

# Using Wavelets based Feature Extraction and Relevance Weighted LDA for Face Recognition

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**Abstract.** In this work, we propose an efficient face recognition system which has two steps. Firstly we take 2D wavelet coefficients as a representation of faces images. Secondly, for recognition module we present a new variant on Linear Discriminant Analysis (LDA). This algorithm combines the advantages of the recent LDA enhancements namely relevance weighted LDA and QR decomposition matrix analysis. Experiments on two well known facial databases show the effectiveness of the proposed method. Comparisons with other LDA-based methods show that our method improves the LDA classification performance.

## 1 Introduction

Face recognition has become a very active research area in the last decade due to the interest in video surveillance, access control and security. Although there are many algorithms for face recognition which work well in constrained environments, various changes in face images present a great challenge, and a face recognition system must be robust with respect to the much variability of face images such as facial expression, pose and illumination. To handle with this problem it is important to choose a suitable representation of face images. In this paper we propose to use a multilevel two dimensional (2D) discrete wavelet transform (DWT) [1] to decompose face images and choose the lowest resolution subband coefficients for robust face representation with regard to lighting changes and ability of extracting important facial features while keeping computational complexity low. These facial features are taken as entry of an LDA [2] algorithm to find the optimal projection so that the ratio of the traces of the between-class and the within-class scatter matrices of the projected samples reaches its maximum. However, a critical issue using LDA, particularly in face recognition area, is the Small Sample Size (SSS) Problem. To overcome this limitation many LDA-based methods have been proposed. Among them, the most popular one is to use principal components analysis (PCA) as a pre-processing step aiming to reduce the dimensionality prior to performing LDA [3], [4]. Recently, Ye et al [5] have presented the so-called LDA/QR algorithm as an alternative way to handle the SSS Problem by using QR decomposition. Moreover, [6] has been shown that the class separability criterion that classical LDA maximize is not necessarily representative of classification accuracy and the resulting projection will preserve the distances of already well-separated classes while

causing unnecessarily overlap of neighbouring classes. To solve this problem Loog et al [6] have proposed an extended criterion by introducing a weighting scheme in the estimation of between class scatter matrix. From the similar standpoint [8] have extended this concept to estimate the within class scatter matrix by introducing the inter-class relationships as relevance weights. He has presented an LDA enhancements algorithm namely relevance weighted LDA (RW-LDA) by replacing the unweighted scatter matrices through the weighted scatter matrices in the classical LDA method. Still, this algorithm cannot directly applied for face recognition because of the singularity of the weighted within class scatter matrix. In this paper we propose a solution to this problem by introducing a QR decomposition matrix analysis on RW-LDA and make it applicable for face recognition. The rest of the paper is organized as follows: In the next section, we briefly review related work on some LDA based methods for linear dimension reduction. Section 3 introduces the new proposed algorithm. Experiments and discussions are presented in Section 4. We draw the conclusion in Section 5.

## 2 Related Works

Firstly some assumptions and definitions will be presented. Given data matrix  $X \in \mathbb{R}^{d \times n}$ , classical LDA aims to find a transformation matrix  $W = [w_1, \dots, w_\ell] \in \mathbb{R}^{d \times \ell}$ , that maps each column  $x_i$  of  $X$ , for  $1 \leq i \leq n$ , in the  $d$ -dimensional space to a vector  $y_i$  in the  $\ell$ -dimensional space as follows:

$$W : x_i \in \mathbb{R}^d \rightarrow y_i = W^t x_i \in \mathbb{R}^\ell (\ell < d).$$

Assume that the original data in  $X$  is partitioned into  $c$  classes.  $X_i \in \mathbb{R}^{d \times n_i}$  is the data matrix containing only the data points from the  $i^{th}$  class, where  $n_i$  is the size of the  $i^{th}$  class and  $n = \sum_{i=1}^c n_i$ . An optimal transformation  $W$  that preserves the given cluster structure can be approximated by finding a solution of the following criterion:

$$J(w) = \text{trace}((W^t S_w W)^{-1} (W^t S_b W)). \quad (1)$$

Where,

$$S_b = \sum_{i=1}^c p_i (m_i - m)(m_i - m)^t = H_b H_b^t. \quad (2)$$

$$S_w = \sum_{i=1}^c p_i \sum_{j=1}^{n_i} (x_{ij} - m_i)(x_{ij} - m_i)^t = H_w H_w^t. \quad (3)$$

$e_i = (1, \dots, 1) \in \mathbb{R}^{d \times c}$ ,  $m_i$  denotes the mean of class  $i$  with prior probability  $p_i = \frac{n_i}{n}$  and  $m$  is the total mean;  $x_{ij}$  is the  $d$ -dimensional pattern  $j$  from class  $i$ . The classical LDA criterion in (1) is not optimal with respect to minimizing the classification error rate in the lower dimensional space. He tends to overemphasize the classes that are more separable in the input feature space. As a result, the classification ability will be impaired. To deal with this problem, Loog et al. [6] have proposed to introduce a weighting function to the discriminant criterion, where a weighted between-class scatter matrix is defined to replace the conventional between-class scatter matrix. Classes

that are closer to each other in the output space, and thus can potentially impair the classification performance, should be more heavily weighted in the input space. According to [6], weighted between-class scatter matrix  $\hat{S}_b$  can be defined as:

$$\hat{S}_b = \sum_{i=1}^{c-1} \sum_{j=i+1}^c w(d_{ij}) p_i p_j (m_i - m_j)(m_i - m_j)^t. \quad (4)$$

Where  $p_i$  and  $p_j$  are the class priors,  $d_{ij}$  is the Euclidean distance between the means of class  $i$  and class  $j$ . The weighting function  $w(d_{ij})$  is generally a monotonically decreasing function defined as in [7]:

$$w(d_{ij}) = d_{ij}^{-2h} \text{ with } d_{ij} = \|m_i - m_j\|, h \in \mathfrak{R}. \quad (5)$$

Recently, [8] has extended the concept of weighting to estimate a within-class scatter matrix. Thus by introducing a so-called relevance weights, a weighted within-class scatter matrix  $\hat{S}_w$  is defined to replace a conventional within-class scatter matrix:

$$\hat{S}_w = \sum_{i=1}^{c-1} p_i r_i \sum_{j=1}^{n_i} (x_{ij} - m_i)(x_{ij} - m_i)^t \text{ with } r_i = \sum_{j \neq i} \frac{1}{w(d_{ij})}. \quad (6)$$

Using the weighted scatter matrices  $\hat{S}_b$  and  $\hat{S}_w$  the criterion in (1) is weighted and the resulting algorithm is referred to as relevance weighted LDA (RW-LDA).

### 3 The Proposed Algorithm

It is easy to verify that when a small sample size problem takes place, such as in the face recognition area, both  $S_w$  and  $\hat{S}_w$  are singular and then RW-LDA cannot be used directly. To overcome this problem, we propose to use QR decomposition matrix analysis [9] as in LDA/QR but with the modified expressions of  $H_b$  and  $H_w$  defined as follows:

$$\hat{H}_b = [\alpha_{12}(m_1 - m_2), \dots, \alpha_{(c-1)c}(m_{c-1} - m_c)]. \quad (7)$$

$$\hat{H}_w = [\beta_1(X_1 - m_1 e_1), \dots, \beta_c(X_c - m_c e_c)]. \quad (8)$$

So that  $\hat{S}_b = \hat{H}_b \hat{H}_b^t$  and  $\hat{S}_w = \hat{H}_w \hat{H}_w^t$ , where  $\alpha_{ij} = \sqrt{p_i p_j w(d_{ij})}$  and  $\beta_i = \sqrt{p_i r_i}$ . with a weighting function  $w(d_{ij}) = ((m_i - m_j)^t (m_i - m_j))^{-h}$ ,  $h \in \mathfrak{R}$ .

The steps for the proposed RW-LDA/QR algorithm are presented by the following pseudo code:

### 4 Experimental Results

The proposed method is tested by two group experiments corresponding to ORL[10] and Yale[11] faces databases. The test protocol is the same in the both experiments.

**First Group:** The ORL face database consists of images from  $c = 40$  different people, using 10 images from each person, for a total of 400 images. Firstly we down

Input: Data matrix  $X$ ,  $h$ .

Output: Discriminant projection matrix  $W$ .

1. Compute the mean of each class  $i$ ,  $m_i$  and the mean of all the classes  $m$ .
2. Construct  $\hat{H}_b, \hat{H}_w$  from (9) and (10).
3. Perform QR decomposition  $\hat{H}_b: \hat{H}_b = QR$ .
4. Compute  $\tilde{S}_b = Q^t \hat{S}_b Q$  and  $\tilde{S}_w = Q^t \hat{S}_w Q$ .
5. Compute the  $t$  eigenvectors  $g_i$  of  $(\tilde{S}_b)^{-1} \tilde{S}_w$  with increasing eigenvalues, where  $t = \text{rank}(\hat{H}_b)$ .  
The projection matrix is  $W = QG$  with  $G = [g_1, \dots, g_t]$ .

sample all images to 56x46 pixels without any other pre-processing. For the test protocol we randomly take  $k$  images from each class as the training data, with  $k \in \{2, \dots, 9\}$ , and leave the rest  $10 - k$  images as the probe. The Nearest Neighbour algorithm was employed with Euclidean distance for classification. Such test is run ten times and we take the average of the results for comparison. At the training phase, we represent each image by a raster scan vector of the intensity value as the column of the input data matrix  $X$ . Moreover, to choose the value of the parameter  $h$  in the weighting function, we have calculate the recognition rate with vary within the range from 1 to 7. Table 1 shows the results over the variation of  $h$ , from which the value  $h = 4$  achieves the max recognition rate. Hence, for the rest of paper we take  $h = 4$ . Average recognition rate for each of the three algorithms is reported in Table 2.

**Table 1.** Recognition rates for different  $h$  in RW-LDA/QR.

h	1	2	3	4	5	6	7
Recognition Rates (percent)	81,90	82,20	82,78	83,62	81,46	80,84	80,09

**Table 2.** Recognition rates (percent) for the ORL dabase.

k	2	3	4	5	6	7	8	9
Fisherface	76,06	86,78	92,95	94,15	95,06	95,75	96,37	97,05
LDA/QR	81,43	90,60	93,20	96,37	96,30	97,50	98,08	99,00
RW-LDA/QR	83,62	92,00	95,20	96,25	97,75	98,91	99,00	99,75

From these results we see that the proposed method RW-LDA/QR performs better performance. In other word, we present the results using wavelets transform as feature extraction. Original images used here have a 112x92 size. The columns of the data matrix are generated by the lowest frequency subimages (LL) from 2D wavelet decomposition on the original images of database.

Table 3 shows the comparisons result of the algorithms with one-level Haar wavelet decomposition. Table 4 list the results with Haar wavelets at level 2.

It is noted that there are a weak increase in the Recognition rate for LDA/QR but not a great change for RW-LDA/QR with the use of the one-level 2D Haar wavelet transform. In addition, one sees in Table 3 that starting from level 2 of the decomposition in

**Table 3.** Recognition rates using one-level 2D Haar wavelet transform.

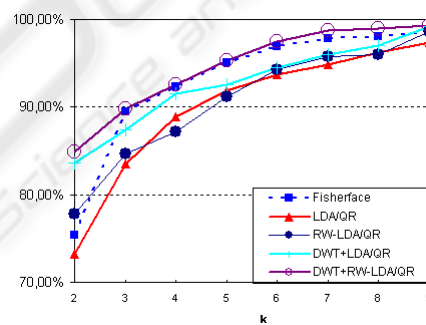
k	2	3	4	5	6	7	8	9
LDA/QR	81,43	90,60	93,20	96,37	96,30	97,50	98,08	99,00
RW-LDA/QR	83,62	92,00	95,20	96,25	97,75	98,91	99,00	99,75

**Table 4.** Recognition rates using tow-level 2D Haar wavelet transform.

k	2	3	4	5	6	7	8	9
DWT+LDA/QR	66,56	79,82	86,12	90,40	94,18	95,66	97,00	98,00
DWT+RW-LDA/QR	71,65	82,25	89,12	92,80	95,87	97,16	97,37	98,50

wavelet, the performance of algorithms LDA/QR and RW-LDA/QR are degraded with superiority for RW-LDA/QR compared to LDA/QR.

**Second Group:** The Yale face database consists of images from different people, using 11 images from each person, for a total of 165 images. For simplicity of computations, we have downsampled the images to 50x50 pixels. Firstly we have performed the Fisherface, LDA/QR an RW-LDA/QR on Yale database without wavelet features. Secondly, using two-level 2D Harr wavelet transform we have performed DWT+LDA/QR and DWT+RW-LDA/QR algorithms. Fig.1 depicts the recognition rate of all algoritms. Hence, as we can see it on Fig.1, the use of wavelets features improves the performance of LDA/QR and RW-LDA/QR algorithms, especially on Yale database that contains images with variations on illumination.

**Fig. 1.** Recognition rate for yale database.

## 5 Conclusion

In this paper, we presented a novel method for face recognition. Our method outperforms others LDA based methods on ORL database and it performs acceptable result for Yale database. The use of wavelets as feature extraction improves the performance of the algorithms presented in this paper. However all algorithms presented in this paper are linear methods. Since facial variations are mostly non linear, LDA, LDA/QR and RW-LDA/QR projections could only provide suboptimal solutions. On future work we plan to extend the algorithm presented in this paper by introducing kernel methods to take account of this drawback.

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