A GUARANTEED STATE BOUNDING ESTIMATION FOR UNCERTAIN NON LINEAR CONTINUOUS TIME SYSTEMS USING HYBRID AUTOMATA

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Abstract: This work is about state estimation in the bounded error context for non linear continuous time systems. The main idea is to seek to estimate not an optimal value for the unknown state vector but the set of feasible values, thus to characterize simultaneously the value of the vector and its uncertainty. Our contribution resides in the use of comparison theorems for differential inequalities and the analysis of the monotonicity of the dynamical systems with respect to the uncertain variables. The uncertain dynamical system is then bracketted between two hybrid dynamical systems. We show how to obtain this systems and to use them for state estimation with a prediction-correction type observer. An example is given with bioreactors.

1 INTRODUCTION

State estimation with continuous dynamic systems is recognized as problem of great importance in practice. Indeed, to apply advanced methods for the control or the diagnosis of dynamical systems one often needs to compute on-line their internal state. Generally the direct measurement of this state by means of sensors may not be available for various reasons such as physical, practical, economic, ...etc. However, it is possible to carry out this task by software sensors, i.e. observers or estimator which can provide on line an estimate of the real state system, under certain conditions of observability (Hermann and J.Krener, 1977; Hermann, 1963).

In fact, there are always uncertainties in the mathematical models used for characterizing the system under study. Consequently, the classical approaches for building observers are insufficient (Dochain, 2003). Thus, a new approach was developed recently in a deterministic set-membership context, which aims to reconstruct all the state trajectories which are consistent with both the uncertain models and the uncertain measurements.

This approach can be used easily when measurements are available at discrete time. It is of *prediction-correction* type: (*i*) The prediction phase consists in computing a guaranteed over (conservative) approximation of the reachable state space generated by the uncertain system, (*ii*) and the correction phase consists in removing from this over approximation all the part which are not consistent with feasible measurements domains, each time a measurement is available.

In the literature, several geometrical forms are used to implement this set-membership approach with linear systems. For example, parallelotopes (Chisci et al., 1996), ellipsoidal (Chernousko, 2005) and zonotopes (Combastel, 2005).

With nonlinear systems, guaranteed numerical integration method for the ordinary differential equation (ODE) (Nedialkov, 1999) based on intervals Taylor models (Moore, 1966) was used recently to solve this estimation problem (Raïssi et al., 2004). Generally, the wrapping effect (Moore, 1966; Nedialkov, 1999) associated with the intervals representation of uncertain variables limits considerably their use in order to deal with practical cases. Thereafter, in order to circumvent the wrapping effect, the authors of (Kieffer and Walter, 2006) used the Müller's existence theorem (Müller, 1926; Walter, 1997) as a tool for deriving a guaranteed enclosure for an uncertain dynami-

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cal system between two deterministic dynamical systems. The main difficulty resides in the definition of the bracketing systems. Our contribution is in the continuation of these work since we show how to build deterministic hybrid dynamical systems as bracketing systems and thus make it possible to use the Müller theorem with a larger class of nonlinear dynamical systems.

This work is organized as flow. In the second section we present the context and the main ideas of set-membership estimation. Then, we will recall the Müller's theorem in section 3. We introduce the hybrid bracketing approach for uncertain dynamical systems in section 4. Finally we illustrate our method on a model drawn from bioreactors domain in section 5.

2 PROBLEM STATEMENT

2.1 Context

Let us consider the uncertain continuous dynamic system (1) where uncertainties are naturally represented by bounded intervals with *a priori* known bounds,

$$\left\{ \begin{array}{c} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, \mathbf{u}, [\mathbf{p}], t) \\ \mathbf{y}(t) = \mathbf{g}(\mathbf{x}, \mathbf{u}, [\mathbf{p}], t) \\ \mathbf{x}(t_0) \in [\mathbf{x}_0] \subset \mathbb{D} \end{array} \right\} \quad (1)$$

where $t \in [t_0, T]$, $\mathbf{f} \in C^{k-1}(\mathbb{D} \times \mathbb{U} \times [\mathbf{p}])$, $\mathbb{D} \times \mathbb{U} \times [\mathbf{p}] \subseteq \mathbb{R}^{n+n_u+n_p}$ is an open set; n, n_u, m and n_p are the dimension of respectively the state vector $\mathbf{x}$, the input vector $\mathbf{u}$, the output vector $\mathbf{y}$ and the parameter vector $\mathbf{p}$. The functions $\mathbf{f} : \mathbb{D} \times \mathbb{U} \times [\mathbf{p}] \to \mathbb{R}^n$ and $\mathbf{h} : \mathbb{D} \times \mathbb{U} \times [\mathbf{p}] \to \mathbb{R}^m$ are possibly nonlinear. The initial state $\mathbf{x}_0$ is assumed to belong to a prior known set $[\mathbf{x}]$. We assume that measurements $\mathbf{y}_j$ of the output vector are available at sampling times $t_i \in \{t_1, t_2, \ldots, t_n\}$ in $I = [t_0, t_{n_T}]$. Note that the sampling interval needs not be constant. The measurement noise is a discrete time signal assumed additive and bounded with known bounds. Denote $\mathbb{E}_j$ a feasible domain for output error at time t_j: the feasible domain for model output at time t_j is then given by

$$\mathbb{Y}_j = \mathbf{y}_j + \mathbb{E}_j \quad (2)$$

Under these considerations, estimating the state vector $\mathbf{x}$ consists in determining an upper approximation of the set $\mathbb{X}(t)$ of all acceptable state trajectories

$$\mathbb{X}(t) = \left\{ \begin{array}{c} \mathbf{x}(t) \mid (\forall t \in I \; \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, \mathbf{u}, [\mathbf{p}], t)) \\ \wedge \\ (\forall t_j \in \{t_1, t_2, \ldots, t_{nT}\}, \\ \mathbf{x}(t_j) \in (\mathbf{g}^{-1}(\mathbb{Y}(t_j), \mathbf{u}, [\mathbf{p}]) \cap [\mathbf{x}_j])) \end{array} \right\} \quad (3)$$

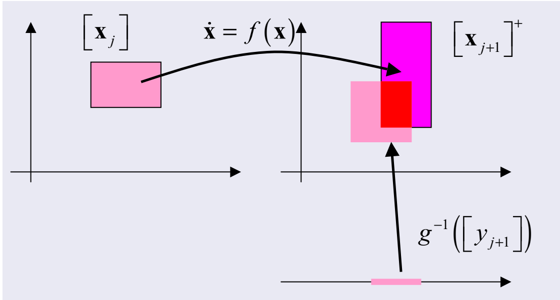

Figure 1: Prediction and correction phases.

2.2 Principle: Prediction-correction Method

Figure 1 shows the principle of the prediction and correction phases for this set-membership approach, between two successive time measurement indexes t_j and t_{j+1}. Indeed, by using one of the guaranteed numerical simulation methods for uncertain ODEs, the *prediction* phase computes an guaranteed over enclosure $[\mathbf{x}_{j+1}]^p$ for all solutions of (1) at the time t_{j+1} with $(t_j, [\mathbf{x}_j])$ as initial conditions.

$$\mathbf{x}(t_{j+1}; t_j, [\mathbf{x}_j]) \subseteq [\mathbf{x}_{j+1}]^p. \quad (4)$$

The *correction* phase uses set inversion, consistency techniques and intervals analysis (Jaulin et al., 2001) to characterize the reciprocal image $[\mathbf{x}_{j+1}]^{inv}$ at time t_{j+1} of the admissible measurement domain $\mathbb{Y}_{j+1}$ by the output model $\mathbf{g}$

$$[\mathbf{x}_{j+1}]^{inv} = \mathbf{g}^{-1}([\mathbf{y}_{j+1}], [\mathbf{p}]) \quad (5)$$

where $[\mathbf{y}_{j+1}] = \mathbb{Y}_{j+1}$.

Thereafter, it contracts the predicted state intervals vector by comparing it with the reciprocal image $[\mathbf{x}_{j+1}]^{inv}$ and eliminating the inconsistent state vectors

$$[\mathbf{x}_{j+1}]^c = [\mathbf{x}_{j+1}]^{inv} \cap [\mathbf{x}_{j+1}]^p \quad (6)$$

Thus for the next measurement the predication phase will be initialized by $[\mathbf{x}_{j+1}] = [\mathbf{x}_{j+1}]^c$. In fact, by repeating these two phases each time a new measurement is available, one improves considerably precision of the guaranteed over approximation of the set $\mathbb{X}(t)$.

The algorithm below shows the process for this set-membership estimation approach

Algorithm : Prediction_Correction_estimation

1. **Input**: $([\mathbf{x}_0], [\mathbf{p}], \mathbf{f}, \mathbf{g}, [\mathbf{y}_1], \ldots, [\mathbf{y}_{nT}])$
2. $t_j = t_0$; $[\mathbf{x}_j] = [\mathbf{x}_0]$;
3. **while** $(t_j < t_{nT})$ **do**
4. $\{t_{j+1}, [\mathbf{x}_{j+1}]^p\}$=**Validated_Integration**$([\mathbf{x}_j], [\mathbf{p}], t_j)$;
5. $[\mathbf{x}_{j+1}]^{inv} = \mathbf{g}^{-1}([\mathbf{y}_{j+1}], [\mathbf{p}])$;
6. $[\mathbf{x}_{j+1}]^c = [\mathbf{x}_{j+1}]^{inv} \cap [\mathbf{x}_{j+1}]^p$;
7. $[\mathbf{x}_{j+1}] = [\mathbf{x}_{j+1}]^c$
8. $j = j+1$;
9. **end**
10. **Output**: $\mathbb{X}(t)$.

3 MÜLLER'S THEOREM

In this section, we introduce an approach for bracketing an uncertain dynamical systems when both the initial state and parameter vectors are defined by boxes (intervals vector). The main idea consists in building a lower and an upper dynamical system which involve no uncertainty and enclose in a guaranteed way, the all state trajectories generated by the original uncertain system. This approach relies on comparison theorems for differential inequalities (Smith, 1995; Hirsch and Smith, 2005), and in particular the work of Müller (Müller, 1926; Marcelli and Rubbioni, 1997).

Theorem:(Müller, 1926; Kieffer and Walter, 2006)
Consider the dynamical system

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, \mathbf{p}, \mathbf{u}(t)), \tag{7}$$

where function $\mathbf{f}$ is continuous over a domain $\mathbb{T}$ defined by

$$\mathbb{T} : \begin{cases} \omega(t) \leq \mathbf{x}(t) \leq \Omega(t) \\ \underline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}} \\ t_0 \leq t \leq t_{n_T} \end{cases} \tag{8}$$

Functions $\omega_i(t)$ and $\Omega_i(t)$ are continuous over $[t_0, t_{n_T}]$ for all i and satisfy the following properties

1. $\omega(t_0) = \underline{\mathbf{x}}_0$ and $\Omega(t_0) = \overline{\mathbf{x}}_0$
2. the left derivatives $D^-\omega_i(t)$ and $D^-\Omega_i(t)$ and the right derivatives $D^+\omega_i(t)$ and $D^+\Omega_i(t)$ of $\omega_i(t)$ and $\Omega_i(t)$ are such that

$$\forall i,\ D^{\pm}\omega_i(t) \leq \min_{\underline{\mathbb{T}}(t)} f_i(\mathbf{x}, \mathbf{p}, t) \tag{9}$$

$$\forall i,\ D^{\pm}\Omega_i(t) \geq \max_{\overline{\mathbb{T}}(t)} f_i(\mathbf{x}, \mathbf{p}, t) \tag{10}$$

where $\underline{\mathbb{T}}(t)$ is the subset of $\mathbb{T}(t)$ defined by

$$\underline{\mathbb{T}}_i : \begin{cases} x_i = \omega_i(t) \\ \omega_j(t) \leq x_j \leq \Omega_j(t),\ j \neq i \\ \underline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}} \end{cases} \tag{11}$$

and where $\overline{\mathbb{T}}(t)$ is the subset of $\mathbb{T}(t)$ defined by

$$\overline{\mathbb{T}}_i : \begin{cases} x_i = \Omega_i(t) \\ \omega_j(t) \leq x_j \leq \Omega_j(t),\ j \neq i \\ \underline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}} \end{cases} \tag{12}$$

Then for all $\mathbf{x}_0 \in [\underline{\mathbf{x}}_0, \overline{\mathbf{x}}_0]$, $\mathbf{p} \in [\underline{\mathbf{p}}, \overline{\mathbf{p}}]$, system (1) admits a solution $\mathbf{x}(t)$ that stays in the domain

$$\mathbb{X} : \begin{cases} t_0 \leq t \leq t_{n_T} \\ \omega(t) \leq \mathbf{x}(t) \leq \Omega(t) \end{cases} \tag{13}$$

and takes the value $\mathbf{x}_0$ at t_0. If, in addition, for all $\mathbf{p} \in [\underline{\mathbf{p}}_0, \overline{\mathbf{p}}_0]$, function $\mathbf{f}(\mathbf{x}, \mathbf{p}, t)$ is Lipschitzian with respect to $\mathbf{x}$ over $\mathbb{D}$ then this solution is unique for any given $\mathbf{p}$.

Finally, an enclosure for the solution of (7) is given by

$$\forall t \in [t_0, t_{n_T}], \quad [\mathbf{x}](t) = [\omega(t), \Omega(t)] \tag{14}$$

The main difficulty is to obtain suitable bracketing functions $\omega(t)$ and $\Omega(t)$ in the general case. However, when the components of $\mathbf{f}$ are monotonic with respect to each parameter and each state vector component, it is quite easy to define these systems (Kieffer et al., 2006), while avoiding possible divergence that may occur when both upper and lower components of the parameter/state vector appear simultaneously in the same expression of the components of the bracketing systems (Ramdani et al., 2006).

Rule 1 - Use of monotonicity property (Kieffer et al., 2006)
In order to build the upper system, i.e. the one which yields the upper solution $\Omega(t)$, one can replace in the formal expression of f_i, x_i by Ω_i, x_j $(j \neq i)$ by Ω_j if $\frac{\partial f_i}{\partial x_j} \geq 0$ or by ω_j if $\frac{\partial f_i}{\partial x_j} \leq 0$ and p_k by $\overline{p}_k$ if $\frac{\partial f_i}{\partial p_r} \geq 0$ or by $\underline{p}_k$ if $\frac{\partial f_i}{\partial p_k} \leq 0$. The components of the lower system, i.e. the one which yields the lower solution $\omega(t)$ are derived by reversing monotonicity conditions.

Obviously $\omega(t)$ and $\Omega(t)$ are in general, solutions of a system of coupled differential equations, i.e.

$$\begin{cases} \dot{\omega}(t) = \underline{\mathbf{f}}(\omega, \Omega, \underline{\mathbf{p}}, \overline{\mathbf{p}}, t) \\ \dot{\Omega}(t) = \overline{\mathbf{f}}(\omega, \Omega, \underline{\mathbf{p}}, \overline{\mathbf{p}}, t) \\ \omega(t_0) = \underline{\mathbf{x}}_0 \\ \Omega(t_0) = \overline{\mathbf{x}}_0 \end{cases} \tag{15}$$

which involves no *uncertain* quantity. Therefore interval Taylor models such as the one presented in (Nedialkov, 1999) can be used for efficiently solving (15). Indeed when these methods are used for solving differential equations with no uncertainty, they are usually able to curb the pessimism induced by the wrapping effect, even over long integration time.

4 HYBRID BRACKETING SYSTEM

Now, one address the case of uncertain dynamical systems (1), for which the signs of the partial derivatives $\partial f_i/\partial p_k$ and $\partial f_i/\partial x_j$ change along the integration time interval $[t_0, t_{n_T}]$. In such a case, the uncertain system (1) admits an enclosure over each time interval where functions f_i are monotonic with respect to variables p_k and x_j. Therefore both upper and lower bounding systems are defined by piecewise nonlinear ODEs and can thus be regarded as hybrid dynamical systems. Thus, they can be modeled by an *hybrid automaton*(Alur et al., 1995).

So to compute a guaranteed enclosure of reachable state space generated by the uncertain system (1), we will built hybrid system of l continuous dynamic modes which satisfied locally conditions imposed by *rule 1*

$$\mathbb{M} = \{M_1, M_2, ..., M_l\} \tag{16}$$

and which given in state space representation by

$$\begin{cases} \dot{\omega}(t) = \underline{\mathbf{f}}_{M_i}(\omega, \Omega, \underline{\mathbf{p}}, \overline{\mathbf{p}}, t) \\ \dot{\Omega}(t) = \overline{\mathbf{f}}_{M_i}(\omega, \Omega, \underline{\mathbf{p}}, \overline{\mathbf{p}}, t) \end{cases} \text{for } i = 1, ..., l \tag{17}$$

Hence, the evolution of this hybrid system is controlled by the sign changes of the partial derivatives $\partial f_i/\partial p_k$ and $\partial f_i/\partial x_j$ which represents the guard conditions which authorize the transitions between the continuous bracketing modes. Thus, for a given initial conditions the execution of this hybrid automata makes it possible to obtain a guaranteed upper approximation of the reachable state space of the continuous time system (1).

Example:
Let us consider the following system

$$\dot{x}(t) = f(x, [p]). \tag{18}$$

According to the sign of $\partial f/\partial p$, the system (18) admitted two possible bracketing modes

$if\ \ \partial f/\partial p \leq 0$

$$then\ \ M_1 = \begin{cases} \dot{\Omega}(t) = f_{M_1}(\Omega, \underline{p}) \\ \dot{\omega}(t) = f_{M_1}(\omega, \overline{p}) \end{cases} \tag{19}$$

$$else\ \ M_2 = \begin{cases} \dot{\Omega}(t) = f_{M_2}(\Omega, \overline{p}) \\ \dot{\omega}(t) = f_{M_2}(\omega, \underline{p}). \end{cases} \tag{20}$$

and its hybrid bracketing automata is represented in the figure 2

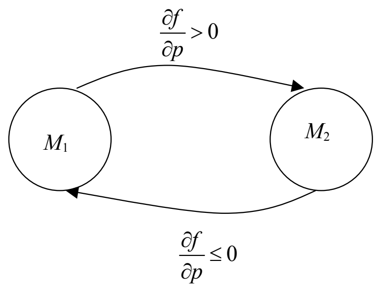

Figure 2: Hybrid automata.

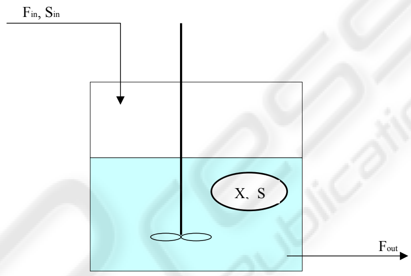

Figure 3: General representation of a bioreactor.

5 APPLICATION

5.1 Model

Generally, the mathematical model of biotechnological processes is difficult to establish with accuracy, that is due to the living behavior of the bacteria represented by a complex poorly known function of the bioreactor state. In this section we consider a simple model where only one population of bacteria is taken into account. In this context, to describe the state of the bioreactor, two state variables are necessary, the first one represents the bacteria concentration called biomass and denoted X, the second one represents the substrate concentration, denoted S. Thus the model below shows the evolution of the biomass by consuming the polluting body S

$$\begin{cases} X(t) = \mu(S)X - \alpha DX \\ S(t) = -k_1\mu(S)X + D(S_{in} - S) \end{cases} \tag{21}$$

where $\mu(s)$ is the growth rate of biomass modeled by the Haldane law:

$$\mu(S) = \mu_0 \frac{S}{S + k_s + S^2/k_i} \tag{22}$$

with the uncertain bounded parameter μ_0

$$\underline{\mu}_0 \leq \mu_0 \leq \overline{\mu}_0.$$

This bioreactor is fed by a solution containing substrate in concentration S_{in} which is not exactly measured

$$\underline{S}_{in}(t) \le S_{in}(t) \le \overline{S}_{in}(t)$$

and we suppose that the biomass is accessible to measurement

$$y(t) = X(t).$$

5.2 Building Hybrid Bracketing System

For this application, according to the sign of the derivative of μ with respect to S,

$$sign(\frac{d\mu(S)}{dS}) \begin{cases} > 0 \text{ if } S < \sqrt{k_s k_i},\ \forall S \in [S] \\ \le 0 \text{ if } S \ge \sqrt{k_s k_i},\ \forall S \in [S] \\ \text{ambiguous if } \sqrt{k_s k_i} \in [S] \end{cases} \quad (23)$$

system (21) allows three possible modes for the bracketing. The first mode, $M_1 = 1$, corresponds to the intervals time when this derivative is negative

$$(M_1 = 1) \begin{cases} \overline{X}(t) = \overline{\mu}(\overline{S})\overline{X} - \alpha D\overline{X} \\ \overline{S}(t) = -k_1\underline{\mu}(\overline{S})\underline{X} + D(\overline{S}_{in} - \overline{S}) \\ \underline{X}(t) = \underline{\mu}(\underline{S})\underline{X} - \alpha D\underline{X} \\ \underline{S}(t) = -k_1\overline{\mu}(\underline{S})\overline{X} + D(\underline{S}_{in} - \underline{S}) \end{cases} \quad (24)$$

and the second mode, $M_2 = 2$, is linked to intervals time where this derivative is positive

$$(M_2 = 2) \begin{cases} \overline{X}(t) = \overline{\mu}(\underline{S})\overline{X} - \alpha D\overline{X} \\ \overline{S}(t) = -k_1\underline{\mu}(\overline{S})\underline{X} + D(\overline{S}_{in} - \overline{S}) \\ \underline{X}(t) = \underline{\mu}(\overline{S})\underline{X} - \alpha D\underline{X} \\ \underline{S}(t) = -k_1\overline{\mu}(\underline{S})\overline{X} + D(\underline{S}_{in} - \underline{S}). \end{cases} \quad (25)$$

Finally, the third mode $M_3 = 0$ is associated to case when the sign of this derivative is ambiguous. In this case either one uses guaranteed integration methods based on interval Taylor models to bracket (21), or if this is possible, one finds a trivial bracketing for $\mu(s)$. For example,

$$\underline{\mu}_0 \frac{\underline{S}}{\underline{S} + k_s + \underline{S}\overline{S}/k_i} \le \mu(S) \le \overline{\mu}_0 \frac{\overline{S}}{\overline{S} + k_s + \underline{S}\overline{S}/k_i}.$$

Hence, one propose the below differential equations system for the third mode $M_3 = 0$

$$(M_3 = 0) \begin{cases} \overline{X}(t) = \overline{\mu}_0 \frac{\overline{S}}{\overline{S}+k_s+\underline{S}\overline{S}/k_i}\overline{X} - \alpha D\overline{X} \\ \overline{S}(t) = -k_1\underline{\mu}(\overline{S})\underline{X} + D(\overline{S}_{in} - \overline{S}) \\ \underline{X}(t) = \underline{\mu}_0 \frac{\underline{S}}{\underline{S}+k_s+\underline{S}\overline{S}/k_i}\underline{X} - \alpha D\underline{X} \\ \underline{S}(t) = -k_1\overline{\mu}(\underline{S})\overline{X} + D(\underline{S}_{in} - \underline{S}). \end{cases} \quad (26)$$

Now, function **Validated_Integration** in algorithm **Prediction_Correction_Estimation** selects on line the local bracketing mode according to the sign of $\frac{d\mu(.)}{dS}$ and then uses guaranteed numerical integration methods for ODE based on intervals Taylor models for solving (24), (25) or (26).

5.3 Results of Simulation

The data considered in this example are as follows: $\alpha = 0.5$, $k = 42.14$, $k_s = 9.28mmol/l$, $k_i = 256mmol/l$, $\mu_0 \in [0.64, 0.84]$, $X_0 \in [0 >, 10]$, $S_0 \in [0, 100]$, $S_{in}(t) \in ([62, 68] + 15\cos(1/5t))$,

$$D(t) = \begin{cases} 2 \text{ si } 0 \le t \le 5 \\ 0.5 \text{ si } 5 \le t \le 10 \\ 1.14 \text{ si } 10 \le t \le 20, \end{cases}$$

and the feasible measurement domain

$$\mathbb{Y}(t_j) = [0.98y_m(t_j), 1.02y_m(t_j)]$$

with a constant measurements time step $t_{j+1} - t_j = 2$.

The red continuous lines curves in figures 4 and 5 show the guaranteed enclosure of all the possible state trajectories of (21) which are compatible with the model and its uncertainties and the acceptable domain of the discrete measurements signal. The discontinuous blue curves represent the real state of (21) which corresponds to the following values of the uncertain parameters and the initial state: $\mu_0 = 0.74$, $X_0 = 5$, $S_0 = 40$ and $S_{in}(t) = 65 + 15\cos(1/5t)$.

So figure 6 shows the actual commutations between the three bracketing modes as obtained during the simulation period. This represents the evolution of the discrete component of the hybrid automata used to bracket the state flow generated by the uncertain system (21).

As a conclusion, guaranteed numerical integration methods for ODE based on the interval Taylor models fail to give non divergent enclosures after few integration step because of the large magnitude in uncertainty in both parameter vector and initial state vector. In addition, *rule 1* is not applicable over all the simulation period because the sign of the partial derivative $\frac{\partial X}{\partial S}$ changes with time. Hence, our hybrid bracketing method is an important alternative to solve set membership estimation problems of this kinds.

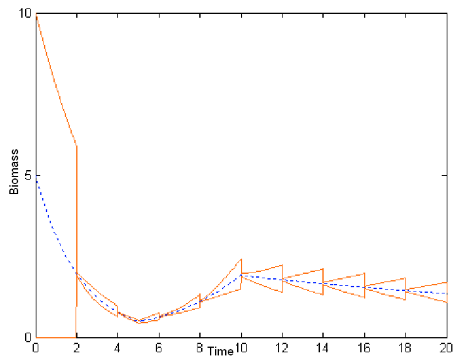

Figure 4: The both real and estimated evolution the biomass.

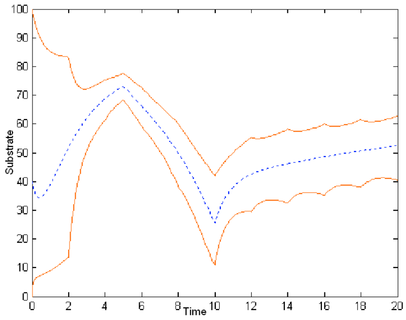

Figure 5: The both real and estimated evolution the substrate.

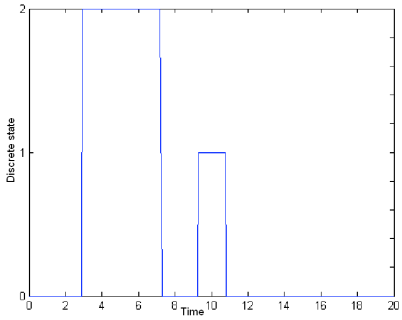

Figure 6: Indices of the three bracketing modes used over the simulation period.

6 CONCLUSIONS

In this communication, we wanted to show that by using the Müller's theorem and by analyzing the monotonicity of the uncertain dynamical system with respect to both the uncertain variables and parameters, one is able to solve the state membership estimation problem for a large class of uncertain dynamical systems. Indeed, the method presented makes it possible to circumvent the propagation of the pessimism due to the wrapping effect which generally causes the divergence of guaranteed numerical integration methods based on interval Taylor models when used with uncertain ODEs. In the future, we wish to extend this approach for systems with higher dimension and also to hybrid dynamical systems.

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