OPTIMAL ENERGY ALLOCATION FOR DETECTION IN WIRLESS SENSOR NETWORKS

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Abstract: The problem of binary hypothesis testing in a wireless sensor network is studies in the presence of noisy channels and for non-identical sensors. We have devised an energy allocation scheme for individual sensors in order to optimize a cost function with a constraint on the total network energy. Two cost functions were considered; the probability of error and the J-divergence distance measure. We have also designed a mathematically tractable fusion rule for which optimal energy allocation can be achieved. Results of optimal energy allocation and the resulting probability of error are presented for different sensor network configurations.

1 INTRODUCTION

In recent years wireless sensor networks (WSN) have attracted a great deal of attention from the research community. Typical applications of WSNs include environmental monitoring, surveillance, intruder detection and denial of access, target tracking, and structure monitoring, among others. Wireless sensor networks can also serve as the first line of detection for various types of hazards, such as toxic gas or radiation.

The nodes in wireless sensor networks are powered by batteries for which replacement, if at all possible, is very difficult and expensive. Thus in many scenarios, wireless sensor nodes are expected to operate without battery replacement for many years. Consequently , constraining the energy consumption in the nodes is a very important design consideration. In (Luo and Giannakis, 2004), the authors consider quantization of sensor data and energy allocation for the purpose of estimation under a total energy constraint. Optimal modulation with minimum energy requirements to transmit a given number of bits with a prescribed bit error rate (BER) is considered in (Cui et al., 2005).

In this paper we consider the problem of binary hypothesis testing using wireless sensor networks under energy constraint. Traditionally, the decentralized detection problem has been investigated assuming identical sensor nodes. For example the work reported in (Zhang et al., 2002; Tsitsiklis, 1988; Varshney, 1997), considers identically distributed observations for all the sensor nodes and error-free transmissions from the nodes to the fusion center. In this paper we do not assume identically distributed observations. In particular the observation noise experienced by each sensor may be different. Furthermore, the wireless channels between the sensor nodes and the fusion center is assumed to be a noisy channel. Specifically, it is assumed that the nodes' decision is transmitted using a modulation scheme over an AWGN channel. Our goal is to design a fusion rule and an energy allocation for the nodes so as to minimize a cost function subject to a limit on the total energy of all the nodes. We consider two types of cost functions. The probability of error at the fusion center as well as the divergence distance measure.

The remainder of this paper is organized as follows. In Section 2 we present the system model. The problem of energy allocation for the probability of error and the *J*-divergence cost functions is studied in Sections 3 and 4, respectively. The results are presented in Section 5 and the conclusions are drawn in Section 6.

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2 SYSTEM MODEL

Let H be a binary random variable with prior probability distribution given by $P(H = H_0) = q_0$ and $P(H = H_1) = q_1$. We consider a network of n wireless sensors with sensor k acquiring a measurement X_k about the state of H. Gaussian observations are assumed although the results can be extended to other cases. With this assumption we have

$$\begin{aligned} p_{X_k}(x|H_0) &\sim \mathcal{N}(0,\sigma_k^2) \\ p_{X_k}(x|H_1) &\sim \mathcal{N}(d,\sigma_k^2) \end{aligned} \tag{1}$$

Sensor k computes a local binary decision u_k according to

$$u_k = \begin{cases} 1, \text{ if} & \ln(\frac{p_{X_k}(x|H_1)}{p_{X_k}(x|H_0)}) \geq \lambda_k \\ 0, \text{ if} & \ln(\frac{p_{X_k}(x|H_1)}{p_{X_k}(x|H_0)}) < \lambda_k \end{cases}$$

For the given distribution in (1) the optimal value of λ_k is given by

$$\lambda_k = \frac{d^2 + {\sigma_k}^2(q_0 - q_1)}{2d} \tag{2}$$

The channel between sensor k and the fusion center is modeled by a binary symmetric channel with cross over probability ε_k. The value of u_k is transmitted to the fusion center over this channel and z_k denotes the received bit. Let E_T denote the total energy available to all the sensors. The fraction of energy allocated to sensor k is given by $x_k E_T$ where $0 \leq x_k \leq 1$ and $\sum_{i=1}^n x_i = 1$. For the sake of concreteness we assume that the sensors use a BPSK modulation scheme. The value of ε_k is then given by $\varepsilon_k = Q(\sqrt{2E_T x_k/N_0})$ where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-x^2/2}\, dx$.

3 ERROR PROBABILITY CRITERION

The fusion center receives the sequence $\mathbf{z} = (z_1, z_2, \cdots, z_n)$ and must decide on the state of H. Finding the optimum decision rule for H based on $\mathbf{z}$ is mathematically intractable. We therefore choose the following fusion rule.

$$\psi(\mathbf{z}) = \begin{cases} H_1, & \sum_{i=1}^n \alpha_i z_i \geq \tau \\ H_0, & \sum_{i=1}^n \alpha_i z_i < \tau \end{cases} \tag{3}$$

Our motivation here is that for $\alpha_i = \frac{1}{\sigma_i^2}, i = 1, 2, \cdots, n$, this is the optimal rule if the fusion center had access to the observations $\{X_k\}$. Our goal is to choose the values of $\alpha_i, x_i, i = 1, 2, \cdots, n$, and τ such that the probability or error is minimized subject to the constraint that the total energy of the sensor network for a single measurement and transmission does not exceed E_T.

Evaluation of the performance of this rule requires the distribution of $Z = \sum \alpha_i z_i$. We invoke the central limit theorem and assume that, given either hypothesis, Z is a Gaussian random variable (Eremin, 1999). The conditional moments of Z are then evaluated as follows.

$$E(Z|H_\ell) = \sum \alpha_i E(z_i|H_\ell), \;\; \ell = 0,1 \tag{4}$$

and

$$\mathrm{var}(Z|H_\ell) = \sum \alpha_i \mathrm{var}(z_i|H_\ell), \;\; \ell = 0,1 \tag{5}$$

Now

$$\begin{aligned} E(z_i|H_0) &= P(z_i = 1|H_0) \\ &= P(z_i = 1|u_i = 0, H_0)P(u_i = 0|H_0) \\ &\quad + P(z_i = 1|u_i = 1, H_0)P(u_i = 1|H_0) \\ &= \varepsilon_i \left[1 - Q\left(\frac{\lambda_i \sigma_i}{d} + \frac{d}{2\sigma_i}\right)\right] \\ &\quad + (1-\varepsilon_i) Q\left(\frac{\lambda_i \sigma_i}{d} + \frac{d}{2\sigma_i}\right) \end{aligned} \tag{6}$$

where $\varepsilon_i = Q(\sqrt{2E_T x_i/No})$ is the crossover probability for the ith channel. Let $E(z_i|H_0) = \omega_{i0}$. Then

$$\mathrm{var}(z_i|H_0) = \omega_{i0}(1 - \omega_{i0}) \tag{7}$$

Similarly for hypothesis H_1, we have

$$\begin{aligned} E(z_i|H_1) &= P(z_i = 1|H_1) \\ &= \varepsilon_i \left[1 - Q\left(\frac{\lambda_i \sigma_i}{d} - \frac{d}{2\sigma_i}\right)\right] \\ &\quad + (1-\varepsilon_i) Q\left(\frac{\lambda_i \sigma_i}{d} - \frac{d}{2\sigma_i}\right) \end{aligned} \tag{8}$$

Let $E(z_i|H_1) = \omega_{i1}$. Then

$$\mathrm{var}(z_i|H_1) = \omega_{i1}(1 - \omega_{i1}) \tag{9}$$

Let $\mathrm{var}(z_i|H_\ell) = {\gamma_{i\ell}}^2$. The probability of false alarm is now given by

$$P_f = P(Z \geq \tau|H_0) = Q\left(\frac{\tau - \sum_{i=1}^n \alpha_i \omega_{i0}}{\sqrt{\sum_{i=1}^n {\alpha_i}^2 {\gamma_{i0}}^2}}\right) \tag{10}$$

and the probability of detection is given by

$$P_d = P(Z \geq \tau|H_1) = Q\left(\frac{\tau - \sum_{i=1}^n \alpha_i \omega_{i1}}{\sqrt{\sum_{i=1}^n {\alpha_i}^2 {\gamma_{i1}}^2}}\right) \tag{11}$$

Finally the probability of error is given by

$$P_e = q_0 P_f + q_1(1 - P_d) \tag{12}$$

Let $\mathbf{x} = (x_1, x_2, \cdots, x_n)$ and $\mathbf{y} = (\alpha_1, \alpha_2, \cdots, \alpha_n)$. Under the probability of error criteria, we can formulate the following optimization problem.

$$\begin{array}{ll} \text{Minimize} & P_e(\tau, \mathbf{x}, \mathbf{y}) \\ \text{Subject to} & \sum x_i = 1 \\ & x_i \geq 0 \end{array} \tag{13}$$

Now the Lagrangian is given by

$$L(\tau, \mathbf{x}, \mathbf{y}, \{\kappa_i\}, \mu) =$$

$$P_e(\tau, \mathbf{x}, \mathbf{y}) + \sum_{i=1}^{n} \kappa_i x_i + \mu(\sum_{i=1}^{n} x_i - 1) \tag{14}$$

The Karush-kuhn-Tucker(KKT) (Boyd and Vandenberghe, 2004) conditions dictate that there must exist $\{\kappa_i\}_{i=1}^n$ and μ such that

$$x_i \geq 0, \gamma_i \geq 0, \kappa_i x_i = 0, i = 1, 2, .., n. \tag{15}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{16}$$

$$\nabla P_e(\tau, \mathbf{x}, \mathbf{y}) + \nabla \sum_{i=1}^{n} \kappa_i x_i + \nabla \mu (\sum_{i=1}^{n} x_i - 1) = 0 \tag{17}$$

where ∇ denotes gradient. By solving this problem we can obtain the optimal energy allocation $\mathbf{x}$, and the decision rule $(\tau, \mathbf{y})$.

4 DISTANCE MEASURE CRITERION

In general we would like to perform the energy allocation using the probability of error as the cost function. However, as noted in the previous section obtaining the optimal detection rule may be intractable. In addition, while currently we are only considering a one bit quantization of the sensor observations, we would like to extend our results allowing the sensors to use more generalized quantizers. In this case obtaining an expression for the error probability that is suitable for energy allocation is difficult. Therefore, in this section we opt for an alternative cost function, namely the J-divergence distance measure which belongs to the class of Ali-Silvey distance measures between probability measures. Theorems relating the maximum distance to the minimum probability of error justify the application of distance measures in our setting (Poor and Thomas, 1977). A lower bound for the error probability in terms of the J-divergence distance measure is given in (Kailath, 1967). For more discussion on the Ali-Silvey class of distance measures and their application for the design of generalized quantizer we refer the reader to (Poor and Thomas, 1977).

The J-divergence distance measure is given by

$$J(\mathbf{x}) = E_{H_1}[T(\mathbf{z})] - E_{H_0}[T(\mathbf{z})] \tag{18}$$

where $T(\mathbf{z})$ is the log-likelihood ratio function given by $\ln \frac{p(\mathbf{z}|H_1)}{p(\mathbf{z}|H_0)}$ and E_{H_ℓ} is expectation operation under the hypothesis H_ℓ. We can write

$$T(\mathbf{z}) = \ln \frac{p(\mathbf{z}|H_1)}{p(\mathbf{z}|H_0)} = \sum_{i=1}^{n} \ln \frac{p(z_i|H_1)}{p(z_i|H_0)} \tag{19}$$

Thus $J(\mathbf{x}) = \sum_{i=1}^{n} j(x_i)$, where

$$j(x_i) = (\omega_{i1} - \omega_{i0}) \ln \frac{\omega_{i1}}{\omega_{i0}} + (\nu_{i1} - \nu_{i0}) \ln \frac{\nu_{i1}}{\nu_{i0}}$$

and where $\nu_{i\ell} = P(z_i = 0|H_\ell) = 1 - \omega_{i\ell}$, for $\ell = 0, 1$.

The optimization problem is now formulated as follows.

$$\text{Maximize} \quad \sum_{i=1}^{n} j(x_i) \tag{20}$$

$$\text{Subject to} \quad \sum_{i=1}^{n} x_i = 1 \tag{21}$$

$$x_i \geq 0 \tag{22}$$

The Lagrangian is given by

$$L(\mathbf{x}, \{\kappa_i\}, \mu) = -\sum_{i=1}^{n} J(x_i) + \sum_{i=1}^{n} \kappa_i x_i + \mu(\sum x_i - 1) \tag{23}$$

The KKT conditions dictate that there must exist $\{\kappa\}_{i=1}^n$ and μ such that:

$$x_1 \geq 0, \kappa_i \geq 0, \kappa x_i = 0, i = 1, 2, ..., n$$

$$\sum_{i=1}^{n} x_i = 0$$

$$-\nabla(\sum_{i=1}^{n} J(x_i)) + \nabla(\sum_{i=1}^{n} \kappa_i x_i) + \nabla(\mu(\sum_{i=1}^{n} x_i - 1)) = 0 \tag{24}$$

By solving this problem we can obtain the optimal energy allocation $\mathbf{x}$. In this case the fusion rule is given by

$$\psi(\mathbf{z}) = \begin{cases} H_1, & T(\mathbf{z}) > \tau \\ P(H_1) = a, & T(\mathbf{z}) = \tau \\ H_0, & T(\mathbf{z}) < \tau \end{cases} \tag{25}$$

where H_1 is chosen with probability a when $T(\mathbf{z}) = \tau$.

5 NUMERICAL RESULTS

5.1 Error-Free Channels

To show the efficacy of the prediction rule in (3), we consider the case of error free channels. The WSN is assumed to have five sensors with the noise variances given in Table 1. We plot the error probability for the optimal values of $\{\alpha_i\}$ (given in Table 2) as a function of τ in Figure 1. For comparison we have also plotted the error probability for $\alpha_i = 1/\sigma_i^2$. It can be seen that both cases result in similarly small error probabilities albeit for different values of τ. This indicates that if optimization over τ is performed then $\alpha_i = 1/\sigma_i^2$ results in good performance.

Table 1: σ values for different sensor index.

Sensor node index(i)	1	2	3	4	5
σ	1	2	3	4	5

Table 2: Optimumα values for different sensor index.

Sensor	1	2	3	4	5
α	2.023	0.52	0.33	0.26	0.23

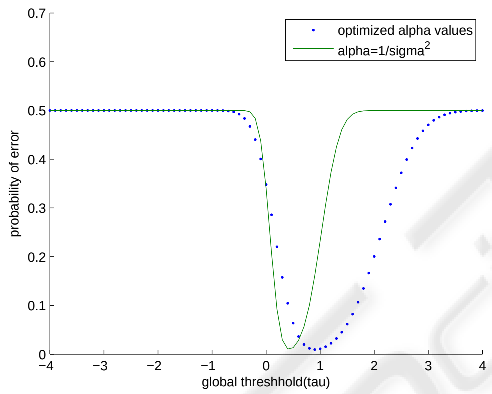

Figure 1: Probability of error vs global threshold.

5.2 Energy Allocation for Noisy Channels

In this case ,we are interested in optimal energy allocation for non-identical sensors. For sensor i the channel is a binary symmetric channel with crossover probability $\varepsilon_i = Q(\sqrt{2E_T x_i/N_0})$. The WSN configurations are given in Tables 1 and 3. The optimal value of energy fractions obtained through analytical formulation are depicted in Figures 2 and 3. In these figures we also show the resulting error probabilities. As expected the sensors with smaller noise variance are allocated a higher fraction of the energy.

We obtained similar results for the case N=8, with following σ_i values.

Fig.3 shows the results for the case N=8.

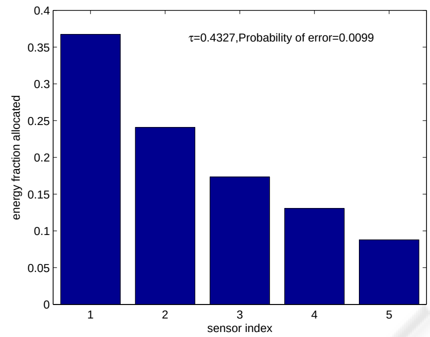

Figure 2: Optimal energy fractions allocated for N=5.

Table 3: σ values for different sensor index.

Sensor	1	2	3	4	5	6	7	8
σ	1	2	3	4	5	4	3	2

5.3 Energy Allocation Using the Distance Measure

For the WSN configurations in Tables 1 and 3 we have obtained the optimal energy allocation using the J-divergence distance measure. The energy allocations are shown in Figures 4 and 5, respectively. It is interesting to note that in these cases the nodes with a large noise variance are not allocated any energy and thus are prevented from transmitting their decision to the fusion center. These nodes are censored. In these figures we also show a lower bound on the error probability obtained from the J-divergence distance measure.

6 CONCLUSION

We have studied the problem of binary hypothesis testing in a wireless sensor network in the presence of noisy channels and for non-identical sensors. We have designed a mathematically tractable fusion rule for which optimal energy allocation for individual sensors can be achieved. The objective is to optimize a cost function with a constraint on the total network energy. Two cost functions were considered; the probability of error and the J-divergence distance measure. Results of optimal energy allocation and the resulting probability of error are presented for different sensor network configurations.

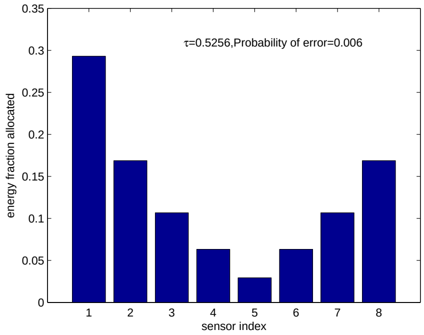

Figure 3: Optimal energy fractions allocated for N=8.

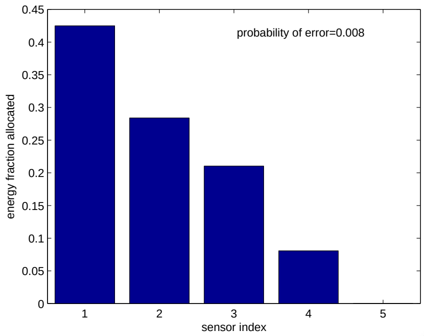

Figure 4: Optimal energy fractions allocated for N=5 using Distance Measure.

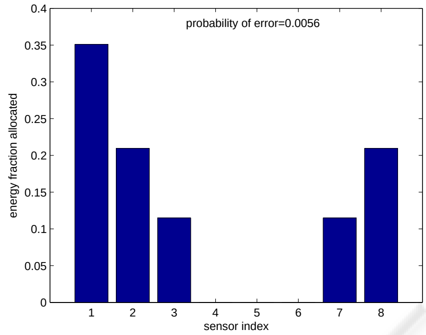

Figure 5: Optimal energy fractions allocated for N=8 using Distance Measure.

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