

TURBO SPACE TIME CODES FOR TAILBITING TRANSMISSION

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Abstract: This paper studies a novel technique that combines turbo tailbiting codes over the ring with symbol interleaved space-time modulation. After initial theoretical introduction and analysis, the simulation results are presented. It is shown that for different antenna array sizes, different packet sizes and two kinds of channels the Turbo space time tailbiting coding have a clear advantage over the transmission method with direct truncation.

1 INTRODUCTION

Driven by the increasing need for capacity and system availability whilst using the same bandwidth, multiple-input-multiple-output (MIMO) technique has become an interesting extension to wireless communication systems (Vucetic, 2003). With the introduction of multiple antenna arrays came the need for proper channel coding and transmission method. Recently, in order to improve the error performance of wireless systems techniques combining space-time (ST) and turbo coding have been proposed. Typically in such systems packet data transmission is used. However, in order to use convolutional codes in the packet transmission we must convert these codes to block codes. There are some well known methods for this conversion (Ma, 1986). One of them is called Direct Truncation. The most popular method is called Tailbiting (TB). In this method we transmit the convolutional coded data in a block form without known tail. By framing coded data we are not adding known bits to the end of the data information stream (Cox, 1994). Simply the binary encoder starts and finishes the encoding process in the same state. This state is not known to the decoder.

In this paper, we propose a novel technique that combines turbo tailbiting codes over the ring with symbol interleaved space-time modulation. The turbo encoder consists of two feedback systematic convolutional encoders over the ring of integers modulo- M . This encoder is connected to s -random

symbol interleaver and next to space-time modulator. We investigate the system which contains a turbo space time encoder over ring modulo-4 and QPSK modulator.

The performance of the proposed system for various packet sizes, different number of transmit antennas and MIMO quasi-static or fast fading channel is evaluated by simulation.

This paper is organised as follows. Section 2 describes the TB encoding procedure which uses the feedback systematic convolutional encoders over ring Z_M . In Section 3, we present the structure of space time turbo encoder over ring Z_M . In Section 4, the MIMO channel model is described. Section 5 explains iterative decoding with symbol interleaver. Section 6 presents the simulation results. Finally, a conclusion is drawn in Section 7.

2 TAILBITING CONVOLUTIONAL CODES OVER RING

In this paper we concentrate on the usage of non-binary codes (Massey, 1989). There are a few reasons for using them. They have larger Euclidean distances than codes over $GF(2)$. Moreover, they make the usage of mapping and set partitioning not necessary. In the analyzed method we encode and decode a block of N (M -ary) symbols without a know tail, thus keeping the effective rate of transmission.

The encoding procedure to achieve this is not difficult if the structure of the used encoder is feedforward. Then the starting state depends only on the m last information symbols in the transmitted packet (m is the number of memory cells in the encoder). In case of using a recursive systematic convolutional encoder (RSC) with feedback (Figure 1) the ending state depends on all of the information symbols in the packet. Finding initial state, wherein the encoder should start its work and after processing N symbols to end encoding in the same state is complex and not always possible (Weiß, 2001).

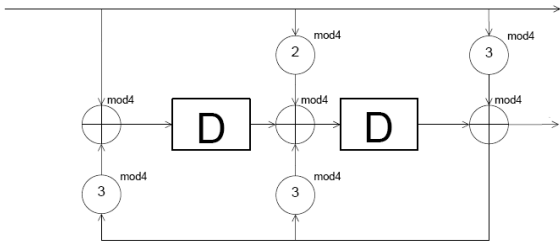


Figure 1: RSC encoder over ring $\mathfrak{R}=\mathbb{Z}_4$.

In Figure 1, we show an example of the RSC encoder over the ring $\mathfrak{R}=\mathbb{Z}_4$ of integers modulo-4 (Remlein, 2003). The code rate is $R=1/2$ ($k=1, n=2$) and the transfer function matrix is:

At instant t an information vector with M -ary

$$G(D) = \begin{bmatrix} 1 & 3 + 2D + D^2 \\ 1 + 3D + 3D^2 \end{bmatrix}$$

symbols belonging to the ring $Z_M = \{0, 1, 2, \dots, M-1\}$, ($\mathfrak{R}=\mathbb{Z}_M$; $M=4$) inputs the encoder. The convolutional encoder generates encoded vector which contains sequence of the symbols elements belonging to the same ring $\mathfrak{R}=\mathbb{Z}_4$.

The encoder coefficients in Fig. 1 are from the set $\{0, \dots, M-1\}$, $M=4$. The memory cells are capable of storing the ring elements. Multiplications and additions are performed in the ring of integers modulo- M .

3 TURBO SPACE TIME ENCODER OVER RING

Model of the system analyzed in this article joins packet transmission tailbiting convolutional encoding over the ring with space time encoding. The convolution codes over the ring Z_M considered in (Remlein, 2003) were not optimized for the multi antennas systems. Authors of this article used these

codes in the system shown in Figure 2. This is a modification of solution considered in (Stefanou, 2001). We propose placing the tailbiting turbo encoder over the ring in the transmitter. In our proposal there is no need to consider set-partitioning and symbol mapping. The usage of M -symbol alphabet in the encoder in a natural way maps symbols into M -PSK signal elements. Every constellation point has its equivalent symbol.

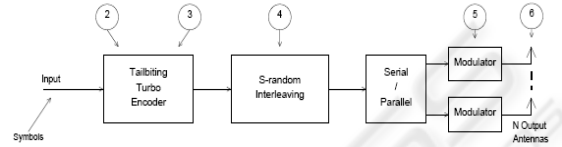


Figure 2: Transmitter diagram.

The transmitter (Fig. 2) is constructed as follows:

1. Construct encoder for M symbol alphabet.
2. Fed the turbo encoder with uninterleaved and interleaved data sequence.
3. Turbo encode the uninterleaved and interleaved input sequence.
4. Interleave the encoder output sequence in S -random interleaver.
5. Modulate the signals.
6. Input symbol to different antenna in the array.

In Figure 3 it is shown the turbo encoder (Berrou, 1993) used in the transmitter.

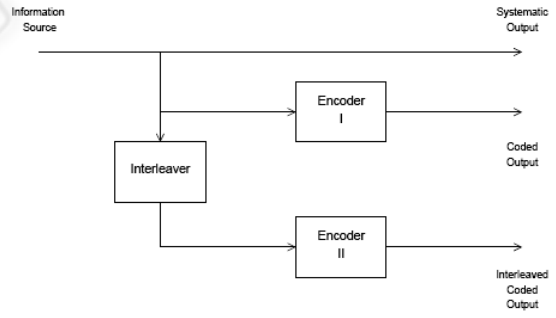


Figure 3: Structure of the turbo encoder.

The encoder I and encoder II are RSC encoders shown in Figure 1. The interleaver is an UMTS type interleaver (3GPP TS 25.212). The interleaver is responsible for decorrelating inputs to encoder I and encoder II.

The S -random interleaver (Dolinar, 1995) reorders the turbo coded sequence to further decorrelate the symbols before feeding the antenna inputs.

The S-random interleaver (S=1,2,...) is a semirandom interleaver. Each randomly selected integer is compared with S previously selected integers. If the difference between them is smaller than S, the current selection is rejected and a new integer is generated. The process repeats until N distinct integers are selected.

4 CHANNEL MODEL

The telecommunication system with n_T transmit and n_R receive antennas we consider can be described by the following formula:

$$y = Hs + v \tag{1}$$

where $s \in C^{n_T \times 1}$ defines the transmission vector, $v \in C^{n_R \times 1}$ defines the additive white Gaussian noise vector, $H \in C^{n_R \times n_T}$ is the channel matrix, that describes the connections between the transmitter and receiver and can be expressed as:

$$H = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1N} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{M1} & \alpha_{M2} & \dots & \alpha_{MN} \end{bmatrix} \tag{2}$$

Where α_{mn} is the complex transmission coefficient between element m at the transmitter (TX) and element n at the receiver (RX). To generate channel matrix H, we used the narrowband Kronecker model presented in (Ozcelik, 2003). In this model it is assumed that the receive correlation matrix is independent of the corresponding transmit matrix and vice-versa. The channel model used in this paper was further simplified according to the assumptions in (Ozcelik, 2003). To calculate the correlation coefficients between the antennas in the transmitting as well as in the receiving array, we followed the approach in (Loyka, 2002).

Figure 4 depicts the influence of distance between array elements on capacity. The simulations were made for the case of 2 element arrays. The solid line presents the idealized case, when no correlation can be observed among the antennas at the mobile station (MS). As we can see, the line representing the distance of 0.5 wavelength, lies very close to the solid line. Hence, we can assume that this distance is sufficient for normal operation. The distance between elements at the Base Station (BS) is normally much larger than in the Mobile Station, and it does not have much influence on the capacity. Therefore, we used for the simulations the

following parameters: BS azimuth spread = 18°, BS distance=10 λ. MS azimuth spread = 90°, MS distance=0.5 λ.

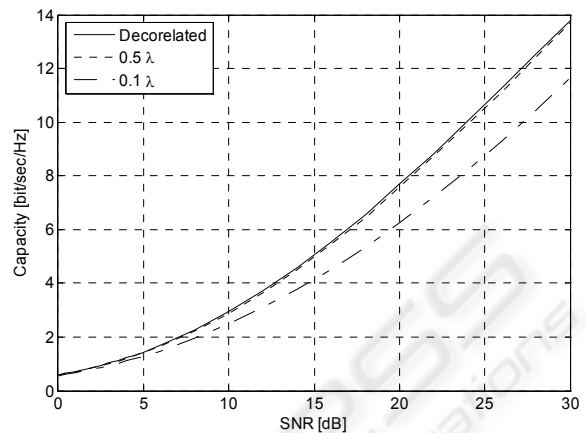


Figure 4: Capacity vs distance between array elements for 2 antennas. BS azimuth spread = 18°. MS azimuth spread = 90°.

5 RECEIVER

The block scheme of the receiver used in the analyzed system is shown in Figure 5.

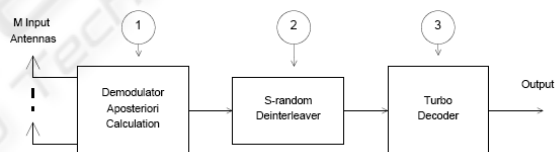


Figure 5: Receiver diagram.

The demodulator performs two operations: it demodulates the signal and it calculates the a posteriori values of the received signal. The demodulated symbols are deinterleaved and fed to the turbo decoder.

The scheme of the turbo decoder is shown in the Figure 6. The decoder uses a well known from the literature connection of two SISO (Soft Input Soft Output) decoders (Hagenauer, 1996) with feedback.

The crucial and most interesting part of the decoding process is the iterative exchange of information between decoders. This is also the reason for calling this method “turbo”. With each iteration, the Bit/Symbol error rate drops. Now we can see, why there is so much research on proper interleavers. Let us consider a situation when the channel undergoes a deep fade. With great probability part of the transmitted signal is lost, but

due to interleaving and trellis decoding, it might be reconstructed.

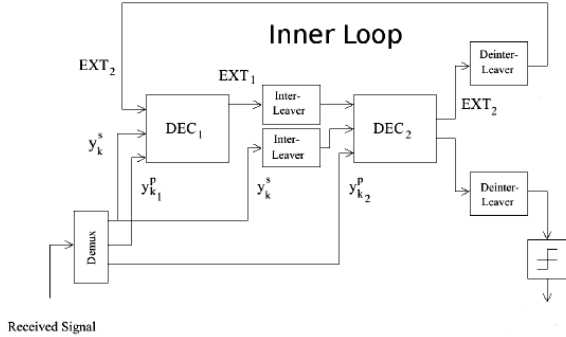


Figure 6: Turbo decoder. EXT- extrinsic information, y_k^s - systematic information, $y_{k_1}^p$ - parity information from the first encoder, $y_{k_2}^p$ - parity information from the second decoder.

The technique that allows the turbo codes to almost achieve the Shannon bound is in fact the exchange of information between the decoding blocks. To process the information in the receiver so called Soft Input Soft Output Decoder (SISO) (Hagenauer, 1989) was developed. Our SISO decoder uses the improved Soft Output Viterbi Algorithm which is able to decode the non-binary codes (Cong, 1999). The processing steps of SISO decoder are explained after the notation is described.

u_i is the i -th symbol from the alphabet $\{0, 1, \dots, M-1\}$;

$\Gamma(s_k)$ is cumulative metric value at state s_k ;
 $\hat{L}(s_k) = [\hat{L}_{j,\mu}(s_k)]$ is $\delta \times M$ reliability measure matrix stored at each state, where $j = k - \delta + 1, \dots, k$, $\mu = 0, 1, \dots, M-1$, and δ is the size of the decoding window;

Δ is reliability difference between the survivor at state s_k and the most likely path terminating in the state s_k with $u_j = \mu$.

At each state s_k corresponding to the decoding time k , SOVA calculates and stores the $\Gamma(s_k)$ as in the normal Viterbi Algorithm, additionally it stores $\hat{L}(s_k)$. At each step $(k+1)$ for every state s_{k+1} the algorithm evaluates cumulative metric candidates

$\Gamma(s_k^0, s_{k+1}), \dots, \Gamma(s_k^{M-1}, s_{k+1})$ originating from s_k and selects the minimum cumulative metric $\Gamma(s_{k+1})$. However, it does not neglect the rest, they are being stored as competitive paths. After finding

$\Gamma(s_{k+1})$, the reliability differences Δ can be calculated in the following way:

$$\Delta_m = \Gamma(s_k^m, s_{k+1}) - \min_{m \in \{0, \dots, M-1\}} \Gamma(s_k^m, s_{k+1}) \quad (3)$$

where $\Delta_m = 0$ for the surviving path at state s_{k+1} .

We first set $\hat{L}_{k+1,\mu}(s_{k+1}) = \Delta_\mu$, because Δ_μ is the reliability difference between the survivor at s_{k+1} and the most likely path terminating in state s_{k+1} with $u_{k+1} = \mu$. The next step is to update the remaining values in $\hat{L}_{j,\mu}(s_{k+1})$, $j = k - \delta + 1, \dots, k$ based on the following rule:

$$\hat{L}_{j,\mu}^m(s_{k+1}) = \min_{m \in \{0, \dots, M-1\}} \{ \hat{L}_{j,\mu}^m + \Delta_m \} \quad (4)$$

where $\hat{L}_{j,\mu}^m \equiv \hat{L}_{j,\mu}(s_k^m)$. The modified SOVA algorithm, shows performance equal to the of Max-Log-MAP algorithm at a lower computational expense (Berrou, 1993).

6 SIMULATION RESULTS

The computer simulations were performed in MATLAB environment. The simulations were conducted for QPSK modulation and RSC encoder over ring Z_4 with the transfer function matrix:

$$G(D) = \begin{bmatrix} 1 & \frac{3+2D+D^2}{1+3D+3D^2} \end{bmatrix}$$

In this paper, both quasi-static and fast fading channels were considered. For quasi static fading, it is assumed that the fading coefficients are constant during a packet transmission. These coefficients vary from one packet to another independently. For fast fading channels the fading coefficients vary from one symbol to another independently.

Basic performance of the turbo decoder for symbol tailbiting over ring transmission after 6 inner iterations with respect to the block length is showed in Figure 7. We observe that using packet of length 500, for $\text{SER} = 10^{-2}$, it is possible to achieve a gain of 2dB with respect to the transmission with packet size equal to 40 symbols.

At first we consider 2 transmit and 2 receive antenna system with a fast fading channel. The simulations were conducted for different block lengths. As expected, the longer the block, the better the SER.

Next simulation was made for 2 different block lengths and for 2 different decoding techniques.

Figure 8 shows a comparison between two decoding methods; tailbiting and direct truncation. As expected, the later method delivers worse results. Another important observation is that for higher SNR, the difference between those two methods is getting larger in favor of tailbiting. We can observe that the tailbiting method improves the performance about 0.3 dB for $SER=10^{-2}$ with respect to the direct truncation method, for transmission with packet size equal to 500.

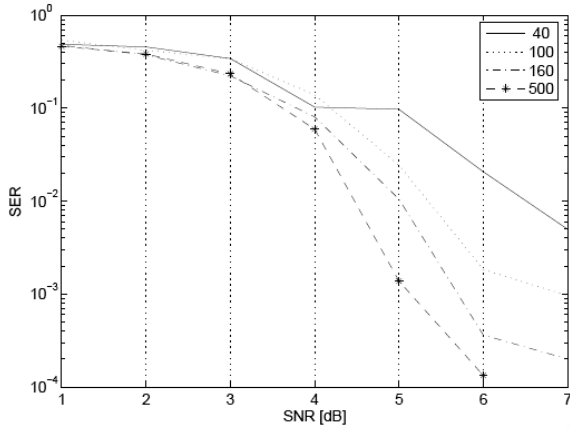


Figure 7: Different block sizes for turbo decoder and tailbiting trellis ending. 2TX, 2RX, 2-symbols/Hz/antennas pair, QPSK, fast fading channel.

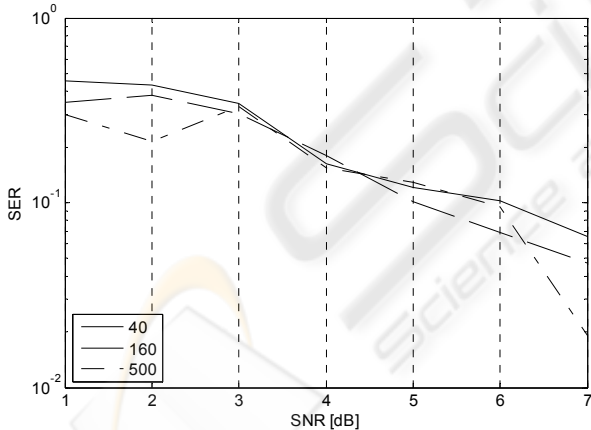


Figure 8: Tailbiting decoding vs Direct truncation trellis ending for 2 block sizes. 2TX, 2RX, 2-symbols/Hz/antennas pair, QPSK, fast fading channel.

For burst channels (with burst length of 130 space-time symbols), (2TX × 2RX) scenario and different block lengths, the situation differs. As is shown in Figure 9, the performance of the system drops significantly.

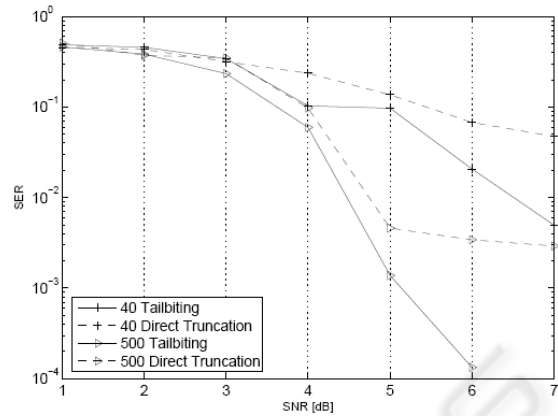


Figure 9: Quasi-static channel (130 space symbols), different block sizes for turbo decoder and tailbiting trellis ending. 2TX, 2RX, 2 symbols/Hz/antennas pair, QPSK.

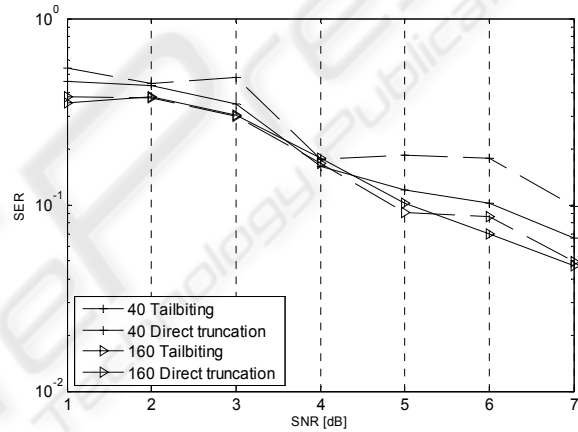


Figure 10: Quasi-static channel (130 space symbols), tailbiting decoding vs direct truncation trellis ending for 2 block sizes. 2TX, 2RX, 2 symbols/Hz/antennas pair, QPSK.

The waterfall region moved from 4 dB in fast fading channel to 6-7 dB in quasi-static channel. For burst channels both decoding methods, tailbiting and direct truncation, yield almost the same performance (still tailbiting performs better), see Figure 10.

Figure 11, shows the relation between SER and different array sizes in a quasi-static channel for 500 block size. At 4 dB there is observable a breakthrough, thus more element antenna arrays deliver better performance. We observe that using 4TX/4RX antennas, for $SER=10^{-2}$, it is possible to achieve a gain about of 0.5 dB with respect to the transmission with 3TX/3RX antennas and the gain about of 1 dB with respect to the system with 2TX/2RX antennas.

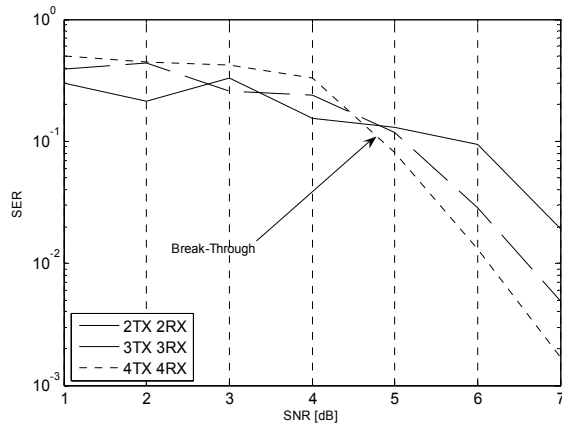


Figure 11: SER performance vs different array sizes, 500 symbols block, quasistatic channel (130 space symbols), for turbo decoder and tailbiting trellis ending. 2-symbols/Hz/antennas pair, QPSK.

7 CONCLUSIONS

New structure of Turbo space time encoder over ring for tailbiting transmission is proposed. This paper aimed at investigating the performance of space time symbol interleaved non-binary turbo tailbiting coded systems. A quasi-static and fast fading channels were considered. In the receiver, were used iterative soft input soft output (SOVA) algorithm.

Analysis of the obtained results shows that the use of tailbiting coding over ring improves the quality of the transmission in comparison to the direct truncation method. An improvement is observed for all packet lengths. We have to mention that the encoding process in tailbiting method is more complex than in direct truncation method.

REFERENCES

- Vucetic, B., Yuan, J., 2003, *Space-Time Coding*, John Wiley & Sons Ltd.
- Ma, H.H., Wolf, J.K., 1986, On Tailbiting Convolutional Codes. *IEEE Trans. Commun.*, vol. 34, Feb. 1986, 104-111.
- Wei, C., Bettstetter, C., Riedel, S., 2001. Code Construction and Decoding of Parallel Concatenated Tail-Biting Codes. *IEEE Trans. Information Theory*, Vol. 47, No.1, January 2001, 366-386.
- 3GPP TS 25.212 version 5.2.0 Release 5
- Remlein, P., 2003. The Encoders with the feedback for the packed transmission without tail symbols. *VIII-th Poznan Workshop on Telecommunication, PWT '03, Pozna 11-12 Dec. 2003*, 165-169, (in polish).
- Ozcelik, H., Herdin, M., Weichselberger, W., Wallace, J., Bonek, E., 2003. Deficiencies of 'Kronecker' MIMO radio channel model. *Electronics Letters*, 7th August 2003, vol.39, No. 16, 1209-1210.
- Loyka, S., Tsoulos, G., 2002. Estimating MIMO System Performance Using the Correlation Matrix Approach. *IEEE Communications Letters*, Vol. 6, NO. 1, January 2002, 19-21.
- Stefanov, A., Duman, T.M., 2001. Turbo-Coded Modulation for Systems with Transmit and Receive Antenna Diversity over Block Fading Channels: System Model, Decoding Approaches, and Practical Considerations. *IEEE Journal on Selected Areas in Communications*, vol. 19, No. 5, May 2001, 958-968.
- Hagenauer, J., Offer, E., Lutz, P., 1996. Iterative Decoding of Binary Block and Convolutional Codes. *IEEE Transactions on Information Theory*, Vol. 42, No. 2, March 1996, 429-445.
- Hagenauer, J., Hoehner, P., 1989. A Viterbi Algorithm with Soft-Decision Outputs and its Applications. In *Proc. Globecom, Dallas, TX, Nov. 1989*, 1680-1686.
- Berrou, C., Glavieux, A., Thitimajshima, P., 1993. Near Shannon Limit Error-Correcting Coding and Decoding: Turbo Codes. *Proc. of the 1993 International Conference on Communications, in Proc. ICC'93, Geneva, Switzerland, May 1993*, 1064-1070.
- Cong, L., Xiaofu, W., Xiaoxin, Y., 1999. On SOVA for Nonbinary Codes. *IEEE Communications Letters*, Vol. 3, No. 12, December 1999, 335-337.
- Cox, R.V., Sundberg, C.W., 1994. An Efficient Adaptive Circular Viterbi Algorithm for Decoding Generalized Tailbiting Convolutional Codes. *IEEE Transactions on Vehicular Technology*, Vol. 43, No. 1, Feb. 1994, 57-68.
- Massey, J. L., Mittelholzer, T., 1989. Convolutional codes over rings. In *Proc. 4th Joint Swedish-USSR Int. Workshop Information Theory*, 14-18.
- Dolinar, S., Divsalar, D., 1995. Weight distribution for turbo codes using random and nonrandom permutations. *JPL Progress report 42-122, Aug. 15*.