HYBRID WAVELET-KALMAN FILTER MULTI-SCALE SEQUENTIAL FUSION METHOD

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Abstract: With the development of automation, multi-scale data fusion has become a hot research topic, however, limited by the constraint that signal to implement wavelet transform must have the length of 2^q, multi-scale data fusion problem involved with non-2^n sampled observation data still hasn't been efficiently solved. In this paper, we develop a hybrid wavelet-Kalman filter multiscale sequential fusion method. First, we develop the hybrid wavelet-Kalman filter multiscale estimation method which combines the advantage of wavelet and Kalman filter to obtain the real time, recursive, multiscale estimation of the dynamic system. Then, a multiscale sequential fusion method is presented. Under the hybrid wavelet-Kalman filter multiscale estimation frame, we can easily fuse information from multiple sensors sequentially without designing other complex fusion algorithm. The multiscale sequential fusion method can fuse non-2^n sampled data just by analyzing the possible observation structure to design the observation model of the stacked dynamic system. Simulation result of three sensors with sampling interval 1, 2 and 3 shows the efficiency of this method.

1 INTRODUCTION

In many fields, such as, automatic control, aerospace, communication, navigation and production industry, more than one sensor is used to gather complete information of the object or process. According to the mechanism of each sensor, they can be placed on different scales and the sampling rate of these sensors may also be different. The research of multi-sensor data fusion for dynamic system is significant both in practice and theoretically (Wen 2002a, Wen 2002b, Lang Hong1994). Especially, in many cases, the sampling interval may not equal to 2^n, thus it is inconvenient for us to fuse information provided by these sensors. Therefore the tracking or estimation accuracy may be strongly reduced.

The main technique used in multi-scale data fusion is Kalman filter and wavelet analysis. Kalman filter can result in real-time, recursive and optimal estimate while it doesn't take the multi-scale character of the object into account. Wavelet transform can implement multi-scale analysis and estimation of the dynamic system, but the estimate is neither real time nor recursive (Wen 2002a).

Using Kalman filter, data fusion algorithm for multi-sensor sampling at same rate has been successfully developed. Coporating with multi-scale theory, multi-scale data fusion algorithm for multi-sensor sampling at 2^n interval has also been developed. Limited by the fact that signals to implement wavelet transform must have the length of 2^q, the method mentioned in (Wen 2002b, Lang Hong1994) can't solve the data fusion problem when the sensors used are not sampling at 2^n interval.

We find that once the dynamic system is stacked in a given length 2^q, sensors not sampled at interval 2^n has different observation structure on each block, that is, the length of the observation vector on each block may be different, and the sampling rule on each observation block is also different.

Based on the hybrid wavelet and Kalman filter sequential fusion method developed in (Wen 2006a, Zhou 2007), we are intend to develop a sequential fusion scheme by designing the stacked observation model to fuse the observation data coming from those sensors sampling at non-2^n interval.

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2 MANUSCRIPT PREPARATION

2.1 Dynamic System

Considering a system involving K sensors

$$x(k+1) = A(k)x(k) + w(k) \quad (1)$$

$$z_i(k_i) = C_i(k_i)x(k_i) + v_i(k_i) \quad i = 1,2,\cdots,K \quad (2)$$

where $k \in N$, $k_i = d_i k$, $d_i \in N$ is the sampling interval of each sensor, $x(k) \in R^n$ is the object's state, $A(k) \in R^{n\times n}$ is the system matrix.

The System's process noise $w(k) \in R^n$ is the Gaussian white noise with the following statistics

$$E\{w(k)\} = 0 \quad (3)$$

$$E\{w(k)w^T(l)\} = Q(k)\delta_{k,l} \quad k,l \geq 0 \quad (4)$$

$Q(k)$ is a nonnegative matrix.

The observation noise $v_i(k_i)$ is also Gaussian white noise

$$E\{v_i(k_i)\} = 0 \quad (5)$$

$$E\{v_i(k_i)v_j^T(l_j)\} = Q(k)\delta_{i,j}\delta_{k,l} \quad i,j = 1,2,\cdots,K \quad (6)$$

$x(0)$ is the initial state of the system,

$$E\{x(0)\} = x_0 \quad (7)$$

$$E\{[x(0)-x_0][x(0)-x_0]^T\} = P_0 \quad (8)$$

$x(0)$, $w(k)$ and $v_i(k_i)$ is independent.

2.2 Stacked System

Rewrite the dynamic model (1) and (2) as a stacked system with block length M

$$\overline{X}(m+1) = \overline{A}(m)\overline{X}(m) + \overline{W}(m) \quad (9)$$

$$\overline{Z}_i(m) = \overline{C}_i(m)\overline{X}(m) + \overline{V}_i(m) \quad i = 1,2,\cdots K \quad (10)$$

where

$$\overline{X}(m) = [x^T((m-1)M+1),\cdots,x^T((m-1)M+M)]^T \quad (11)$$

$$\overline{A}(m) = \mathrm{diag}[\prod_{j=0}^{M-1} A(mM-j),\cdots,\prod_{j=0}^{M-1} A(mM+M-j)] \quad (12)$$

$$\overline{Z}_i(m) = [z_i^T(M(m-1)+d_i-r_i(m)), z_i^T(M(m-1)+2d_i-r_i(m)), \cdots; z_i^T(M(m-1)+S_i(m)d_i-r_i(m))]^T \quad (13)$$

where $\overline{Z}_i(m)$ is the observation of m th block observed by sensor i . $\overline{C}_i(m)$ in equa.(14) is the observation matrix, $r_i(m) = m(M-1) \bmod d_i$, $e(a)$ is the unit vector whose a th element is 1, while other elements are all 0.

$$\overline{C}_i(m) = \begin{bmatrix} C_i(M(m-1)+d_i-r_i(m))\cdot e(d_i-r_i(m)) \\ C_i(M(m-1)+2d_i-r_i(m))\cdot e(2d_i-r_i(m)) \\ \vdots \\ C_i(M(m-1)+S_i(m)d_i-r_i(m))\cdot e(S_i(m)-r_i(m)) \end{bmatrix} \quad (14)$$

Section 4.2 shows the detailed designing of $\overline{C}_i(m)$.

$\overline{V}_i(m)$ is the observation noise with statistics

$$E\{\overline{V}_i(m,s)\} = 0 \quad (15)$$

$$E\{\overline{V}_i(m,s)\overline{V}_j^T(m,t)\} = R_i\delta_{i,j}\delta_{s,t} \quad s,t = 1,2,\cdots,S_i(m) \quad (16)$$

where $S_i(m)$ is the length of the observation vector on the m th block. $\overline{W}(m)$ is the process noise

$$\overline{W}(m) = B(m)\widetilde{W}(m) \quad (17)$$

$$\widetilde{W}(m) = [w^T((m-1)M+1),\cdots,w^T((m-1)M+2M-1)]^T \quad (18)$$

$$E[\overline{W}(m)] = 0 \quad (19)$$

$$\overline{Q}(m) \equiv E[\overline{W}(m)\overline{W}^T(m)] = B(m)\widetilde{Q}(m)B^T(m) \neq 0 \quad (20)$$

$$B(m) = \begin{bmatrix} \prod_{j=0}^{M-2} A(mM-j) & \prod_{j=0}^{M-3} A(mM-j) & \cdots & I & 0 & \cdots & 0 \\ 0 & \prod_{j=0}^{M-2} A(mM+1-j) & \prod_{j=0}^{M-2} A(mM+1-j) & \cdots & I & 0 & 0 \\ \cdots & \cdots & \cdots & & \cdots & & \\ 0 & 0 & \cdots & 0 & \prod_{j=0}^{M-2} A(mM+M-1-j) & \cdots & I \end{bmatrix} \quad (21)$$

$$\widetilde{Q}(m) = diag[Q((m-1)M+1),\cdots,Q((m-1)M+2M-1)]^T \quad (22)$$

in equations (14)-(22), $diag[X,Y,\cdots,Z]$ is the blocked diagonal matrix.

2.3 Multiscale Stacked System

Implementing wavelet transform on equation (9)

$$W_X\overline{X}(m+1) = W_X\overline{A}(m)\overline{X}(m) + W_X\overline{W}(m) \quad (23)$$

That is

$$\overline{\gamma}(m+1) = \overline{A}_w(m)\overline{\gamma}(m) + \overline{\mu}(m) \quad (24)$$

where W_X is the operator matrix of wavelet transform, satisfying[Wen 2002 a, Lang Hong1994]

$$W_X^* W_X = I$$

$$\bar{\gamma}(m) = W_X \bar{X}(m) \tag{25}$$

$$\bar{\mu}(m) = W_X \bar{W}(m) \tag{26}$$

$$\bar{A}_w(m) = W_X \bar{A}(m) W_X^* \tag{27}$$

$$\bar{Q}_w(m) = W_X \bar{Q}(m) W_X^* \tag{28}$$

It is easy to prove that the process noise of the new stacked system (24) is statistically independent, that is $\bar{Q}_w(m) = 0$, which is also one of the advantages of hybrid wavelet-Kalman filter: de-correlating the correlation between blocks (Wen 2006a).

With the orthogonality of W_X , we can rewrite the observation equation as

$$\bar{Z}_i(m) = \bar{C}_i(m) W_X^* W_X \bar{X}(m) + \bar{V}_i(m) \tag{29}$$

$$\bar{Z}_i(m) = \bar{C}_i(m) W_X^* \bar{\gamma}(m) + \bar{V}_i(m) \quad i = 1,2,\cdots,K \tag{30}$$

$$\bar{\gamma}(m) = W_X \bar{X}(m) \tag{31}$$

That is

$$\bar{Z}_i(m) = \bar{C}_i(m) W_X^* \bar{\gamma}(m) + \bar{V}_i(m) \quad i = 1,2,\cdots,K \tag{32}$$

3 HYBRID WAVELET-KALMAN FILTER MULTI-SCALE ESTIMATION FOR A SINGLE SENSOR

The following state transition equation and the observation equation of the wavelet transform coefficient of the m th block can be established(Tong 2000)

$$\bar{\gamma}(m, s+1) = \bar{\gamma}(m, s) + \bar{w}(m, s) \text{ , } s = 0,1,2,\cdots,S_1 - 1 \tag{33}$$

$$\bar{Z}_1(m, s) = H(m, s)\bar{\gamma}(m, s) + \bar{V}_1(m, s), \quad s = 1,2,\cdots,S_1 \tag{34}$$

where $\bar{Z}(m,s)$ is the observation at time s of block m . In equa.(34),

$$H(m) \equiv \bar{C}(m) W^T \tag{35}$$

where $H(m,s)$ is the s th row of the matrix $H(m)$.

The main idea of hybrid wavelet-Kalman filter method includes two steps (Wen 2006 a, Wen 2006 b, Zhou 2007):

(1) Wavelet transform coefficients prediction based on stacked dynamic system $\hat{\bar{\gamma}}(m \mid m-1) = \bar{A}_w(m)\hat{\bar{\gamma}}(m-1 \mid m-1)$.

(2) Use each observation on this block to update the estimation of wavelet transform coefficient.

Implement Kalman filter on the system given by equa. (33) and (34). In each block, the original state can be derived by a prediction process

$$\hat{\bar{\gamma}}_{0|0}(m) = \bar{A}_w(m)\hat{\bar{\gamma}}(m-1 \mid m-1) \tag{36}$$

$$\bar{P}_{0|0}(m) \equiv E[\hat{\bar{\gamma}}_{0|0}(m)\hat{\bar{\gamma}}_{0|0}^T(m)] = \bar{A}_w(m)\bar{P}_w(m-1 \mid m-1)\bar{A}_w^T(m) + \bar{Q}_w(m) \tag{37}$$

The filter process follows as

$$\hat{\bar{\gamma}}_{s+1|s+1}(m) = \hat{\bar{\gamma}}_{s+1|s}(m) + K(s+1)\tilde{Z}(m, s+1)$$
$$s = 0,1,2,\cdots,S_1 - 1 \tag{38}$$

$$\hat{\bar{\gamma}}_{s+1|s}(m) = \hat{\bar{\gamma}}_{s|s}(m) \tag{39}$$

$$P_{s+1|s}(m) = P_{s|s}(m) \tag{40}$$

$$K(s+1) = P_{s+1|s}(m) H^T(m,s+1)[H(m,s+1) P_{s+1|s}(m) H^T(m,s+1) + \bar{R}(m,s+1)]^{-1} \tag{41}$$

$$\tilde{Z}(m, s+1) = \bar{Z}(m, s+1) - H(m, s+1)\hat{\gamma}_{s+1|s}(m) \tag{42}$$

$$P_{s+1|s+1}(m) = [I - K(s+1)H(m, s+1)]P_{s+1|s}(m) \tag{43}$$

This filter process is essentially the gradually updating of the prediction $\hat{\bar{\gamma}}_{0|0}(m)$ .The final updating as the estimation of this block

$$\hat{\bar{\gamma}}(m \mid m) = \hat{\bar{\gamma}}_{S_1|S_1}(m) \tag{44}$$

$$\bar{P}_w(m \mid m) = \bar{P}_{S_1|S_1}(m) \tag{45}$$

The whole process of hybrid wavelet-Kalman filter method can be shown in figure 1.

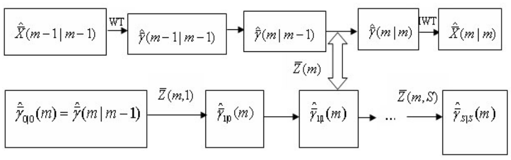

Figure 1: Hybrid wavelet-Kaman filter Algorithm.

4 NON-2^n SAMPLED DATA'S SEQUENTIAL FUSION

4.1 Sequential Fusion based on Hybrid Wavelet-Kalman Filter

To fuse the observation data coming from multiple sensors we can simply cascade these data sequentially. Then use the cascaded data to update the prediction $\hat{\bar{\gamma}}_{0|0}(m)$ more times than only one sensor case. The total updating times is

$$S = \sum_{i=1}^{K} S_i \tag{46}$$

This S times updating is the fusion estimation of the wavelet transform coefficient. The sequential fusion process can be shown in fig.2.

The main advantage of this sequential fusion is that the fusion estimate process uses the same mechanism with that of the hybrid wavelet-Kalman filter in one single sensor case without designing a new complex fusion rule.

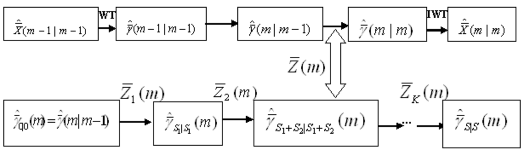

Figure 2: Hybrid wavelet-Kalman filter Sequential fusion.

This sequential fusion algorithm doesn't require that the sampling interval of the observation is 2^n .Thus we can manage to process the fusion problem involving non-2^n sampling data

4.2 Blocked Observation Model for Non-2^n Sampling Case

One crucial step in hybrid wavelet-Kalman filter is to determine the structure of the stacked observation matrix $\bar{C}_i(m)$ especially for the non-2^n sampling case since the observation structure and observation vector of each block are different.

By analyzing, we find that $\bar{C}_i(m)$ varies periodically with m .The varying rule is determined by the sampling interval d_i and the block length M . The varying period is the minimum common multiple of M and d_i .

For clarity, we display the observation structure in the case $M = 4$ and $d_i = 3$, $M = 8$ and $d_i = 3$ for the system $n = 1$, $A(k) = A$, $C_i(k_i) = C_i$.

For $M = 4$ and $d_i = 3$, the period of $\bar{C}_i(m)$ is 12, that is 3 blocks. In these 3 blocks, sensor i samples 4 observation data in total.

$$\bar{C}_i(m) = \begin{cases} C_i \cdot [0\,0\,1\,0] & m = 3j-2 \\ C_i \cdot [0\,1\,0\,0] & m = 3j-1 \\ C_i \cdot [1\,0\,0\,0; 0\,0\,0\,1] & m = 3j \end{cases} \tag{47}$$

where semicolon denotes another row in the matrix. Equation (59) means that in the $m = 3j-2$ block, sensor i sampled 1 observation data; in the $m = 3j-1$ block, sensor i sampled 1 observation data; in the $m = 3j$ block, sensor i sampled 2 observation data.

For the case $M = 8$ and $d_i = 3$, the period of $\bar{C}_i(m)$ is 24, that is 8 data blocks. In these 8 blocks, sensor i samples 8 observation data in total. The resulted stacked observation matrix is

$$\bar{C}_i(m) = \begin{cases} C_i \cdot [e_3; e_6] & m = 3j-2 \\ C_i \cdot [e_1; e_4; e_7] & m = 3j-1 \\ C_i \cdot [e_2; e_5; e_8] & m = 3j \end{cases} \tag{48}$$

in the $m = 3j-2$ block, sensor i sampled 2 observation data; in the $m = 3j-1$ and $m = 3j$ block sensor i sampled 3 observation data.

More generally, for $M = 2^q$ and d_i without $u \in N$ s.t. $d_i = 2^u$, the structure of $\bar{C}_i(m)$ is

$$\bar{C}_i(m) = \begin{bmatrix} C_i \cdot e(d_i - r_i(m)) \\ C_i \cdot e(2d_i - r_i(m)) \\ \vdots \\ C_i \cdot e(S_i(m)d_i - r_i(m)) \end{bmatrix} \tag{49}$$

where $S_i(m)$ is the number of matrix rows which is the maximum integer s.t.

$$[S_i(m)d_i - r_i(m)] \le mM$$

5 SIMULATION

This section gives the simulation of the algorithm developed in this paper to demonstrate its validity. Multiscale sequential fusion result of 3 sensors whose sampling interval are respectively 1,2 and 3 are compared with that of one single sensor using Kalman filter method.

The parameters used in the simulation are $A(k) = 0.96$, $Q(k) = 1$, the initial state is $x_0 = 1$, $P_0 = 1$ .Stacking the system with block length $M = 4$, then use the Haar wavelet to implement wavelet transform. The observation parameters are $\bar{C}_1(m) = I_4$ and $R_1 = 0.5$; $\bar{C}_2(m) = 0.5 \cdot [0\,1\,0\,0; 0\,0\,0\,1]$ and $R_2 = 0.1$;

$$\overline{C}_3(m)=\begin{cases}2\cdot[0\ 0\ 1\ 0] & m=3j-2\\ 2\cdot[0\ 1\ 0\ 0] & m=3j-1\\ 2\cdot[1\ 0\ 0\ 0;0\ 0\ 0\ 1] & m=3j\end{cases},$$

$R_3=0.01$.

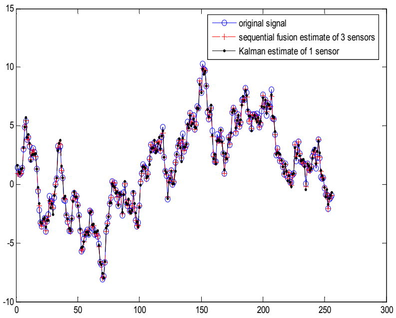

Figure 3: Sequential fusion result via single sensor estimate.

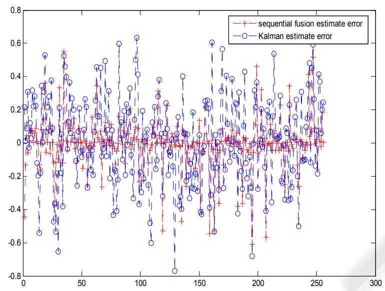

Figure 4: Sequential fusion error via single sensor estimate error.

It is easy to see that using the method mentioned in section 4 to design the stacked observation model can realize the multiscale sequential fusion of non-2^n sampling data. Compare the fusion estimate using this multiscale sequential fusion method and one single sensor estimate using Kalman filter,we conclude that it is an efficient method to process fusion problem with non-2^n sampling observation data, which is an obstacle of multi-scale data fusion.

The mean of absolute error (MAE) displayed in table 1 compare the estimate error accuracy based on one single sensor 1 using Kalman filter method and that based on sensor multi-sensor using the multiscale sequential fusion. We find that the estimation accuracy improved 2.53 times.

Table 1: MAE of sequential fusion and single KF.

	MAE
single sensor Kalman filter	0.2169
3 sensor sequential fusion	0.0857

6 CONCLUSIONS

Hybrid wavelet-Kalman filter method can obtain the real time multi-scale estimate of dynamic system. The multiscale sequential fusion algorithm based on it can easily fuse information from multiple sensors sequentially without designing other complex fusion algorithm. In addition, the hybrid wavelet-Kalman filter multiscale sequential fusion method can used to fuse non-2^n sampled data just by designing the periodically varied stacked observation matrix.

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