SLIDING MODE CONTROL
Is it Necessary Sliding Motion?

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Abstract: Sliding mode control has been recognized to be insensitive to exogenous perturbations if the reachability conditions is warranted. Two phases follow the dynamics of the closed-loop perturbed system: 1) Finite-time convergence to the sliding surface, and 2) Sliding motion along the sliding surface. In the sliding motion the system has a reduced-order dynamic behavior. But, is it really necessary to have sliding motion to warranty the robustness property of the sliding mode controller? The main objective of this position paper is to theorize this important question.

1 INTRODUCTION

The most distinguished feature of sliding mode control it its ability to issue very robust control systems facing exogenous perturbations. Moreover, sliding mode control has been applied to a wide variety of control objectives such as regulation, tracking control, model following, adaptive control, observer design, among others (Perruqueti and Barbot, 2002; Edwards and Spurgeon, 1998). However, in all issues, sliding motion is secure to keep operable the sliding mode controller. Basically, the sliding controller involves two steps design. Ones is the finite-time convergence to the discontinuity manifold (the sliding surface), and the second one is the design of the sliding dynamics. However, and according with the examples given here, sliding motion is not necessary to warranty the main property of sliding controllers: insensitiveness to external perturbations. The outline of this position paper is as follows. The basic idea of sliding mode control using a second-order system is studied in Section two. Section three, two examples where not sliding motion exits all the time are granted and applied to a perturbed system illustrating that the insensitiveness property is not loss. Finally, Section five the conclusions are stated.

2 SLIDING MODE CONTROL

The basic idea of sliding mode control can be illustrated using second-order system (J. Y. Hung and HUng, 1993; Edwards and Spurgeon, 1998; Perruqueti and Barbot, 2002). At this point, consider the following system (Perruqueti and Barbot, 2002):

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_2 + u. \end{aligned} \tag{1}$$

This system represents a dc-motor model, where u is the scalar input control. Let us assume that the sliding surface is specified as:

$$s(x_1, x_2) = x_2 + \alpha x_1 = 0, \quad \alpha > 0. \tag{2}$$

The control objective consists to find a control law u such that the sliding surface is attractive and reachable in finite time. In the sliding surface, the sliding mode motion then takes place (Perruqueti and Barbot, 2002; Edwards and Spurgeon, 1998; J. Y. Hung and HUng, 1993). The reachability in finite time is warranted if

$$s\dot{s} \leq \eta |s|, \quad \eta > 0, \tag{3}$$

called the $\eta-$reachability condition (Edwards and Spurgeon, 1998; Perruqueti and Barbot, 2002). Then, from (2), we have

$$s\dot{s} = s(\dot{x}_2 + \alpha \dot{x}_1). \tag{4}$$

Invoking (1), we get

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$$
\begin{aligned}
s\dot{s} &= s(-x_2 + u + \alpha x_2) \\
&= s((\alpha - 1)x_2 + u).
\end{aligned} \tag{5}
$$

If we set

$$u = -(\alpha - 1)x_2 - \eta sgn(s), \quad \eta > 0, \tag{6}$$

we arrive to

$$s\dot{s} = s(-\eta sgn(s)) = -\eta|s|. \tag{7}$$

So, any trajectory of the closed-loop system (1) and (6), reaches the sliding surface $s = 0$ in finite time. Furthermore, in the sliding surface, the dynamic motion yields:

$$x_2 + \alpha x_1 = \dot{x}_1 + \alpha x_1 = 0 \Rightarrow \dot{x}_1 = -\alpha x_1 \tag{8}$$

which is asymptotically stable because $\alpha > 0$. Observe that the trajectory in the sliding surface can not scape from it. In resume, slide mode control involves two steps. Design a control law such as the sliding manifold be attractive and reachable in finite time, and design the sliding mode dynamics such as it is asymptotically stable. Also, it is recognizable that in sliding motion the order of the system is reduced from two to one. Let us now assume exogenous perturbation in our system. So, let it be as:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -x_2 + u + sin(t)
\end{aligned} \tag{9}
$$

where the exogenous perturbation is bounded. Then, using (6), we have:

$$s\dot{s} = -\eta|s| + sin(t)s \leq -\eta|s| + |s| = -|s|(\eta - 1). \tag{10}$$

So, the η-reachability condition is satisfied with $\eta > 1$ and any trajectory of the closed-loop perturbed system (9) and (6) reaches the sliding surface in finite time, where its sliding mode dynamics ($\dot{x}_1 = -\alpha x_1$) is unaltered in spite of the external perturbation. Simulations results are given in Fig. 1 with $\alpha = 0.5$ and $\eta = 2$.

3 MODIFIED SLIDING MODE CONTROL

Here, our main contributions are stated. The objective is to eliminate the persistency of chattering in the control law keeping the benefits of the sliding

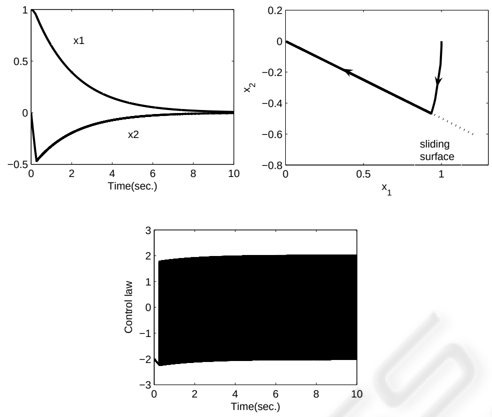

Figure 1: Simulations results of the perturbed system with $x_1(0) = 1$ and $x_2(0) = 0$.

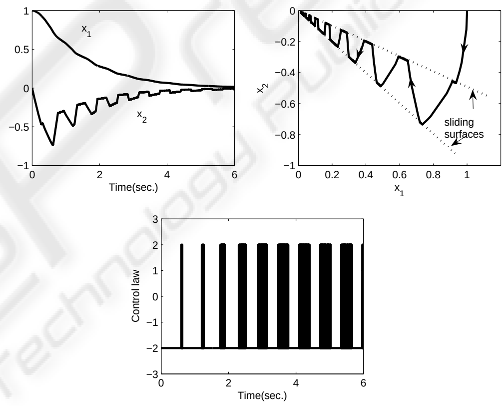

Figure 2: Simulations results of the perturbed system with $x_1(0) = 1$ and $x_2(0) = 0$ utilizing the modified sliding mode control.

mode control. From Fig. 1, when the system is perturbed and the η-reachability conditions is satisfied, the closed-loop system is insensitive to external perturbations, but chattering is sustained almost all the time. One way to reduce persistency on chattering is by means of changing the value of α in (6) and (2). For instance, employing the same perturbed system (9), with $\eta = 2$, and the same initial conditions; changing α from 0.5 to 1 at a frequency of $5/\pi Hz$ (a square signal), the simulation results are shown in Fig. 2. Here, the property of insensitiveness to external perturbations is evident and the chattering persistency is reduced. The commutation in the value of α is similar to commuting between two sliding surfaces (see Fig. 2). This works because the η-reachability condition ensures insensitiveness to exogenous perturbations. Here, sliding motion just occurs a certain time. So, the price is that sliding motion is not preserved all the time; i.e., the reduced order dynamic motion is not sustained all the time. But, it is not so important from the robust control of view (mitigation of the external perturbation). Moreover, we can commutate between these sliding surfaces avoiding sliding motion; i.e., when the trajectory hits a sliding surface (warranted by the η-reachability condition), commutate to the other one (see Fig. 3). In Fig. 3, chattering occurs as a Zeno behavior, that is, the number of switching in the control law tends to infinite in finite time.

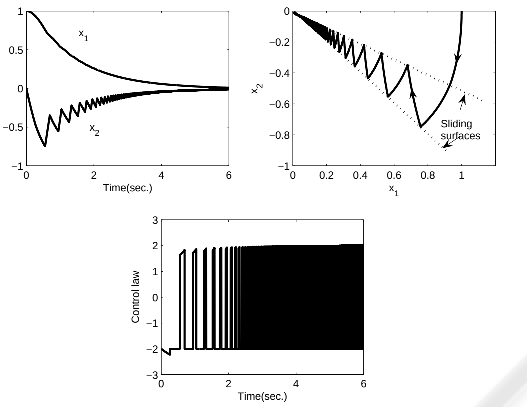

Figure 3: Simulations results of the perturbed system with $x_1(0) = 1$ and $x_2(0) = 0$ utilizing a second modified sliding mode control.

4 CONCLUSIONS

Employing second-order dynamic systems, we evidenced that sliding motion is not required to keeping insensitive property of sliding mode control. Also, with the examples shown here, chattering is not persistently exhibited. So, we have presented an important observation about this topic, that we think it will open further comments on it.

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