TRACKING CONTROL OF WHEELED MOBILE ROBOTS WITH A SINGLE STEERING INPUT
Control Using Reference Time-Scaling

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Abstract: This paper presents a time-scaling based control strategy of the kinematic model of wheeled mobile robots with one input which is the steering angle. The longitudinal velocity of the mobile robot cannot be influenced by the controller but can be measured. Using an on-line time-scaling, driven by the longitudinal velocity of the robot and its time derivatives, one can achieve exponential tracking of any sufficiently smooth reference trajectory with non-vanishing velocity. The price to pay is the modification of the traveling time along the reference trajectory according to the time-scaling. The measurement of the time derivatives of the velocity is no longer necessary if the tracking controller is designed to the linearized tracking error dynamics.

1 INTRODUCTION

The kinematic model of a wheeled mobile robot (WMR) has generally two inputs namely the longitudinal velocity of the rear axis midpoint and the steering angle of the front wheels. Several strategies are applied to control such WMRs with these two inputs including the tracking error transformation based control reported by (Dixon et al., 2001), the sliding mode controller based solution proposed by (Benalia et al., 2003), and the behavior based control strategy studied by (Gu and Hu, 2002). An important property of the model is its differential flatness (Fliess et al., 1995; Fliess et al., 1999) implying its dynamic feedback linearizability (with a singularity at zero velocities).

However, situations may occur where the longitudinal velocity of the WMR is not generated by a feedback controller, but by an external source. A practical example of this scenario is the tracking problem related to a passenger car without automatic gear. In such a situation the human driver needs to generate the velocity of the car with an appropriate management of the gas, clutch, and break pedals while the tracking controller may influence only the angle of the steered wheels. The kinematic model obtained in such a situation is no longer differentially, but orbitally flat (Respondek, 1998; Guay, 1999), since one of the inputs is lost.

The control problem is still the tracking of the reference trajectory but this tracking may become impossible if the velocity of the reference WMR moving along the reference path is always superior to the real velocity generated by the driver. The opposite is also possible such that the velocity of the reference WMR is always inferior to the real velocity generated by the driver. Nevertheless it is expected that the path of the controlled WMR joins the path of the reference WMR for any velocity profile generated by the driver. To achieve exponential tracking in both cases, this paper suggests a time-scaling of the reference path. This time-scaling uses the measurement of the velocity generated by the driver and eventually its time derivatives. A practical mean to obtain these measurements is the use of the ABS signals available on the CAN bus of the vehicle or the use of alternative sensors (e.g. accelerometers).

Recall that time-scaling is a commonly used concept to find optimal trajectories, to cope with input saturation, to reduce tracking errors, and to establish equivalence classes of dynamical systems.

One may use off-line time-scaling methods to find the time optimal trajectories for robot manipulators (Hollerbach, 1984) or for autonomous mobile vehicles (Cuesta and Ollero, 2005). The problem with these off-line methods is that no sufficient control input margins are always assured for the closed loop

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control during the tracking. Other algorithms use therefore on-line trajectory time-scaling for robotic manipulators to change the actuator boundaries such that sufficient margin is left for the feedback controller (Dahl and Nielsen, 1990).

Another concept is to use the tracking error instead of the input bounds in order to modify the time-scaling of the reference path (Lévine, 2004; Szádeczky-Kardoss and Kiss, 2006). These methods change the traveling time of the reference path according to the actual tracking error by decelerating if the movement is not accurate enough and by accelerating if the errors are small or vanish.

Time-scaling is also introduced related to the notion of orbital flatness defined in (Fliess et al., 1999) where a Lie-Bäcklund equivalence of dynamical systems is established such that the transformation involved may change the time according to which the systems evolve. Another approach that relates time-scaling to feedback linearization is reported in (Sampei and Furuta, 1986).

The remaining part of the paper is organized as follows. The next section presents the kinematic models of the WMRs. Section 3 studies briefly the flatness properties of the models. Section 4 presents a simple motion planning method. The time-scaling concept is introduced in Section 5. Two tracking feedback laws, both using time-scaling are reported in Section 6. Simulation results are presented in Section 7 and a short conclusion terminates the paper.

2 KINEMATIC WMR MODELS

Let us introduce some notations first. Figure 1 depicts a WMR in the xy horizontal plane. Let us suppose that the Ackermann steering assumptions hold true, hence all wheels turn around the same point (denoted by P) which lies on the line of the rear axle. It follows that the kinematics of the robot can be fully described by the kinematics of a bicycle fitted to the longitudinal symmetry axis of the vehicle (see Figure 1). The coordinates of the rear axle midpoint are given by x and y. The orientation of the car with respect to the x axis is denoted by the angle θ, hence the WMR evolves on the configuration manifold $\mathbb{R}^2 \times \mathbb{S} = SE(2)$. The angle of the front wheel of the bicycle with respect to the longitudinal symmetry axis of the robot is denoted by φ. We consider $u_2 = \varphi$ as an input. The longitudinal velocity of the rear axle midpoint is denoted by u_1 if it is a control input (two input case) and by v_{car} if not (one input case).

The distance l between the front and rear axles equals to one. Then the model equations can be obtained after some elementary considerations which result (see also for example (Rouchon et al., 1993; Cuesta and Ollero, 2005))

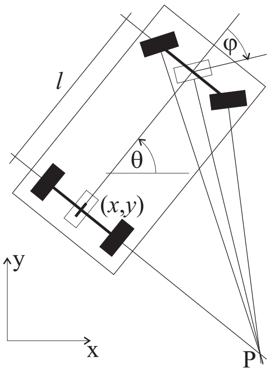

Figure 1: Notations of the kinematic model.

$$\dot{x} = u_1 \cos\theta \quad (1)$$
$$\dot{y} = u_1 \sin\theta \quad (2)$$
$$\dot{\theta} = u_1 \tan u_2. \quad (3)$$

Since time-scaling is involved in the sequel, we precise that the time in this system is denoted by t and $\dot{x}$ denotes the time derivative of the function $x(t)$ such that $\dot{t} = 1$.

Consider now the case where the longitudinal velocity is not a control input but an *external signal* v_{car} which is generated by the driver or by any other mean such that the controller has no influence on its evolution. The corresponding model with one input is defined by the equations

$$\dot{x} = v_{car} \cos\theta \quad (4)$$
$$\dot{y} = v_{car} \sin\theta \quad (5)$$
$$\dot{\theta} = v_{car} \tan u_2. \quad (6)$$

Consider now a time different from the time t, denoted by τ. Based on (1)-(3), let us define a model evolving with the time τ and given by the equations

$$x'_\tau = u_{\tau,1} \cos\theta_\tau \quad (7)$$
$$y'_\tau = u_{\tau,1} \sin\theta_\tau \quad (8)$$
$$\theta'_\tau = u_{\tau,1} \tan u_{\tau,2} \quad (9)$$

where the subscript τ denotes the dependence on the time τ and x'_τ is the derivative of the function $x_\tau(\tau)$ with respect to τ. (No subscript is used for variables dependent on time t except cases where the distinction is necessary). It is obvious that $\tau' = 1$ as $\dot{t} = 1$.

3 FLATNESS OF THE MODELS

One can easily verify or check in the literature (Rouchon et al., 1993; Fliess et al., 1995) that the model (1)-(3) (respectively (7)-(9)) is differentially flat, hence it can be linearized by a dynamic feedback and a coordinate transformation. The flat output is given by x and y (respectively x_τ and y_τ).

The differential flatness property of the models can be exploited both for motion planning and tracking purposes. Given a reference trajectory $x_{ref}(t)$ and $y_{ref}(t)$ for the flat output variables x and y, which are at least two times differentiable with respect to the time t, one can determine the time functions of θ_{ref}, $u_{1,ref}$, and $u_{2,ref}$ which satisfy (1)-(3) according to a mapping

$$\{x_{ref},\ldots,\ddot{x}_{ref},y_{ref},\ldots,\ddot{y}_{ref}\} \rightarrow \{\theta_{ref},u_{1,ref},u_{2,ref}\}.$$

The same holds true for the model (7)-(9) evolving with the time τ hence there exists a mapping

$$\{x_{\tau,ref},\ldots,x''_{\tau,ref},y_{\tau,ref},\ldots,y''_{\tau,ref}\} \rightarrow \{\theta_{\tau,ref},u_{\tau,1,ref},u_{\tau,2,ref}\}. \quad (10)$$

The model (4)-(6) is not differentially, but orbitally flat for $v_{car} \equiv 1$ as shown by (Guay, 1999).

4 MOTION PLANNING

The motion planning is done for the system (7)-(9) exploiting its differential flatness property. The motion planning realizes the mappings

$$\tau \rightarrow \{x_{\tau,ref},x'_{\tau,ref},x''_{\tau,ref}\} \quad (11)$$

$$\tau \rightarrow \{y_{\tau,ref},y'_{\tau,ref},y''_{\tau,ref}\} \quad (12)$$

for $\tau \in [0,T]$ where T is the desired traveling time along the path such that the mapping (10) allows then to calculate the time functions of the remaining variables of the model.

Several motion planning schemes can be used to realize (11) and (12). One may want to solve an obstacle avoidance problem in parallel with the generation of the references (Cuesta and Ollero, 2005). For the sake of simplicity, seventh degree polynomial trajectories are considered in this paper such that

$$x_{\tau,ref} = \sum_{i=0}^{7} a_{x,i}\tau^i, \qquad y_{\tau,ref} = \sum_{i=0}^{7} a_{y,i}\tau^i. \quad (13)$$

The coefficients are obtained as solutions of a set of linear algebraic equations determined by the constraints that the polynomials and their three successive derivatives must satisfy at $\tau = 0$ and $\tau = T$. Notice that the non-zero constraints are no longer respected in the scaled time t for the derivatives of the references unless $\dot{\tau} \equiv 1$. It follows in particular that the constraints imposed on the longitudinal velocities at $\tau = 0$ (respectively at $\tau = T$) will be scaled by $\dot{\tau}(0)$ (respectively by $\dot{\tau}(t(T))$.

The motion planning can be done off-line prior to the tracking and the time-scaling does not need the redesign of the reference. It follows that more involved methods can be also applied including the one involving continuous curvature pathes with Fresnel integrals (Fraichard and Scheuer, 2004).

5 TIME-SCALING

A time-scaling law, which is defined by the mapping $t \mapsto \tau(t)$ or by its inverse $\tau \mapsto t(\tau)$ can be obtained based on the following consideration. Rewrite (4)-(6) as

$$dx = v_{car}dt\cos\theta \quad (14)$$

$$dy = v_{car}dt\sin\theta \quad (15)$$

$$d\theta = v_{car}dt\tan u_2. \quad (16)$$

Similarly, rewrite also (7)-(9) as

$$dx_\tau = u_{\tau,1}d\tau\cos\theta_\tau \quad (17)$$

$$dy_\tau = u_{\tau,1}d\tau\sin\theta_\tau \quad (18)$$

$$d\theta_\tau = u_{\tau,1}d\tau\tan u_{\tau,2}. \quad (19)$$

Consider now the model equations and the following relations obtained from (14)-(16) and (17)-(19) for the inputs of the models

$$\frac{dt}{d\tau} = t' = \frac{u_{\tau,1}}{v_{car}} \qquad u_2 = u_{\tau,2}. \quad (20)$$

This allows to determine a unique trajectory of the system (4)-(6) for a trajectory of the system (7)-(9) if the time function $v_{car}(t)$ and the initial conditions are given, and one supposes non-vanishing velocity functions v_{car} and $u_{\tau,1}$. As far as the initial conditions are considered one may, for instance, suppose that $x(0) = x_\tau(0)$, $y(0) = y_\tau(0)$, $\theta(0) = \theta_\tau(0)$. The relations in the other direction are similar and read

$$\frac{d\tau}{dt} = \dot{\tau} = \frac{v_{car}}{u_{\tau,1}} \qquad u_{\tau,2} = u_2. \quad (21)$$

The following proposition summarizes the properties of the time-scaling for trajectories with strictly positive (respectively negative) velocities.

Proposition 1 *Suppose that one considers trajectories of the different models of the kinematic car such that the velocities v_{car} and $u_{\tau,1}$ are both strictly positive (respectively strictly negative). Then the time-scaling $t \mapsto \tau(t)$ and $\tau \mapsto t(\tau)$ defined by* (20)-(21) *satisfying $t(0) = \tau(0)$ are such that the functions $\tau(t)$ and $t(\tau)$ are strictly increasing functions of their arguments (the scaled time never rewinds).*

This property is a general requirement for meaningful time-scaling and it is satisfied both for forward and backward motions of the car. The time-scaling has a singularity if the car is in idle position.

Note that for a fixed velocity time function $v_{car}(t)$, the time-scaling can be influenced by $u_{\tau,1}$, one of the inputs of the flat model evolving with τ. Observe moreover that for fixed v_{car} these relations do not define a one-to-one correspondence between the sets of trajectories of the respective systems and the number of inputs is not preserved, hence they are not Lie-Bäcklund isomorphisms (Fliess et al., 1999).

Suppose that the references are obtained for (7)-(9) and one disposes of the time functions $x_{\tau,ref}$, $x'_{\tau,ref}$, $x''_{\tau,ref}$, $y_{\tau,ref}$, $y'_{\tau,ref}$, $y''_{\tau,ref}$. (A simple method is given in preceding section for the planning of the reference motion.) The time-scaling is defined by the mappings

$$\{x_{\tau,ref},\ldots,x''_{\tau,ref}\} \rightarrow \{x_{ref}(t),\ldots,\ddot{x}_{ref}(t)\} \quad (22)$$

$$\{y_{\tau,ref},\ldots,y''_{\tau,ref}\} \rightarrow \{y_{ref}(t),\ldots,\ddot{y}_{ref}(t)\} \quad (23)$$

which can be determined using (21) since

$$\tau(t) = \int_0^t \frac{v_{car}}{u_{\vartheta,1}} d\vartheta \qquad \tau(0) = t(0) = 0 \quad (24)$$

$$v_{car} = \dot{\tau} u_{t,1} \quad (25)$$

$$\dot{v}_{car} = \ddot{\tau} u_{t,1} + \dot{\tau}\dot{u}_{t,1} \quad (26)$$

allow to express $\tau(t)$, $\dot{\tau}$, and $\ddot{\tau}$. Then

$$x_{ref}(t) = x_{\tau,ref}(\tau(t)) \quad (27)$$

$$\dot{x}_{ref}(t) = x'_{\tau,ref}(\tau(t))\dot{\tau} \quad (28)$$

$$\ddot{x}_{ref}(t) = x''_{\tau,ref}(\tau(t))\dot{\tau}^2 + x'_{\tau,ref}(\tau(t))\ddot{\tau} \quad (29)$$

and one obtains similar expressions for the higher order time derivatives and for the mapping (23).

Suppose that a reference trajectory is calculated according to the time τ and that one is looking for an open loop control of the real car to follow the geometry of the reference trajectory. Assume moreover that the reference trajectory is calculated based on the initial conditions of the real car. Then the open loop control signal $u_2(t)$ can be calculated from the reference $u_{\tau,2,ref}$ using the time-scaling defined by (20). Notice however that the real traveling time for a reference trajectory according the time t will be obtained as $t(T)$. If the driver generating v_{car} accelerates with respect to the reference trajectory than $T > t(T)$. If he/she is more careful than the algorithm providing the value for T than $T < t(T)$.

6 TRACKING FEEDBACK DESIGN

Two tracking controllers are presented in this section. The first one is based on the flatness property of the model with two inputs and requires the measurement of the car velocity and its two successive time derivatives. Measurement of the acceleration and its time derivative may be prohibitive for real applications. Therefore another tracking feedback is also suggested which is designed for a system obtained by linearizing the tracking error dynamics around the reference trajectory achieving only local stability of the reference trajectory.

6.1 Flatness-Based Tracking using Time-Scaling

The system (1)-(3) can be linearized by dynamic feedback in virtue of its differential flatness property. The resulting linear system is two chains of integrators

$$x^{(3)} = \omega_x \qquad y^{(3)} = \omega_y. \quad (30)$$

Suppose that one specifies the tracking behavior in terms of the tracking errors $e_x = x - x_{ref}$ and $e_y = y - y_{ref}$ such that the differential equations

$$e_x^{(3)} + k_{x,2}\ddot{e}_x + k_{x,1}\dot{e}_x + k_{x,0}e_x = 0 \quad (31)$$

$$e_y^{(3)} + k_{y,2}\ddot{e}_y + k_{y,1}\dot{e}_y + k_{y,0}e_y = 0 \quad (32)$$

hold true. The coefficients $k_{a,i}$ ($a \in \{x,y\}$, $i = 0,1,2$) are design parameters and have to be chosen such that the corresponding characteristic polynomials have all their roots in the left half of the complex plane. These linear differential equations define another (tracking feedback) for (30)

$$\omega_x = x_{ref}^{(3)} - k_{x,2}\ddot{e}_x - k_{x,1}\dot{e}_x - k_{x,0}e_x \quad (33)$$

$$\omega_y = y_{ref}^{(3)} - k_{y,2}\ddot{e}_y - k_{y,1}\dot{e}_y - k_{y,0}e_y. \quad (34)$$

Consider now the model described by (4)-(6). This single input model is not differentially flat, hence cannot be linearized by feedback. It follows that the flatness property cannot be (directly) used to solve the tracking problem.

Let us study the possibility to use the time-scaling defined above to achieve the desired tracking behavior for the non-differentially flat model (4)-(6) with one input.

The idea is to use the differentially flat model (7)-(9) to solve the motion planning problem with the time τ. Then one would use a tracking feedback controller designed again for the flat model which produces $u_{\tau,1}$ and $u_{\tau,2}$. The signal $u_{\tau,1}$ produced by the controller is used to drive the time-scaling of the reference trajectory designed for the time τ according to (21). The control loop is illustrated in Figure 2 for the model (4)-(6) where the controller provides $u_2 = \varphi$ to the single input model. The tracking feedback is

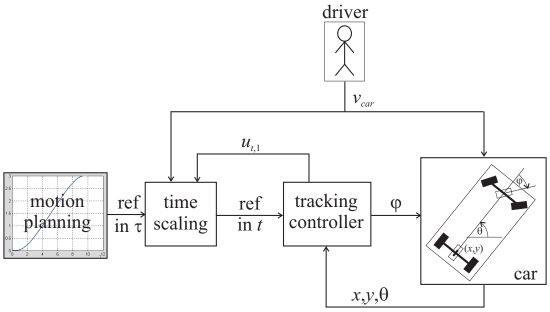

Figure 2: Tracking controller with time-scaling.

designed using the flatness property of the model with two inputs. Define first the dynamics for the feedback as

$$\dot{\zeta}_1 = \zeta_2 \qquad \dot{\zeta}_3 = v_2 \tag{35}$$

$$\dot{\zeta}_2 = v_1 \qquad \varphi = \zeta_3 \tag{36}$$

$$u_1 = \zeta_1 \tag{37}$$

where ζ_1, ζ_2, and ζ_3 are the (inner) states of the feedback. Observe that ζ_2 and v_1 give precisely the derivatives of u_1 which need to realize the time-scaling in (24)-(26), hence no numerical differentiation is needed.

The inputs v_1 and v_2 of the feedback dynamics must be determined such that the tracking errors $e_x(t)$ and $e_y(t)$ satisfy (31) and (32), respectively.

For, one needs to determine first x, $\dot{x}$, $\ddot{x}$, $x^{(3)}$, y, $\dot{y}$, $\ddot{y}$, and $y^{(3)}$ as functions of x, y, θ, ζ_1, ζ_2, and ζ_3 which are the states of the closed loop system including the measured states of the kinematic car model, and the states of the feedback (35)-(37). After some cumbersome but elementary differentiations one obtains

$$\dot{x} = \zeta_1 \cos\theta \tag{38}$$

$$\ddot{x} = \zeta_2 \cos\theta - \zeta_1^2 \sin\theta \tan\zeta_3 \tag{39}$$

$$\begin{aligned} x^{(3)} = {} & \frac{v_1 \cos\theta \cos^2\zeta_3 - 3\zeta_1\zeta_2 \sin\theta \sin\zeta_3 \cos\zeta_3}{\cos^2\zeta_3} \\ & - \frac{\zeta_1^3 \cos\theta - \zeta_1^3 \cos\theta \cos^2\zeta_3 + v_2\zeta_1^2 \sin\theta}{\cos^2\zeta_3} \end{aligned} \tag{40}$$

$$\dot{y} = \zeta_1 \sin\theta \tag{41}$$

$$\ddot{y} = \zeta_2 \sin\theta + \zeta_1^2 \cos\theta \tan\zeta_3 \tag{42}$$

$$\begin{aligned} y^{(3)} = {} & \frac{v_1 \sin\theta \cos^2\zeta_3 + 3\zeta_1\zeta_2 \cos\theta \sin\zeta_3 \cos\zeta_3}{\cos^2\zeta_3} \\ & - \frac{\zeta_1^3 \sin\theta - \zeta_1^3 \sin\theta \cos^2\zeta_3 - v_2\zeta_1^2 \cos\theta}{\cos^2\zeta_3}. \end{aligned} \tag{43}$$

These expressions allow to calculate e_x, $\dot{e}_x$, $\ddot{e}_x$, e_y, $\dot{e}_y$, and $\ddot{e}_y$ using the reference trajectory (scaled with t) and the states of the closed loop system. Plugging in these expressions into (31) and (32), and using (30) one gets

$$\begin{bmatrix} \cos\theta & -\frac{\zeta_1^2 \sin\theta}{\cos^2\zeta_3} \\ \sin\theta & \frac{\zeta_1^2 \cos\theta}{\cos^2\zeta_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \omega_x - A \\ \omega_y - B \end{bmatrix} \tag{44}$$

with

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \frac{-3\zeta_1\zeta_2 \sin\theta \sin\zeta_3 \cos\zeta_3 - \zeta_1^3 \cos\theta + \zeta_1^3 \cos\theta \cos^2\zeta_3}{\cos^2\zeta_3} \\ \frac{3\zeta_1\zeta_2 \cos\theta \sin\zeta_3 \cos\zeta_3 - \zeta_1^3 \sin\theta + \zeta_1^3 \sin\theta \cos^2\zeta_3}{\cos^2\zeta_3} \end{bmatrix} \tag{45}$$

where the inverse of the coefficient matrix can be calculated symbolically. One obtains

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ \frac{-\sin\theta \cos^2\zeta_3}{\zeta_1^2} & \frac{\cos\theta \cos^2\zeta_3}{\zeta_1^2} \end{bmatrix} \begin{bmatrix} \omega_x - A \\ \omega_y - B \end{bmatrix}. \tag{46}$$

The tracking feedback law is defined by (33)-(34), (35)-(37), and by (46). A singularity occurs if $\zeta_1^2 = u_1^2 = 0$ which corresponds to zero longitudinal velocity. Another singular situation corresponds to $\zeta_3 = \varphi = u_2 = \pm\pi/2$ which may occur if the steered wheels are perpendicular to the longitudinal axis of the car. Singularities imply the loss of controllability of the kinematic car model.

6.2 Linearized Error Dynamics

The above method needed the time derivatives of the velocity to carry out the time-scaling which may be difficult to measure or estimate in real application. The method presented in this section uses a transformation of the tracking error expressed in the configuration variables, and the non-linear model obtained is linearized around the reference trajectory. The linearized model is controlled by a state feedback similar to the one reported in (Dixon et al., 2001). The lost input, which is the longitudinal velocity of the WMR is again replaced by a virtual input which depends on the time-scaling of the reference trajectory.

A slightly different kinematic model is used for this method such that the longitudinal velocity of the rear axle midpoint and the tangent of the steering angle ($u_3 = \tan\varphi$) are the inputs of the mobile robot. If the one input case is considered, $u_3 = \tan\varphi$ is the single control input.

Suppose, that the desired behavior of the robot is given by the time functions $x_{\tau,ref}(\tau)$, $y_{\tau,ref}(\tau)$, $\theta_{\tau,ref}(\tau)$ such that these functions identically satisfy (7)-(9) for the corresponding reference input signals $u_{\tau,1,ref}$ and $u_{\tau,3,ref} = \tan u_{\tau,2,ref}$.

We suggest to scale this reference trajectory according to the time t. The scaled reference trajectory is given by $x_{ref}(t) = x_{\tau,ref}(\tau)$, $y_{ref}(t) = y_{\tau,ref}(\tau)$, and

$\theta_{ref}(t) = \theta_{\tau,ref}(\tau)$ and similarly to (7)-(9)

$$x'_{\tau,ref} = u_{\tau,1,ref}\cos\theta_{\tau,ref} \quad (47)$$
$$y'_{\tau,ref} = u_{\tau,1,ref}\sin\theta_{\tau,ref} \quad (48)$$
$$\theta'_{\tau,ref} = u_{\tau,1,ref}u_{\tau,3,ref}. \quad (49)$$

The tracking errors are defined for the configuration variables as $e_x = x - x_{ref}$, $e_y = y - y_{ref}$, and $e_\theta = \theta - \theta_{ref}$. Let us now consider the transformation

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} \quad (50)$$

of the error vector (e_x, e_y, e_θ) to a frame fixed to the car such that the longitudinal axis of the car coincides the transformed x axis. Differentiating this equation w.r.t. time t and using the general rule $\dot{a}(\tau) = \frac{da(\tau)}{dt} = \frac{\partial a}{\partial \tau}\frac{\partial \tau}{\partial t} = a'\dot{\tau}$ we get the differential equation

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 & v_{car}u_3 & 0 \\ -v_{car}u_3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \\ + \begin{bmatrix} 0 \\ \sin e_3 \\ 0 \end{bmatrix} u_{\tau,1,ref}\dot{\tau} + \begin{bmatrix} w_1 & 0 \\ 0 & 0 \\ 0 & w_2 \end{bmatrix} \quad (51)$$

which describes the evolution of the errors with respect to the reference path. The inputs w_1 and w_2 are

$$w_1 = v_{car} - \dot{\tau}u_{\tau,1,ref}\cos e_3 \quad (52)$$
$$w_2 = v_{car}u_3 - \dot{\tau}u_{\tau,1,ref}u_{\tau,3,ref}. \quad (53)$$

Notice that from these inputs w_1 and w_2 the first derivative of the time scaling ($\dot{\tau}$) and the real input $u_3 = \tan\varphi$ can be calculated as

$$\dot{\tau} = \frac{v_{car} - w_1}{u_{\tau,1,ref}\cos e_3} \quad (54)$$
$$u_3 = \frac{w_2 + \dot{\tau}u_{\tau,1,ref}u_{\tau,3,ref}}{v_{car}} \quad (55)$$

if the reference value for the longitudinal velocity $u_{\tau,1,ref} \neq 0$, the error of the orientation $e_3 \neq \pm\pi/2$, and the longitudinal velocity of the car $v_{car} \neq 0$.

This system can be linearized along the reference trajectory, i.e. for $[\ e_1\ \ e_2\ \ e_3\ ]^T = 0$. The linearized system is controllable if at least one of the reference control inputs $(u_{\tau,1,ref}, u_{\tau,3,ref})$ is nonzero. The setpoint of the linearized system obtained from (51) can be locally stabilized by a state feedback of the form

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = -K \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \quad (56)$$

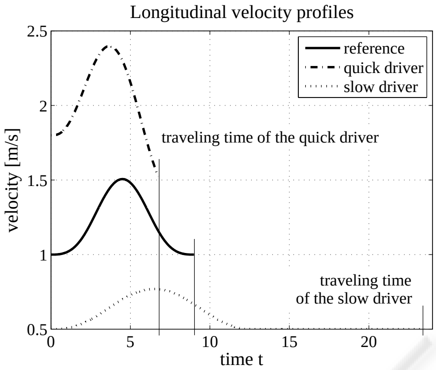

Figure 3: Velocity profiles for the reference, for the quick driver, and for the slow driver. The traveling times are obtained in closed loop.

such that the gain matrix K puts the eigenvalues of the closed loop system in the left half of the complex plane.

The way of calculations is as follows. One suppose that the tracking errors of the configuration variables (e_x, e_y, e_θ) are measured, hence the the error (e_1, e_2, e_3) can be determined using (50). Then the state feedback (56) allows to calculate w_1 and w_2. From the actual value of w_1 one can determine $\dot{\tau}$ using the current value of v_{car}, e_3, and the value of $u_{\tau,1,ref}$ according to the time t obtained by scaling the reference. The input $u_3 = \tan\varphi$ is calculated according to (55) using w_2. The function $\tau(t)$ is obtained by the on-line integration of $\dot{\tau}$ determined by (54) using the initial condition $\tau(0) = 0$. The time distribution of the reference trajectory is finally modified according to τ and $\dot{\tau}$.

Since a linearized model was used for the controller design only local stability is guaranteed. (E.g. if $w_1 \approx 0$ is not fulfilled, $\dot{\tau}$ in (54) can get a negative value, which is not allowed since time cannot rewind.)

7 SIMULATIONS

Examples are shown to demonstrate the functioning of both time-scaling based tracking controllers for the one input case.

7.1 Results of Flatness-Based Solution

We use the feedback described in Subsection 6.1, such that $u_{\tau,1}$ generated by the feedback law drives the time-scaling given in Section 5 together with the measured v_{car} and its two successive time derivatives. The reference trajectory starts from the point $(x_{\tau,ref}(0) = 0, y_{\tau,ref}(0) = 0, \theta_{\tau,ref}(0) = 0)$ and arrives to the point $(x_{\tau,ref}(T) = 10, y_{\tau,ref}(T) = 3.5, \theta_{\tau,ref}(T) = 0)$, all

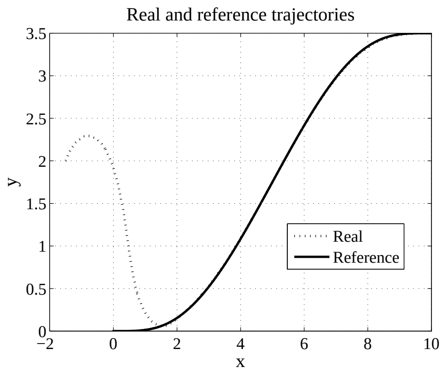

Figure 4: Real and reference trajectories in the horizontal plane – slow driver.

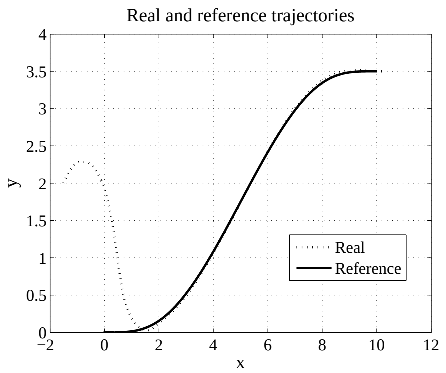

Figure 5: Real and reference trajectories in the horizontal plane – quick driver.

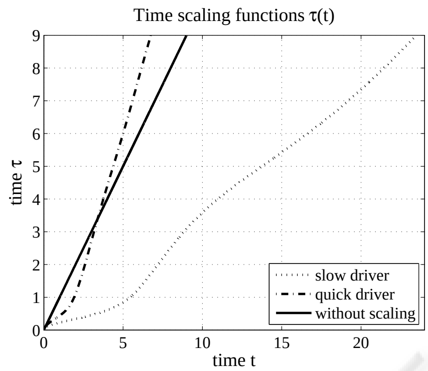

Figure 6: The time-scaling functions $\tau(t)$ along the path for the slow and quick drivers.

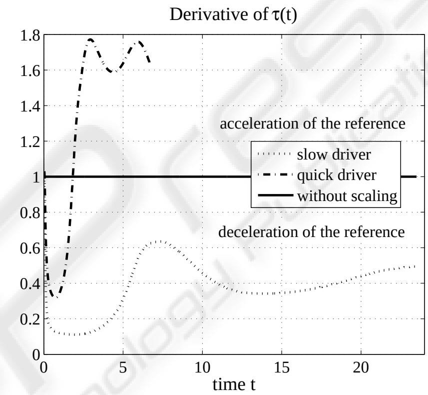

Figure 7: The derivative of the time-scaling functions $\dot{\tau}$.

distances are given in meters and the orientation is given in radians. The traveling time of the reference trajectory is $T = 9$ seconds.

The real initial configuration of the WMR differs from the one used for motion planning, since $x(0) = -1.5$, $y(0) = 2$, and $\theta(0) = \pi/4$.

Two cases are presented such that the geometry of the reference trajectory and the reference velocity profile obtained are the same. The driver's behavior is different for the two cases. In the first case, referred to as the *slow driver* case, the driver imposes considerably slower velocities than those obtained by the motion planning. In the second case, referred to as the *quick driver* case, the driver generates higher velocities than the reference velocity profile. All velocity profiles are given in Figure 3.

The geometries of the reference trajectories and the real trajectories in the horizontal plane are depicted in Figure 4 (slow driver) and in Figure 5 (quick driver). Exponential tracking of the reference trajectory is achieved for each scenario with similar geometry of the real path. Figure 6 and Figure 7 show the effects of on-line time-scaling. If the car is driven by a slow driver it needed more than 23 seconds according to time t to achieve the traveling time $T = 9$ sec which is given for the reference trajectory according to the time τ. The reference in τ was decelerated all along the trajectory ($\dot{\tau} < 1$). The deceleration is also accentuated at low values of t which corresponds to large tracking errors. The time-scaling is completely different for the quick driver who reaches the end of the trajectory faster according to the time t than according to the time τ which means that the reference was accelerated except a short section at the beginning where the tracking error elimination slows down the time-scaling despite the driver's efforts.

7.2 Results Obtained by State Feedback

Here we use the feedback law described in the subsection 6.2 such that the same reference trajectory and initial configuration were used as in the previous subsection.

The reference trajectory and the real path are shown in Figure 8 for the velocity profiles depicted in Figure 9. We achieved exponential tracking.

If the difference between the real and reference initial configurations is larger, the linearized model is

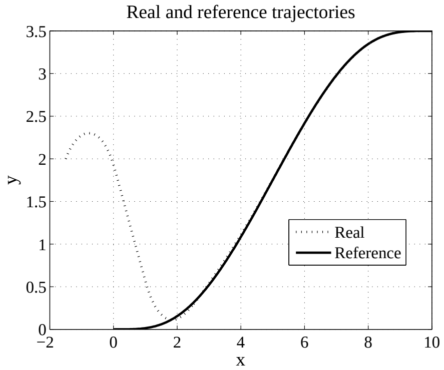

Figure 8: Real and reference trajectories in the horizontal plane – simulation 1.

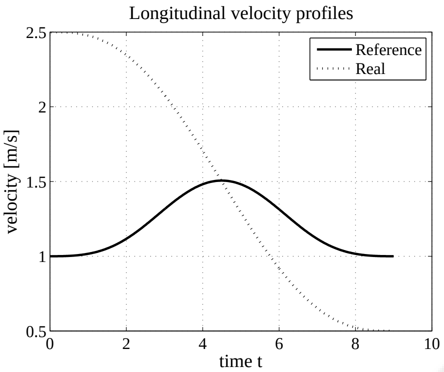

Figure 9: Velocity profiles for the reference and for the car in simulation 1.

no longer valid and the time-scaling may rewind.

8 CONCLUSION

The paper presented two time-scaling based tracking control methods for WMRs with one input such that the longitudinal velocity of the vehicle is generated externally and cannot be considered as a control input. The time-scaling involves the car velocity and its derivatives which need to be measured or estimated. For the tracking controller designed for the linearized error dynamics the time derivatives of the velocity are not needed. The exponential decay of the initial error along the trajectory can be ensured. The results can be extended for the *n*-trailer case.

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