

# Intelligent Refueling Advisory System

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**Abstract.** Recent advances in wireless communication technologies have led to numerous new developments that take advantage of network access, ranging from real-time information delivery essential for many driving decisions to harvesting off-board computing power for remote vehicle diagnostics. Specifically, with the ever increasing fuel cost, there is growing popularity of services that provide information to drivers on current gas prices and alerts on upcoming changes to gas prices. This paper demonstrates how to take currently available technology one step further by providing a proactive refueling advisory system based on minimizing fuel costs over routes and time. The system integrates vehicle data, a navigation system, and internet connectivity to supplant the existing vehicle low fuel warning with a comprehensive decision support system on refueling choices.

## 1 Introduction

Recent advances in wireless communication technologies have led to numerous developments that take advantage of internet connectivity. These developments support a wide range of industries and applications. Focusing on the personal transportation sector, harvesting computing power accessible over the net can be used for remote vehicle diagnostics. Another example is real-time information delivery used onboard vehicles to assist drivers with decisions such as where to find the nearest gas station. A popular extension of this is an emerging service that offers access to current retail gasoline prices. With ever increasing fuel prices, drivers are paying closer attention to their refueling strategies, making when and where to refuel a more important and complex question than in the past. Many drivers are willing to drive a few extra miles to get a cheaper gas price. Drivers may even be willing to only purchase a few gallons of gas today when refueling is necessary with expectations of lower gas prices the next day; this results in more frequent stops, but lower total fuel costs.

The model we introduce in this paper offers users substantial benefits by identifying the best possible refueling strategy over an entire route and multiple time periods, as opposed to the best strategy based on current gas prices in a small local area. It includes how to determine the best refueling strategy for a driver by considering their route, the gas stations and gas prices along the route, the starting fuel level, good estimates of fuel consumption along the route, and user preferences.

Inputs to the system can come from several different sources. A driver's route can be determined in either of two ways: it can be manually input or for frequently driven routes it can be predicted from past driving patterns (see [8] and [16]). Once the route has been determined, gas stations along the route can be identified along with current gas prices. Current gasoline prices can also be used to estimate future fuel prices by a forecasting model. By connecting to the vehicle's internal network, the current fuel level can be obtained along with the vehicle's average fuel economy, which can be used to form a good estimate of fuel consumption for new routes. For frequently driven routes, previously collected data from the vehicle internal network can be used determine route specific fuel consumption.

We will first present background in Section 2 and then give an overview of our system in Section 3. We then provide details on how we forecast future gas prices and determine the refueling strategy in Sections 4 and 5, respectively. We present an example in Section 6 and discuss future work and conclude in Section 7.

## 2 Background

In the United States, there are currently two main sources of current gasoline prices: credit card transactions and networks of gas price spotters. GasBuddy [10] and GasPrice-Watch [11] are examples of the latter approach. Both companies offer a web site and mobile application where the user can look up gas prices for free. The gas prices reported are collected from a network of gas price spotters who are usually volunteers. Consequently, neither company can guarantee the accuracy of the information they provide. Oil Price Information Services (OPIS) [19] obtains their data from credit card transactions, which makes it more reliable than other sources.

When a driver is deciding whether or not to stop to purchase gas, they usually contemplate if the gas prices will be going up or down in the next couple of days. Gas prices at the pump largely depend on wholesale prices and are subject to cyclical fluctuations. While some people follow fluctuations of retail gasoline prices and may have reasonable predictions on when prices will rise or fall, the majority of drivers do not know the likelihood of gas prices going up or down. There are a number of popular web sites that offer the general population an educated guess on the future direction of gas prices; one example is The Gas Game [20].

To assist drivers in making more informed refueling decisions, many car navigation system manufacturers offer gas price services delivered to the vehicle through mobile phone data services or satellite radio broadcast. Examples include the TomTom 920T [17], Dash express [3], and nüvi 780 by Garmin [9] (most get their prices from OPIS). Vehicle manufacturers, like Ford Motor Company, are even taking this one step further by offering their factory installed navigation system with SIRIUS Travel Link [7]. In addition, some researchers have explored notifying drivers of the cheapest place to refuel when their current gas level drops below a certain threshold (see [13] and [15]).

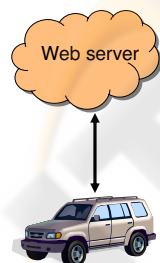
In comparison to the privately owned vehicle sector, the commercial trucking industry has considered complex decisions about when and where truck drivers should stop for gas as a part of their route planning process for quite some time due to the tight correlation between profits and fuel cost. They have developed optimization models to

assist with refueling decisions as each truck's driving route is known and network infrastructures are in place to communicate with truck drivers on the road: an example is Expert Fuel developed by Integrated Decision Support [14].

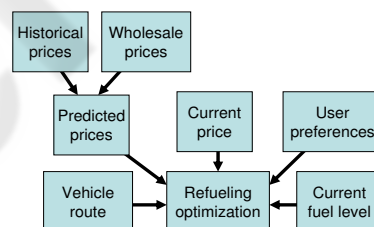
Our approach differs from the above in the several ways. First, we use forecasted gas prices, so that we can consider future days on the route, making the best decision over multiple days. While forecasting has been used for other consumer oriented help applications, such as predicting when online customers should purchase airline tickets [6], it is a new approach for providing refueling strategies. Another way that our approach is different is that our routes do not have to be predetermined, as we can get routes directly from the vehicle GPS. In addition, we get other data directly from the vehicle internal network, such as current fuel level and fuel consumption. Also, we can recalculate on the fly a new strategy if the driver deviates from the original plan. Finally, our system acts as a proactive fuel gauge that notifies the driver before they are approaching the gas station where they should stop (current low on fuel gauge warnings provide data only relative to the internal fuel tank).

### 3 System Overview

Our system combines web and in-vehicle components to assist the driver in minimizing their fuel purchase costs, as seen in Figure 1. To use the system, a driver must first register on a web portal and enter their fuel purchase preferences, such as how many times per week they are willing to stop for gas and which brands they prefer. The web site also assists the driver in mapping out their frequently traveled routes. The driver then creates an approximate travel plan for the next week, which the system uses to identify potential refueling stations. Future versions of our system will predict the driver's future driving routes without any input from the driver



**Fig. 1.** System consisting of web to vehicle connection.



**Fig. 2.** This figure illustrates the data flow used in the refueling strategy optimization algorithm.

To get the most out of our system, the driver's vehicle should include a navigation system and a means of connecting the vehicle to the internet, such as a telematics platform (e.g., OnStar [18], BMW Assist [1]) or a cell phone with data service (e.g. GPRS or EVDO). The vehicle maintains a connection to the web server and periodically uploads the vehicle's current fuel level and GPS location. An optimization algorithm (see Section 5.1) on the web server uses the fuel level to calculate when and where the driver

should refuel and how many gallons to purchase. If the algorithm determines that the driver should stop, it sends this information to the vehicle's navigation system and alerts the driver.

Figure 2 shows the main data sources our system uses to calculate a refueling plan. Future fuel prices for individual fueling stations are predicted using a forecasting algorithm (see Section 4) that takes into account historical prices as well as wholesale prices. Historical and current prices are obtained from a database maintained by OPIS. User preferences and vehicle routes are obtained from the web portal, and the current fuel level is directly obtained from the vehicle.

## 4 Forecasting Model

As previously mentioned, if the consumer had more information on gasoline prices at each gas station along the specified route and predicted prices for the next few days, determining a refueling plan that would minimize the total fuel cost over the trip would be achievable. One important aspect in determining the consumer's refueling strategy is an econometrical model that reasonably predicts daily gasoline retail prices at each station on the route. Many studies show that the retail gasoline price is highly correlated with wholesale gasoline price, according to the production and distribution system of gasoline in the U.S [2]; this provides the foundation for our approach.

We present a simple forecasting approach that predicts future retail gas prices at individual stations when the following limited data is available: wholesale regional gas prices, one-month future market wholesale prices, current station retail gas prices, and historical retail station gas prices. To determine which wholesale price to use as the explanatory variable in our forecasting model, we considered several wholesale prices to see which provided the best correlation to retail gas prices. There are four regional wholesale prices in the U.S.: mid-continent, Los Angeles, New York, and Gulf coast markets. These four regions and the one-month future market wholesale prices have the following correlation with retail prices at gasoline stations in Michigan: 0.89, -0.71, -0.89, 0.92, and -0.91, respectively. In addition, we tested the correlations between the average of two or more wholesale prices and the retail gasoline prices: all values were less than 0.92. As a result, we chose the Gulf coast wholesale price as our explanatory variables in the model, which yielded the best correlation to retail gas prices.

The Energy Information Administration (EIA) reported in 1999 [5] that the downstream gasoline price responses caused by upstream price changes can be represented by the following equation:

$$D_t = \beta_0 + \sum_i (\beta_i U_{t-i}) + \varepsilon. \quad (1)$$

Here  $D_t$  is the price offered to each gas station by the supplier at time  $t$ ; it is a moving average of the wholesale prices  $U_{t-i}$  at time  $t$  for the previous days  $i$  such that  $i = 1, \dots, 6$ . The error term is  $\varepsilon$ , which also represents the downstream markup.

The markup of each gasoline station  $\pi_t$  at time  $t$  is the retail gasoline price  $p_t$  at time  $t$  at each gasoline station minus the price  $D_t$  paid to its suppliers; that is,

$$\pi_t = p_t - D_t. \quad (2)$$

In this study, we do not use the coefficients  $\beta$  provided by the EIA report, but estimate coefficients  $\bar{\beta}$  by applying an Ordinary Least Square (OLS) linear regression [12]: the retail gasoline price at time  $t$  is the dependent variable, and explanatory variables are the wholesale prices  $U_{t-i}$  for  $i = 1, \dots, 6$ , and the monthly dummy prices,  $M_k$ , where  $k = 1, \dots, 12$  represents January through December, respectively, for any day  $t$ . The results of this OLS linear regression are in Table 1. Applying the regression coefficients,  $\bar{\beta}$  and  $\bar{\theta}$ , we can obtain the estimated suppliers' prices  $\bar{D}_t$ :

$$\bar{D}_t = \bar{\beta}_0 + \sum_i (\bar{\beta}_i U_{t-i}) + \sum_k (\bar{\theta}_k M_k). \quad (3)$$

**Table 1.** Relationship between retail gas and mid-continent wholesale prices:  $R^2 = 0.97$ , \*\*\*Statistically significant at 99% confidence level, \*Statistically significant at 90% confidence level.

Variable	Coefficient ( $\bar{\beta}$ )	Standard Error
$U_{t-1}$	0.002 ***	0.001
$U_{t-2}$	0.001	0.001
$U_{t-3}$	0.001	0.001
$U_{t-4}$	0.001	0.001
$U_{t-5}$	4.531 e-04	0.001
$U_{t-6}$	0.003***	0.001
$M_2$	-0.180 ***	0.014
$M_3$	-0.090***	0.020
$M_4$	-0.039*	0.031
$M_5$	0.020	0.038
$M_6$	0.089***	0.033
$M_7$	0.088***	0.032
$M_8$	0.102 ***	0.025
$M_9$	-0.012	0.030
$M_{10}$	-0.012	0.030
$M_{11}$	0.001	0.041
$M_{12}$	0.009*	0.006
constant	0.950***	0.059

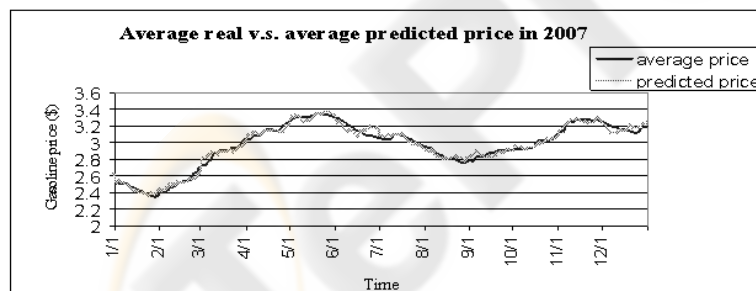
So, the estimated markup of each gasoline station at time  $t$  is  $\bar{\pi}_t = p_t - \bar{D}_t$ . However, we randomly chose some gasoline stations to study their markup and found some interesting markup patterns. For each day of the week, even if the markup  $\bar{\pi}_t$  is different for every  $t$ , it is always higher at the end of a week and lower at the beginning of a week. Since it is not feasible to test each gasoline station's markup, we make the assumption from our comprehensive testing that gasoline prices at each gasoline station have a weekly pattern; without loss of generalization, we calculate the average markup for each weekday, providing us with seven average markups for each gasoline station. We call this average markup for each weekday  $\bar{\pi}_n$  for  $n = 1, \dots, 7$ ; i.e. for each day of the week  $n$ , we calculate the average markup for  $n$  using historical data for  $n$ . For example, to calculate  $\bar{\pi}_n$  for  $n = 1$  (Monday), we take the average markup over all Mondays ( $\bar{\pi}_t$  such that  $t$  is a Monday) in our historical data. Hence, we can predict the gas price for any station one day in advance by

$$\bar{p}_{t+1} = \bar{D}_{t+1} + \bar{\pi}_n, \quad (4)$$

where  $n = 1, \dots, 7$  represents Monday through Sunday, respectively, for any day  $t$ . When retail gas price for time  $t$  is not available for a specific station, we can also use equation (4) to estimate the missing price.

To forecast the gasoline prices at one specific gasoline station more than one day in advance, we can do so using  $\bar{p}_{t+i} = \bar{D}_{t+1} + \pi_{n+i}$ , for  $i = 1$  to  $m$ , where  $m$  is the number of days ahead for which the forecast is desired. Because we do not know the wholesale or suppliers' prices on days  $t+i$ , when  $i > 1$ , the accuracy of the forecasting results will decrease as  $i$  increases.

The graphs below show the prediction results from the simple price forecasting model described above for gas purchases using 2007 daily data. Figure 3 and 4 display average daily retail gasoline prices (averages include prices for 931 stations) and predicted gasoline prices in Michigan. Figure 3 shows the average daily retail gasoline prices compared to the predicted gasoline prices based on the one-day ahead prediction for Michigan over 2007. Figure 4 includes five predicted prices which are one-day ahead, two-day ahead, three-day ahead, four-day ahead and five-day ahead, respectively, for the month of November 2007. For example, when using the three-day ahead predictions, we would use November 1, 2007 data to predict the price of gas on November 4, 2007. The mean of absolute residual values (i.e., the absolute difference between real price and predicted price) for the one to five day ahead predictions are approximately 3.2 cents, 2.8 cents, 2.9 cents, 3.1 cents, and 3.3 cents; the standard deviation of the absolute residual values is 2.3 cents, 2 cents, 2.1 cents, 2.2 cents and 2.4 cents, respectively. Figure 5 show the predicted daily gasoline prices at one stations in Michigan as an example to demonstrate the accuracy of predicted prices for an individual gas station.



**Fig. 3.** Average retail price compared to average predicted retail price for 931 gasoline stations.

Some papers discuss factors that impact daily or weekly gasoline price fluctuation [4], but few researchers have studied forecasting models for daily gasoline prices. Compared to those models, our approach for price prediction provides a simple way when limited data is available to estimate daily gasoline price at each gasoline station with forecasting results that are within a reasonable range to the retail prices. In future research, we plan to develop a more advanced forecasting model that will require more data, but will be able to include effects of other factors on retail gasoline prices, such as a gasoline station's brand, distance to competitors, station characteristics, regional income level, etc. Moreover, we will use an autoregressive integrated moving average

(ARIMA) model [12] to better understand the lag terms  $t - i$  (the number of days before  $t$  that are considered) and their coefficients.

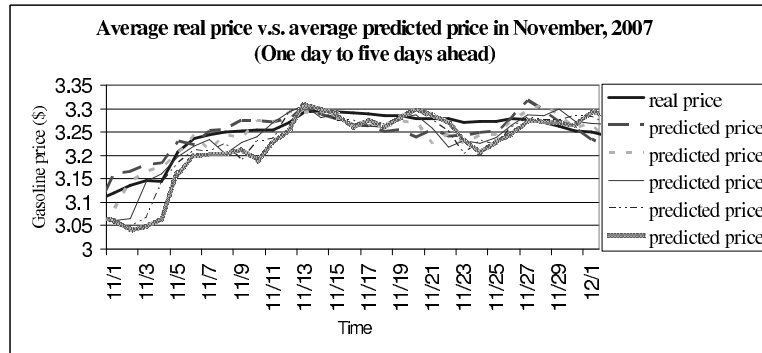


Fig. 4. Average retail price and 5 average predicted prices of 931 stations in November, 2007.

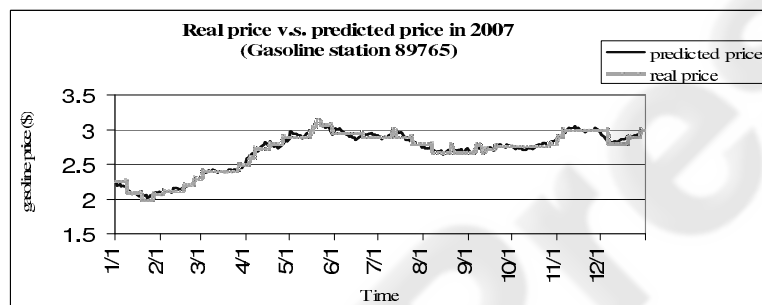


Fig. 5. Real retail price and predicted retail price at gasoline station 89765 in 2007

## 5 Solution Techniques

Once we have the forecasted gas prices, we are ready to use these as cost inputs to the objective function in our optimization model. There are a number of different approaches for solving the optimal refueling strategy problem from heuristical approaches to discrete optimization methods. In this paper, we first describe how to model the refueling strategy optimization problem as a Mixed Integer Program (MIP) [21]. Next, we briefly explore two simple heuristics: the first has the driver stop at the cheapest gas station in the area whenever the driver runs out of gas, and the second recommends a refueling stop within a certain tank capacity range.

### 5.1 Mixed Integer Program

We first describe the MIP optimization technique, introducing the input information, discussing the variables, the objective function, and ending with a detailed description of the constraints and overall model formulation.

**Input Information.** In this section, we describe the parameters that are inputs to the MIP model. We divide these inputs into the following categories: inputs obtained from vehicle internal network, inputs implicit from route specifications, inputs defined by user to specify preferences, and inputs from the forecasting model. We define a route to be a multi-day trip plan of around one week. After a week, the accuracy of the forecasted gas prices diminishes. However, the week can be a rolling horizon, where the model is solved every day for a new week time period.

We get the following information from the vehicle internal network:

- $Max$  = maximum number of gallons the gas tank holds
- $MPG$  = miles per gallon (note that this could be different depending on the route)
- $G_0$  = initial amount of gas in the vehicle at the beginning of the route

As discussed in Section 3, driver routes are continually being collected on the vehicle internal network to populate the user's most frequently driven routes or can be set up on the web portal by the driver for routes not previously driven.

Knowing these routes, we can determine the following inputs:

- $n$  = number of gas stations in route
- $S$  = set of gas stations in route =  $\{1, 2, \dots, n\}$
- $m$  = number of driving days or time periods in the route
- $D$  = set of days =  $\{1, 2, \dots, m\}$
- $d_i$  = the distance from the starting point of the route to gas station  $i$
- $\xi_i$  = the distance from each station  $i$  back to the original route
- $NPI_t$  = the new period index  $\forall t \in D$ . The gas stations are sorted in the order in which the driver will encounter them along the route.  $NPI_t$  identifies the first gas station encountered on day  $t$ . For example,  $NPI_1 = 1$  is the index for the first gas station in the first time period. If on the second day ( $t = 2$ ) the first gas station to visit is station 256, then  $NPI_2 = 256$ .

In addition to the routes, the user specifies the following preferences through the web portal:

- $Min$  = minimum number of gallons of gas the driver wants to have in the fuel tank at any given time
- $MST$  = maximum number of stops over the entire route (note that this should take into consideration the number of miles driven so that the problem does not become infeasible)
- $MSD$  = maximum number of stops in one day of the trip (note that this should take into consideration the number of miles driven so that the problem does not become infeasible)

The last input involves the cost of gas at each station. If the gas station occurs along the route on the current day, then the cost of gas is the current gas price. If the gas station occurs along the route on a future day or the current day's price is not available, then it comes from the forecasting model.

- $c_i$  = cost of gas at station  $i$ .

We keep track of the first station in each time period, so we get the corresponding forecasted gas prices  $\bar{p}_{t+i-1}$  for days  $i = 2, \dots, m$  for each station.



**Variables.** This model contains both binary and continuous variables. The binary variables help make choices of when to stop, whereas the continuous variables determine how much gas to get at a station.

- The first variable,  $x_i$ , is a binary variable that determines whether or not the driver should stop at gas station  $i$ . In other words,

$$x_i = \begin{cases} 1 & \text{if stop at gas station } i \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in \mathcal{S}.$$

Recall that the time period is embedded in this variable as we keep track of it by the value  $NPI_t$ .

- The second variable,  $y_i$ , is the amount (in gallons) of gas purchased from station  $i$  for every  $i \in \mathcal{S}$ .

**Objective Function.** The objective is to minimize the total amount spent on gas when traveling on a specific route over a certain number of days. The user preferences of how many times they are willing to stop also comes into play by adding a weighted penalty to the number of stops. If the user does not set a preference for the number of stops, then we let  $\alpha = 0$ ; that is, we don't impose a penalty. The objective function is as follows

$$\min \sum_i (c_i y_i + \alpha x_i) \quad (5)$$

**Constraints.** In this section, we will discuss the constraints that are involved in guaranteeing that the driver does not run out of gas, that the driver does not get more gas than the fuel tank can hold, and if the driver stops at a station than the driver must get gas.

The first constraint specifies that the vehicle must never run out of gas. So, when the vehicle gets to station  $i$ , the amount of gas that the vehicle had at station  $i - 1$  needs to be enough to get to station  $i$  and still have the minimum gallons required. Note that we add the distance  $\xi_j$  to account for the distance that the driver takes if they stop at some station  $j < i$  to get back to their route.

$$\text{Min} \leq \mathcal{G}_0 - \frac{d_i}{\text{MPG}} + \sum_{j < i} y_j - \sum_{j < i} \frac{\xi_j}{\text{MPG}} x_j \quad \forall i \in \mathcal{S}. \quad (6)$$

The second constraint guarantees that the vehicle will never have more gas than the amount held by the fuel tank. So, for every station visited,

$$\mathcal{G}_0 - \frac{d_i}{\text{MPG}} + \sum_{j \leq i} y_j - \sum_{j < i} \frac{\xi_j}{\text{MPG}} x_j \leq \text{Max} \quad \forall i \in \mathcal{S}. \quad (7)$$

We also constrain the number of stops per route; this accounts for how much gas the vehicle's tank can hold so that the problem does not become infeasible.

$$\sum_{i \in \mathcal{S}} x_i \leq \text{MST} \quad (8)$$

Or, if we only want a maximum number of stops per day, we address it by the following two constraints. The first constraint is for all time periods except for the last one, and the second constraint covers the last time period:

$$\sum_{i \in \mathcal{S}: i \geq NPI_{t-1} \& i < NPI_t} x_i \leq MSD \quad \forall t \in \mathcal{D} \setminus 1, \quad \sum_{i \in \mathcal{S}: i \geq NPI_m \& i \leq n} x_i \leq MSD. \quad (9)$$

The last two constraints are linking constraints that guarantee if the driver does not stop at station  $i$  to get gas then no gallons should be purchased; that is,

$$y_i \leq (Max - Min)x_i \quad \forall i \in \mathcal{S} \text{ and } x_i \leq y_i \quad \forall i \in \mathcal{S}. \quad (10)$$

**Bounds.** The variable  $x_i$  is a binary variable. The variable  $y_i$  is a real number that must be greater than or equal to zero, but less than or equal to the size of the fuel tank minus the preference of how much fuel should always be left in the tank. Note that the upper bound on the  $y_i$  variable is implied by the first linking constraint in (10). The bounds are as follows:  $x_i \in \mathcal{B} \quad \forall i \in \mathcal{S}$  and  $0 \leq y_i \leq Max - Min$  such that  $y_i \in \mathcal{R} \quad \forall i \in \mathcal{S}$ .

### Problem Formulation.

$$\begin{aligned} & \min_{x_i, y_i \quad \forall i \in \mathcal{S}} \sum_i (c_i y_i + \alpha x_i) \\ \text{s.t.} \quad & Min \leq \mathcal{G}_0 - \frac{d_i}{MPG} + \sum_{j < i} y_j - \sum_{j < i} \frac{\xi_j}{MPG} x_j \quad \forall i \in \mathcal{S} \\ & \mathcal{G}_0 - \frac{d_i}{MPG} + \sum_{j \leq i} y_j - \sum_{j < i} \frac{\xi_j}{MPG} x_j \leq Max \quad \forall i \in \mathcal{S} \\ & \sum_{i \in \mathcal{S}} x_i \leq MST \\ & \sum_{i \in \mathcal{S}: i \geq NPI_{t-1} \& i < NPI_t} x_i \leq MSD \quad \forall t \in \mathcal{D} \setminus 1 \\ & \sum_{i \in \mathcal{S}: i \geq NPI_m \& i \leq n} x_i \leq MSD \\ & y_i \leq (Max - Min)x_i \quad \forall i \in \mathcal{S} \text{ and } x_i \leq y_i \quad \forall i \in \mathcal{S} \\ & x_i \in \mathcal{B} \quad \forall i \in \mathcal{S} \text{ and } 0 \leq y_i \leq Max - Min \end{aligned}$$

## 5.2 Heuristics

Two different heuristic approaches are considered as possible solution techniques. The first approach has the driver stop for gas whenever the fuel gage hits a minimum allowable fuel level. For example, if this minimum level is 2 gallons then the driver will stop at the gas station right before the fuel level drops to 2 gallons. When stopping for gas the driver always refuels to the maximum tank capacity. This method may not give the lowest total fuel cost over a trip, however it will result in the minimum number of stops possible to complete the trip. The second method is a Greedy heuristic where the driver continues along their given route until the fuel gage registers a predetermined amount. As an example, let this predetermined amount be half a tank of gas. At this point, the heuristic checks all of the given gas stations between half a tank and the minimum allowable fuel level. The station with the lowest gas price in this range is selected as the

refueling point, and when the driver stops, they fill their tank to the maximum capacity. The driver then continues along the route until the fuel level reaches half a tank, at which point the process repeats itself. This method should show some improvement in fuel costs compared with the previous heuristic and will eliminate the possibility of purchasing small amounts of gas at any given stop.

## 6 Example Scenario

A sample scenario was created to demonstrate the various solution techniques and to compare resulting MIP solutions when user preferences are considered. In this scenario, a five day trip has been planned in which there are 1532 possible gas stations to stop at along the route. For each gas station, the forecasted price of gas and the distance from the starting point is known. The driver's vehicle averages 22 miles per gallon, holds a maximum 15 gallons of gas, and is starting the trip with 5 gallons of fuel. The driver preferences are not to let the fuel level drop below 2 gallons and no stopping for gas more than twice in one day or more than six times over the entire trip.

**Table 2.** Scenario Results for Optimization Methods.

Optimization					Optimization with Penalty				
Day	ID	Distance	Price (\$)	Buy	Day	ID	Distance	Price (\$)	Buy
1	28	15.571	2.799	10.708	1	41	26.685	2.799	11.213
1	171	115.498	2.799	4.542	2	367	284.007	2.837	10.544
3	484	373.707	2.821	6.507	4	853	544.649	2.719	8.098
4	853	544.649	2.719	8.098	5	1240	722.803	2.698	7.539
5	1240	722.803	2.698	7.539	N/A				
Total Trip Fuel Cost: \$ 103.398					Total Trip Fuel Cost: \$ 103.655				

Table 2 displays the following results for the scenario: a listing of on which days to stop, at which gas stations to stop, the price of gas at each station, and how much gas should be purchased at each station. The first column (Optimization) provides the solution when the objective function is to minimize to cost of fuel over the trip and  $\alpha$  is set to zero such that there is no penalty for the number of stops. The second column (Optimization with Penalty) provides the results for the optimization model that includes a penalty in the objective function each time the driver has to stop for gas; this enforces user preferences on how many times a day and per trip they are willing to stop (see equation 5). Here we set  $\alpha = 1$ , but the value of  $\alpha$  could vary depending on the number of gas stations considered in the route. If we compare the results for the optimization models with different objective functions, we see the difference caused by the penalty in the objective function: it eliminates an additional stop on the first day for a slight increase in overall cost of around \$.25.

Table 3 shows the results for both heuristic methods when applied to this scenario. The first column gives the solution when the driver stops for gas right before their fuel gage registers the minimum gas allowed (2 gallons in this scenario). The second column gives the results of the greedy heuristic, which looks for the lowest priced gas between when the fuel gage registers half a tank and 2 gallons.

Table 4 compares the results of the four approaches. The heuristic, which has the driver stop at the closest gas station when they reach 2 gallons of gas in their tank, results in the fewest stops and eliminates only partially filling up the gas tank when stopping. However, this technique does result in the highest total fuel cost which is 6.44% higher than the original optimization technique.

**Table 3.** Scenario Results for Heuristic Methods.

Stop At 2 Gallons					Greedy Heuristic				
Day	ID	Distance	Price (\$)	Buy	Day	ID	Distance	Price (\$)	Buy
1	101	65.892	2.999	12.995	1	41	26.685	2.799	11.213
2	432	350.930	2.946	12.956	2	381	290.172	2.837	11.977
4	1061	636.879	2.877	11.442	4	924	563.492	2.719	12.424
		N/A			5	1446	829.094	2.698	1.78
Total Trip Fuel Cost: \$ 110.06					Total Trip Fuel Cost: \$ 103.945				

**Table 4.** Technique Comparison.

	Optimization	Optimization with Penalty	Stop at 2 Gallons	Greedy
# Stops	5	4	3	4
Stops Fill < 10 Gallons	4	2	0	1
Total Cost (\$)	103.398	103.655	110.06	103.945
% Over Lowest Cost	N/A	0.25%	6.44%	0.53%

## 7 Conclusions

With fuel prices on the rise in the United States, drivers are increasingly concerned with their refueling strategy. This paper presented a model to assist drivers with decisions regarding where and when to refuel along with how much fuel to purchase. The model takes into account various factors, such as user preferences (e.g. how often the driver is willing to stop for gas) and vehicle specific information (e.g. the current fuel level). Gas stations along a specified route can be identified and gas prices corresponding to future days of the route can be determined using the forecasting method discussed in Section 4. Once this information has been aggregated, it is used by the Mixed Integer Programming model presented in Section 5.1 to provide the driver with an optimal refueling strategy. As shown in section 5.2 heuristical approaches can also be used to determine a refueling strategy.

Future versions of this model can be extended to include additional driver preferences or economic factors. For example, by considering flex-fuel vehicles, the refueling decision becomes even more complex as the analysis would include tradeoffs between the price per gallon of fuel, average miles per gallon associated with a particular route and vehicle, and possibly the CO2 effects for multiple fuel types. In addition, we could move towards a more advanced forecasting model and provide a refueling strategy that would take into consideration multiple routes for a single origin destination pair.

The proposed model offers substantial benefits over existing approaches. First, it provides an optimal refueling strategy using forecasted fuel prices over multiple time

periods and locations as opposed to a local strategy based on a small geographical region and the current fuel prices in that area. In addition, the vehicle internal network can be used to monitor the current fuel level and consumption to offer proactive advice instead of waiting for the driver to request help from the system. Finally, over time the system can learn a driver's regular routes and associated fuel consumption to provide more accurate recommendations in the proposed refueling strategy.

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