nonlinear system (1)-(3) subject to Assumptions 1-3. Then, the following control law*

$$u = -\rho sgn(\upsilon) \tag{8}$$

$$\dot{\upsilon} = \ddot{x}_0 \tag{9}$$

solves the control objective if

$$\rho \geq (1-\alpha)DK\bar{\rho}_z + F.$$

Proof. This proof is straightforward by considering direct integration of equation (9). ■

Remark 3 *In the practical implementation of this control law, ν may drift due to unmodeled dynamics, measure errors and disturbance. To avoid this, the following σ-modification (Ioannou and Kokotovic, 1983; Koo and Kim, 1994) can be used,*

$$u = -\rho sgn(\upsilon), \tag{10}$$

$$\dot{\upsilon} = -\sigma\nu + \ddot{x}_0, \tag{11}$$

where σ is a positive constant.

As in the previous Section, a smooth version of the control law in equations (10)-(11) is considered in the following Corollary.

Corollary 2 *Consider the nonlinear system (1)-(3) subject to Assumptions 1-3. Then, the following control law*

$$u = -\rho\frac{\upsilon}{|\upsilon| + \delta} \tag{12}$$

$$\dot{\upsilon} = -\sigma\upsilon + \ddot{x}_0 \tag{13}$$

solves the control objective if

$$\rho \geq (1-\alpha)DK\bar{\rho}_z + F,$$

where $\sigma > 0$ and δ are sufficiently small positive scalar.

5 NUMERICAL SIMULATIONS

In order to investigate the efficiency of the proposed controllers, we set $m = 156 \times 10^3$ kg, $K = 6 \times 10^6$ N/m, $c = 2 \times 10^4$ Ns/m, $\alpha = 0.6$, $D = 0.6$ m, $\lambda = 0.5$, $\beta = 0.1$, $n = 3$, and $A = 1$ (Ikhouane et al., 2005). A set of numerical experiments was performed on the system using information recorded during the 1940 El Centro earthquake. Figure 2 shows the ground acceleration for this earthquake. The open loop base displacement can also be seen in Figure 2. It can be seen that $\rho = 2 \cdot 10^5$ is an upper bound for the expression $(1-\alpha)DK\bar{\rho}_z + F$ in equation (5).

Figures 3, 4 and 5 display the time histories of the motion of the base and the control signal force for different values of ρ and δ, when the control law in equation (7) is used. In an equivalent manner, the time histories of the motion of the base and the control signal force when the control law in equations (12)-(13) is used are depicted in Figures 6, 7 and 8. In both cases, the controlled base displacements are significantly reduced compared to the uncontrolled case. It is worth noting that, when $\sigma = 0.1$ in Figure 8, the results are similar to those in Figure 3.

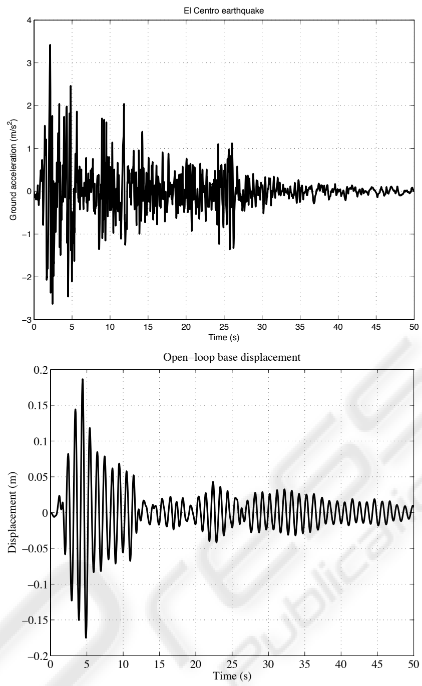

Figure 2: 1940 El Centro earthquake, ground acceleration (top); open loop base displacement (bottom).

6 CONCLUSION

A robust control scheme to attenuate the consequences of seismic events on base-isolated structures has been proposed. It has been shown that a simple controller can fulfill the control objectives, using just velocity measurements or just acceleration information. Simulation results showed the good performance of the controllers. In civil engineering, the controller that just uses acceleration information is of a great interest, due to the fact that accelerometers are easily available. Also, the simplicity of the proposed controllers makes them attractive for a real implementation.

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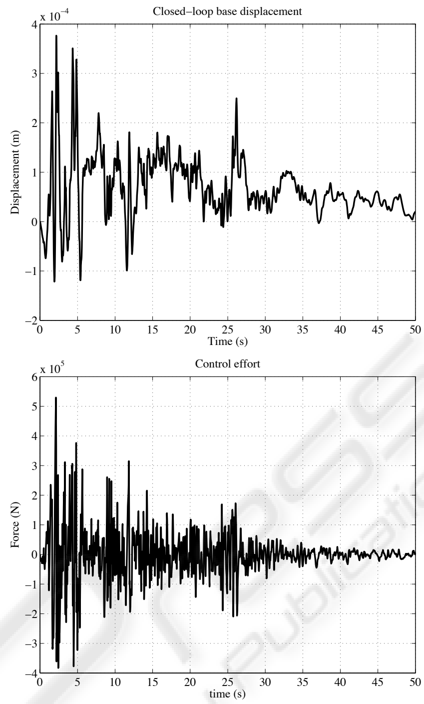

Figure 3: Closed loop base displacement (top) and control signal force (bottom) with control law in equation (7) and parameters $\rho = 2 \cdot 10^6$ and $\delta = 0.01$.

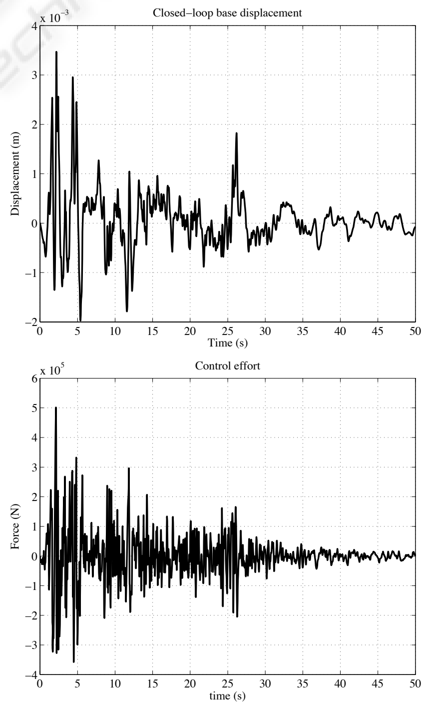

Figure 4: Closed loop base displacement (top) and control signal force (bottom) with control law in equation (7) and parameters $\rho = 2 \cdot 10^6$ and $\delta = 0.1$.

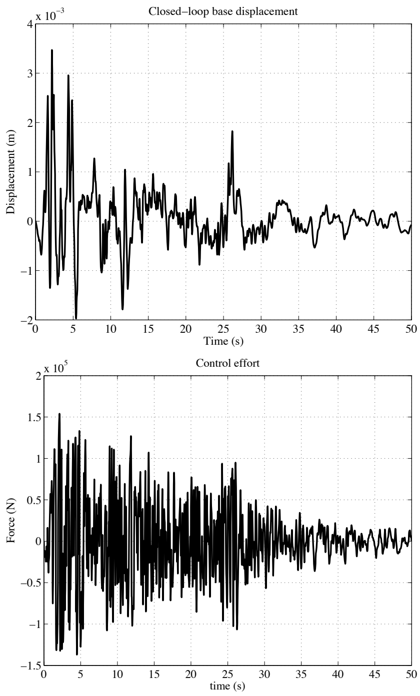

Figure 5: Closed loop base displacement (top) and control signal force (bottom) with control law in equation (7) and parameters $\rho = 2 \cdot 10^5$ and $\delta = 0.01$.

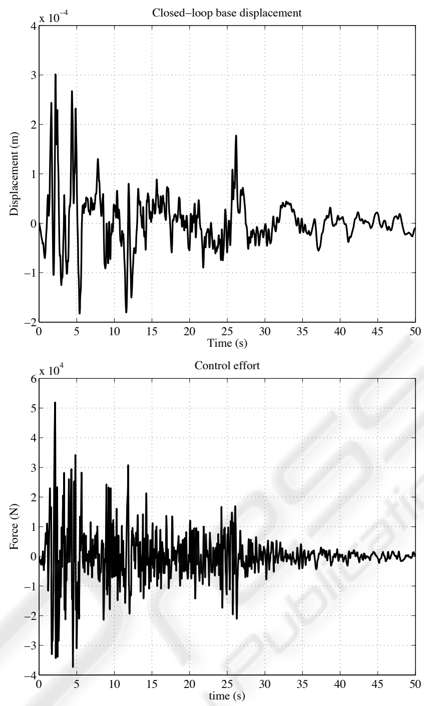

Figure 7: Closed loop base displacement (top) and control signal force (bottom) with control law in equations (12)-(13) and parameters $\rho = 2 \cdot 10^6$, $\delta = 0.1$ and $\sigma = 0.1$.

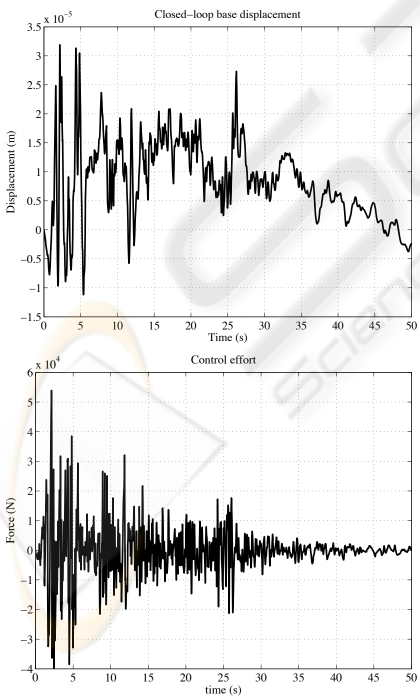

Figure 6: Closed loop base displacement (top) and control signal force (bottom) with control law in equations (12)-(13) and parameters $\rho = 2 \cdot 10^6$, $\delta = 0.01$ and $\sigma = 0.1$.

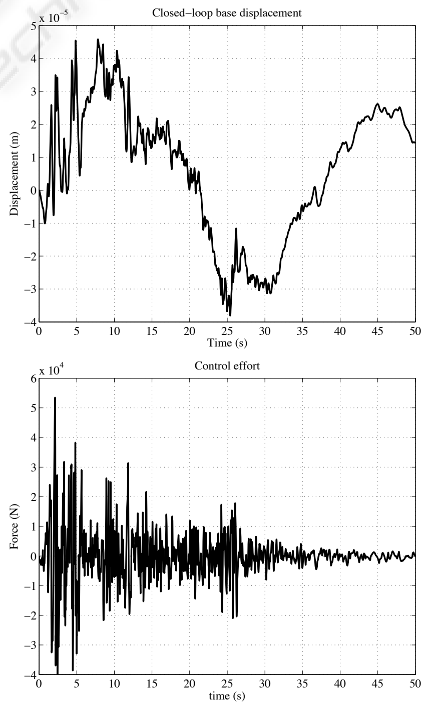

Figure 8: Closed loop base displacement (top) and control signal force (bottom) with control law in equations (12)-(13) and parameters $\rho = 2 \cdot 10^6$, $\delta = 0.01$ and $\sigma = 1$.