A CLOSED-FORM SOLUTION FOR THE GENERIC SELF-CALIBRATION OF CENTRAL CAMERAS FROM TWO ROTATIONAL FLOWS

Ferran Espuny
Departament d'Àlgebra i Geometria, Universitat de Barcelona, Spain

Keywords: Self-calibration, generic camera, non-parametric sensor, optical flow.

Abstract: In this paper we address the problem of self-calibrating a differentiable generic camera from two rotational flows defined on an open set of the image. Such a camera model can be used for any central smooth imaging system, and thus any given method for the generic model can be applied to many different vision systems.
We give a theoretical closed-form solution to the problem, proving that the ambiguity in the obtained solution is metric (up to an orthogonal linear transformation). Based in the theoretical results, we contribute with an algorithm to achieve metric self-calibration of any central generic camera using two optical flows observed in (part of) the image, which correspond to two infinitesimal rotations of the camera.

1 INTRODUCTION

The first proposed generic camera model consisted of a finite set of pixels and imaging rays in a one-to-one correspondence; its calibration-from-pattern was already solved in a quite pleasant way (Sturm and Ramalingam, 2004; Grossberg and Nayar, 2001), although some questions remain open. This model can be used for any vision system with little assumption, in contrast with the classical approaches that impose a parametric restriction to estimate a model (Hartley and Zisserman, 2000).

In (Ramalingan et al., 2005) a first metric self-calibration (calibration without scene o motion knowledge) algorithm was presented from at least two rotations and one translation of a generic central camera, i.e. with a single effective viewpoint. The authors explicitly admitted that the model should be changed to a continuous one with infinitely many rays.

We consider the continuous (resp. differentiable) generic central camera model to be described by a continuous (resp. differentiable) bijective map φ between an sphere and the image plane. An image is obtained by composing the central projection on the sphere (the ideal central camera) with this *warping map* from the sphere onto the image plane. Note that φ gives us a one-to-one correspondence between image points and projection rays, and thus our definition is consistent with the discrete generic camera model.

The differentiable model was introduced in (Nistér et al., 2005), where a closed-form formula was given for the projective self-calibration (i.e. recovering φ up to projective ambiguity) from at least three observed optical flows corresponding to three infinitesimal rotations. In (Grossmann et al., 2006), a first method for metric self-calibration from only two rotational flows was given. The method gave a experimentally unique solution, which was shown to be extremely sensitive to noise and even to fail with certain exact simulated flows.

We give a theoretical closed-form solution to the problem of self-calibrating a differentiable generic camera from two rotational flows defined on an open set of the image. We also proof that the solution is unique up to an orthogonal linear transformation. Our main contribution is an algorithm to achieve metric self-calibration of any central smooth imaging system using two optical flows observed in (part of) the image, which correspond to two linearly independent rotations. We use simulated data to show that the proposed method performs well with both exact and noisy optical flows.

Espuny F. (2007).
A CLOSED-FORM SOLUTION FOR THE GENERIC SELF-CALIBRATION OF CENTRAL CAMERAS FROM TWO ROTATIONAL FLOWS.
In *Proceedings of the Second International Conference on Computer Vision Theory and Applications - IFP/IA*, pages 26-31

2 PROBLEM FORMULATION

2.1 Differentiable Generic Camera

We will describe the *calibration* of a general central camera by a regular 2-differentiable map f between the image plane and the unit sphere:

$$\begin{array}{cccc} f: & \mathbb{R}^2 & \to & S^2 \\ & (u,v) & \mapsto & f(u,v). \end{array} \tag{1}$$

We take as ideal central camera the standard spherical projection

$$\begin{array}{cccc} \pi: & \mathbb{R}^3 & \to & S^2 \\ & p & \mapsto & p/\|p\|. \end{array} \tag{2}$$

According to (Nistér et al., 2005) we model any central camera as the composition of π with $\varphi = f^{-1}$, which warps the spherical image onto the image plane. Note that the calibration map f gives us a locally one-to-one correspondence between image points and projection rays, agreeing with the calibration concept introduced by (Sturm and Ramalingam, 2004) for the (discrete) generic camera model.

2.2 Self-Calibration Problem

We assume that we know on the image two 2-differentiable optical flows

$$\begin{array}{cccc} v_i: & \mathbb{R}^2 & \to & \mathbb{R}^2 \\ & (u,v) & \mapsto & v_i(u,v) \,, \end{array} \tag{3}$$

both defined on a common open subset. We also suppose that the observed flows v_i correspond to infinitesimal rotations of the camera with respective (unknown) linearly independent angular velocities ω_i.

Our problem consists in determining the possible angular velocities ω_i and calibration map f that are compatible with the image flows v_i.

Since each infinitesimal Euclidean rotation with angular velocity ω_i induces on the unit sphere a tangent vector field defined by

$$p \in S^2 \mapsto \omega_i \wedge p \in T_p S^2 \,, \tag{4}$$

following (Nistér et al., 2005) the problem can be formulated as that of finding f and ω_i, $i = 1, 2$, such that

$$\boxed{\mathrm{D}f(u,v) \cdot v_i(u,v) = \omega_i \wedge f(u,v)} \,, \tag{5}$$

being $\mathrm{D}f = (f_u | f_v)$ the 3×2 differential matrix of f.

Observe that, although the rotation axes are required to be linearly independent, the induced flows on the sphere will always be linearly dependent along a circle (see Figure 1). Therefore, when this circle is mapped by f^{-1} on an image curve, the optical flows will be linearly dependent along that set of points.

Thus, we restrict ourselves to solve (5) for those points (u,v) in an open connected subset of image points where the optical flows are linearly independent. Once we have determined f on the open connected subsets with independent flows, it can be extended by continuity to the whole image.

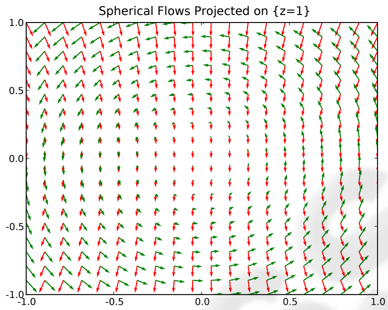

Figure 1: Projection on the plane $\{z = 1\}$ of two spherical flows, with angular velocities $\omega_1 = (1,0,0)$ (going down) and $\omega_2 = (0,0,1)$ (rotating anti-clockwise). The projected flows are linearly dependent along the line $\{y = 0\}$, which corresponds to a circle through the flow singular points.

2.3 Notations

We define the 2×2 matrix $V := (v_1 | v_2)$, and take $\tilde{V} := V^{-1} = \begin{pmatrix} \tilde{v}_{11} & \tilde{v}_{21} \\ \tilde{v}_{12} & \tilde{v}_{22} \end{pmatrix}$, where it exists. Observe that, with these notations, equation (5) says

$$\boxed{\mathrm{D}f = -[f]_\times (\omega_1 | \omega_2) \tilde{V}} \,. \tag{6}$$

For any differentiable function $\varphi : \mathbb{R}^2 \to \mathbb{R}^m$ we denote by $\mathrm{D}\varphi = (\varphi_u | \varphi_v)$ its $m \times 2$ differential matrix.

3 A CLOSED-FORM SOLUTION

3.1 Theoretical Results

For a given *flow matrix* V we want to determine the possible f and ω_i's satisfying the matrix equation (6). First we show that we can reduce the problem to that of determining the angular velocities ω_i.

Let $\tilde{\Delta} = (\tilde{\Delta}_1, \tilde{\Delta}_2)$ be the vector function defined by

$$\begin{pmatrix} \tilde{\Delta}_1 \\ \tilde{\Delta}_2 \end{pmatrix} := \frac{\partial}{\partial v} \begin{pmatrix} \tilde{v}_{11} \\ \tilde{v}_{12} \end{pmatrix} - \frac{\partial}{\partial u} \begin{pmatrix} \tilde{v}_{21} \\ \tilde{v}_{22} \end{pmatrix} \,. \tag{7}$$

Theorem 1. *Assume that we know* ω_1 *and* ω_2*, the angular velocities of the infinitesimal rotation motions. We consider*

$$\tilde{g} := \tilde{\Delta}_1\omega_1 + \tilde{\Delta}_2\omega_2 + \det\tilde{V}\omega_1\wedge\omega_2 \neq 0\ , \qquad (8)$$

Then, the calibration map f *can be computed as*

$$f = \pm\frac{\tilde{g}}{\|\tilde{g}\|}.$$

Proof. If we know ω_1 and ω_2 we can compute the following 3×2 functions:

$$a := (\omega_1|\omega_2)\begin{pmatrix}\tilde{v}_{11}\\ \tilde{v}_{12}\end{pmatrix}\ , \quad b := (\omega_1|\omega_2)\begin{pmatrix}\tilde{v}_{21}\\ \tilde{v}_{22}\end{pmatrix}\ . \qquad (9)$$

By (6), we are looking for f such that

$$\left\{\begin{array}{lcl} f_u & = & [a]_\times\ f\ , \\ f_v & = & [b]_\times\ f\ . \end{array}\right. \qquad (10)$$

Since f is 2-differentiable, it must hold

$$\begin{aligned}
0 &= f_{uv} - f_{vu} = (f_u)_v - (f_v)_u \\
&\overset{(10)}{=} [a_v]_\times\ f + [a]_\times\ f_v - [b_u]_\times\ f - [b]_\times\ f_u \\
&\overset{(10)}{=} [a_v - b_u]_\times\ f + ([a]_\times[b]_\times - [b]_\times[a]_\times) f \\
&= [a_v - b_u + a\wedge b]_\times\ f \\
&\overset{(8)}{=} [\tilde{g}]_\times f\ ,
\end{aligned}$$

from what it follows that f and $\tilde{g}$ must be proportional. Observe that, since the vectors ω_1, ω_2 and $\omega_1\wedge\omega_2$ form a basis of $\mathbb{R}^3$, and $\det\tilde{V}\neq 0$, the function $\tilde{g}$ never vanishes. Finally, the result can be obtained by imposing $\|f\| = 1$. □

Remark 1. In order to improve the stability of the numerical computation of f, we will use the formula $f = \pm g/\|g\|$, with $g = (\det V\tilde{g})$ expressed as follows:

$$g = \Delta_1\omega_1 + \Delta_2\omega_2 + \omega_1\wedge\omega_2\ , \qquad (11)$$

where $\Delta = (\Delta_1, \Delta_2)$ is given by

$$\begin{pmatrix}\Delta_1\\ \Delta_2\end{pmatrix} := \begin{pmatrix} \frac{\partial v_{21}}{\partial u} + \frac{\partial v_{22}}{\partial v} - \frac{\det V_u}{\det V}v_{21} - \frac{\det V_v}{\det V}v_{22} \\ -\frac{\partial v_{11}}{\partial u} - \frac{\partial v_{12}}{\partial v} + \frac{\det V_u}{\det V}v_{11} + \frac{\det V_v}{\det V}v_{12} \end{pmatrix}\ . \qquad (12)$$

Next we give a closed-form formula to find ω_i, the rotation flow angular velocities, using only the image flows v_i. We also determine the ambiguity in the given solution.

Theorem 2. *The matrix* $G_\omega := (\omega_1|\omega_2)^t(\omega_1|\omega_2)$ *can be determined from* V *using the formula*

$$G_\omega = \begin{pmatrix}\Delta_2\\ -\Delta_1\end{pmatrix}(-\Delta_2|\Delta_1) - \begin{pmatrix}D\Delta_2\\ -D\Delta_1\end{pmatrix}V\ . \qquad (13)$$

Thus, given V *the angular velocities* ω_i *can be determined up to an orthogonal transformation of the Euclidean space* $\mathbb{R}^3$.

Proof. By imposing (6) to the function $f = \pm\frac{g}{\|g\|}$, where g is defined in (11), we obtain (note that the sign ambiguity cancels out in both sides):

$$\begin{aligned}
-\left[\frac{g}{\|g\|}\right]_\times(\omega_1|\omega_2)V^{-1} &= \mathrm{D}(\frac{g}{\|g\|}) \\
&= g\mathrm{D}(\frac{1}{\|g\|}) + \frac{1}{\|g\|}\mathrm{D}g \\
&= -\frac{1}{\|g\|}(g\frac{g^t\mathrm{D}g}{g^tg} - \mathrm{D}g)\ ,
\end{aligned}$$

where in the last step we have used that $\|g\| = \sqrt{g^tg}$. Thus, taking $A := \frac{g^t\mathrm{D}g}{g^tg}$, we have obtained the following relation:

$$[g]_\times(\omega_1|\omega_2) = (gA - \mathrm{D}g)V\ . \qquad (14)$$

By (11) the left-hand side term in (14) is

$$[\Delta_1\omega_1 + \Delta_2\omega_2]_\times(\omega_1|\omega_2) + [\omega_1\wedge\omega_2]_\times(\omega_1|\omega_2)\ ,$$

which can be simplified as

$$\omega_1\wedge\omega_2\ (-\Delta_2|\Delta_1) + (\omega_1|\omega_2)\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}G_\omega\ . \qquad (15)$$

Now, (14) can be decomposed in two equalities. The first one corresponds to the components on the direction of $\omega_1\wedge\omega_2$: $(-\Delta_2|\Delta_1) = A\,V$, which gives us

$$A = (-\Delta_2|\Delta_1)V^{-1}\ . \qquad (16)$$

A second equality can be obtained by comparing the parts in (14) on the plane generated by ω_1 and ω_2:

$$\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}G_\omega = (\Delta\,A - \mathrm{D}\Delta)V\ . \qquad (17)$$

The formula for G_ω in the Theorem follows from this last equality using (16). Since we can only know the norm and scalar product of the ω_i, they are determined up to an orthogonal transformation $M\in\mathrm{O}(\mathbb{R}^3)$. Observe that the remaining ambiguity is inherent to the problem: if f and ω_i satisfy (5), then Mf and $M\omega_i$ also give a solution. □

3.2 Self-Calibration Algorithm

Assume that we know two optical flows v_1, v_2 defined on the whole image, corresponding to two infinitesimal rotations of unknown angular velocities ω_1, ω_2.

We fix two directions $d_1, d_2\in\mathbb{R}^3$, which we will use to remove the ambiguity in the determination of the solution.

We propose the following algorithm:

1) Compute $\varepsilon(u,v)$ a measure of linear dependence of the flows at each image pixel:

$$\varepsilon := \frac{\det(v_1|v_2)}{\|v_1\|^2 + \|v_2\|^2} \ . \qquad (18)$$

2) Compute the functions $\Delta_i(u,v)$ defined in (12), and use the formula for G_ω in (13) to compute matrices $G(u,v) = \begin{pmatrix} A & B_1 \\ B_2 & C \end{pmatrix}$.

3) Compute $\mathcal{C}$ the set of pixels (u,v) such that:

i) $A(u,v) > 0$,

ii) $C(u,v) > 0$,

iii) $|B_1(u,v) - B_2(u,v)| < \text{median}(|B_1 - B_2|)$,

iv) (u,v) is not in the border of the image,

v) $\varepsilon(u,v) > \text{median}(\varepsilon)$.

Using $B := (B_1 + B_2)/2$, take the means of A, B and C inside $\mathcal{C}$ as the coefficients of G_ω.

4) Compute ω_1,ω_2 such that $\omega_1 = \lambda_1 d_1$ and $\omega_2 = \mu_1 d_1 + \mu_2 d_2$, with $\lambda_1 > 0$, $\mu_2 > 0$ and $(\omega_1|\omega_2)^t(\omega_1|\omega_2) = G_\omega$.

5) Take $g(u,v)$ as defined in (11), and finally

$$f(u,v) := \text{sign}(g_3(u,v)) \ \frac{g}{\|g\|} \ , \qquad (19)$$

unless fixing $f_3(u,v) > 0$ is not convenient.

3.3 Comments

Since in practice we only know the optical flow on a grid of pixels, the computation of the Δ_i, which requires first derivatives, and the computation of G_ω, involving second derivatives of the flow, will be less accurate at the borders of the image. Note also that the error (5) in a few pixels could be big due to the division in (12); the estimation of f in those pixels can be improved by imposing the smoothness of f in a neighborhood containing pixels with lower error.

Besides that, the formula (13) gives us as many estimators for G_ω as image points. We select in step 3 those points inside the image where the matrices are (closer to be) symmetric definite positive and the optical flows are (closer to be) linearly independent. Alternative criteria can be used, specially if the suggested conditions turn out to be too restrictive.

Finally, reversing the order in the flows (and rotations) changes the sign of the $\Delta_i(u,v)$ in (12), and thus the sign of g in (11). Fixing $f_3(u,v) > 0$ in step 5 makes the algorithm independent of the flows order.

4 EXPERIMENTAL RESULTS

We simulated different generic calibration maps $f : S^2 \to \mathbb{R}^2$ by composing $T : S^2 \to \mathbb{R}^2$, the central projection from the unit sphere onto the plane $\{z = 1\}$, with rectifying maps $F : \mathbb{R}^2 \to \mathbb{R}^2$ defined on the plane $\{z = 1\}$. We considered different maps (Grossmann et al., 2006):

1. $F(u,v,1) = K^{-1}(u,v,1)$, *pinhole sensor*,
2. $F(u,v,1) = (u, \frac{1}{2}(v + \sin(\frac{3\pi u}{4})), 1)$, *sine sensor*,
3. $F(u,v,1) = (10^{\frac{u-1}{2}} \cos(\pi v), 10^{\frac{u-1}{2}} \sin(\pi v), 1)$, *log-polar sensor*,
4. $F(u,v) = (\frac{\tan(\theta\sqrt{u^2+v^2})}{2\tan(\frac{\theta}{2})\sqrt{u^2+v^2}}u, \frac{\tan(\theta\sqrt{u^2+v^2})}{2\tan(\frac{\theta}{2})\sqrt{u^2+v^2}}v, 1)$, a *fish-eye model* with angular field of view θ (Devernay and Faugeras, 2001),

with $(u,v) \in (-1,1) \times (-1,1)$. See Figure 2 for a 20×20 discrete representation of the sensors.

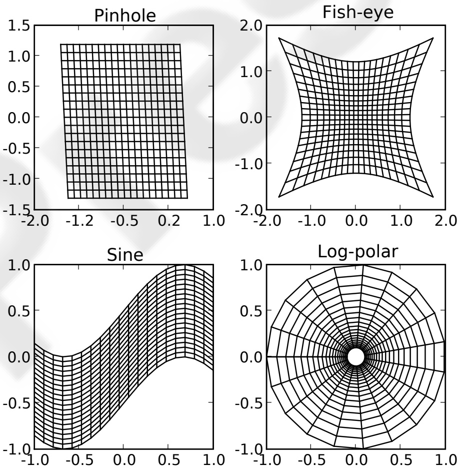

Figure 2: Examples of the considered "unwarping" maps f.

In a first experiment, we wanted to study the behavior of Algorithm 3.2 with exact data. For each sensor, we simulated two optical flows in a 20×20 discrete image, like those in Figure 3, and computed the estimations for G_ω and f. The goodness of the obtained f with the Pinhole and Sine models can be observed in the upper-left picture in figures 5 and 6 respectively, for $\omega_1 = (0.2, 0, 0)$ and $\omega_2 = (0, 0, 0.2)$.

Since our algorithm needs to estimate numerically the derivatives of the optical flows, it is expected to work better with very dense flows. The improvement in the solution with respect to the size of the image flow grid (taken to have $N \times N$ pixels, with N ranging from 20 to 300) can be observed in Figure 4. We

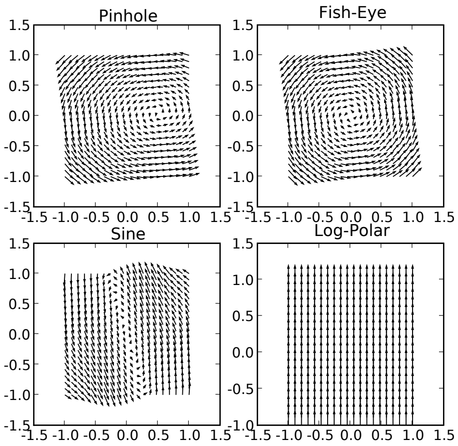

Figure 3: Image optical flows with $\omega = (0,0,0.2)$.

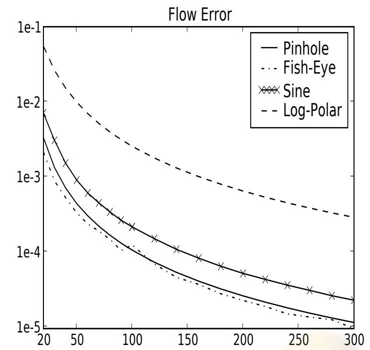

Figure 4: Errors with exact data for different image grid sizes.

used as an over-all error measure the mean of all the differences in the Flow equation (5).

In a similar way, we varied the field of view in the Pinhole and Log-Polar sensors, and observed that our closed-form formulas gave better results with bigger field of view, i.e. bigger image changes.

In order to simulate real (regular) optical flow data, we perturbed exact 300×300 image rotational flows with gaussian noise s relative to the flows (i.e. $v_{ij}^{\mathrm{sim}} = (1+s)v_{ij}^{\mathrm{exact}}$), then smoothed the flow with a Gaussian convolution and finally downsampled it into a 20×20 flow. In Figures 5 and 6 we show examples of typical self-calibration results with noise for $\omega_1 = (0.2,0,0)$ and $\omega_2 = (0,0,0.2)$.

We observed that, specially in presence of noise, not only the product matrix G_ω was important, but also the initial directions of the ω_i. As an example, and to summarize, we show in figures 7 and 8 the be-

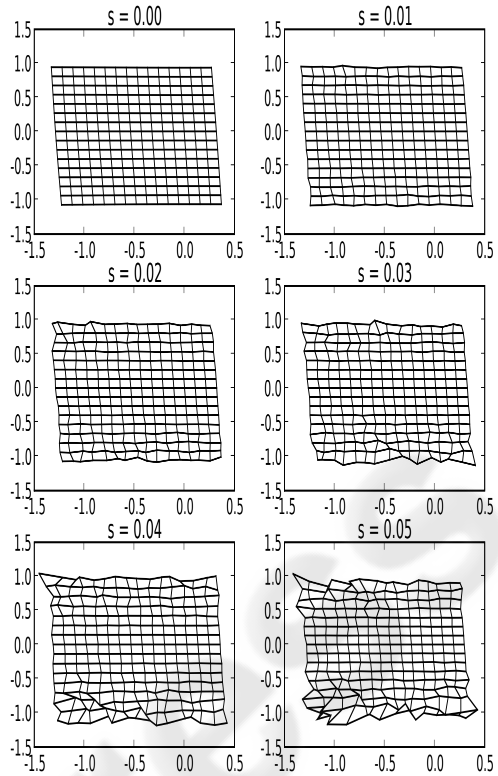

Figure 5: Calibration example of the Pinhole sensor with noise s relative to optical flow; x and z rotation axes.

havior of the considered models with $\omega_1 = (0.2,0,0)$ and either $\omega_2 = (0,0.2,0)$ or $\omega_2 = (0,0,0.2)$. Note that in both cases G_ω is the same. The relative error in the estimation of G_ω (Figure 7) corresponds to the matrix norm of the difference with its true value divided by four times the norm of the true matrix.

5 CONCLUSIONS

In this paper we have shown that it is possible to solve in a closed-form way the problem of self-calibrating any differentiable generic central camera from only two rotational flows, not necessarily observed on the whole image. We have proved that the only remaining ambiguity in the solution is an orthogonal displacement, which affects both the estimation of the rotation angular velocities and the calibration map.

We have also given a self-calibration algorithm based on the previous results. Using simulated data, we have shown that it works quite well with noisy regular optical flows, and that its performance improves with dense image flows and big fields of view. In the future, strongly encouraged by the obtained results,

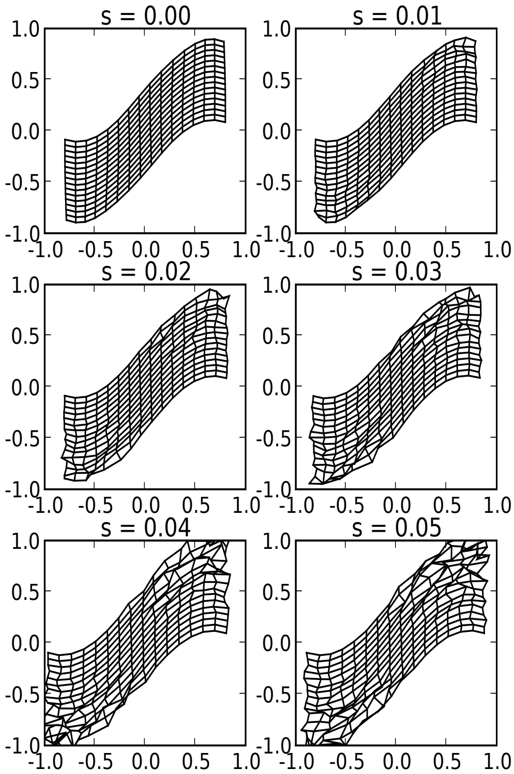

Figure 6: Calibration example of the Sine sensor with noise *s* relative to optical flow; x and z rotation axes.

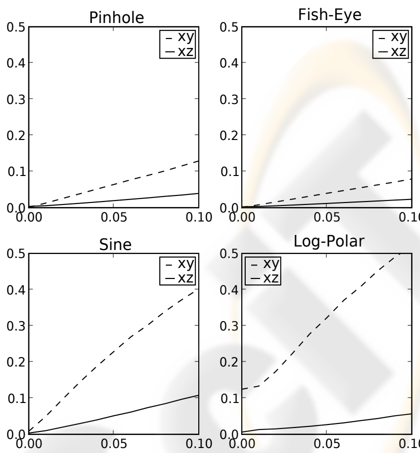

Figure 7: Relative error in G_ω with respect to noise relative to the flows; two different rotation pairs are shown.

we will use our algorithm with pairs of real rotational flows and also adapt it to have a robust self-calibration method able to work with more than two flows.

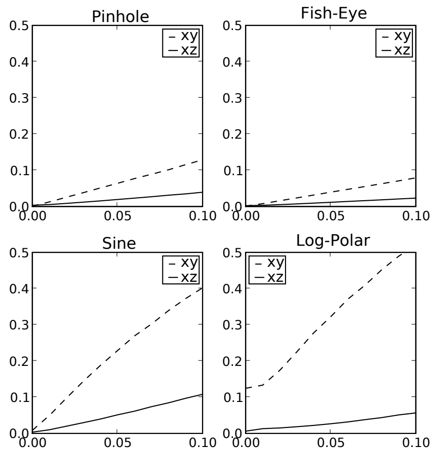

Figure 8: Flow mean error with respect to noise relative to the flows; two different rotation pairs are shown.

ACKNOWLEDGEMENTS

We would like to thank Dr. Peter Sturm for suggesting the problem and hosting the author at INRIA Rhône-Alpes. This work has also been possible thanks to the advice of Dr. José Ignacio Burgos and the financial support of the project BFM2003-02914 from the Spanish Ministerio de Ciencia y Tecnología.

REFERENCES

Devernay, F. and Faugeras, O. D. (2001). Straight lines have to be straight. *Machine Vision and Applications*, 13(1):14–24.

Grossberg, M. D. and Nayar, S. K. (2001). A general imaging model and a method for finding its parameters. In *Proc. ICCV, Vol. 2, pp. 108–115*.

Grossmann, E., Lee, E., Hislop, P., Nistér, D., and Stewénius, H. (2006). Are two rotational flows sufficient to calibrate a smooth non-parametric sensor? In *Proc. CVPR, pp. 1222–1229*.

Hartley, R. I. and Zisserman, A. (2000). *Multiple View Geometry in Computer Vision*. Cambridge University Press, ISBN: 0521623049.

Nistér, D., Stewénius, H., and Grossmann, E. (2005). Non-parametric self-calibration. In *Proc. ICCV, pp. 120–127*.

Ramalingan, S., Sturm, P., and Lodha, S. (2005). Towards generic self-calibration of central cameras. In *Proc. OMNIVIS Workshop, pp. 20–27*.

Sturm, P. and Ramalingam, S. (2004). A generic concept for camera calibration. In *Proc. ECCV, Vol. 2, pp. 1–13*.