# SUBJECTIVE PREFERENCES IN FINANCIAL PRODUCTS

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Keywords: Fuzzy Sets, Finance, Subjective Preferences.

Abstract: When the decision maker invests in the banking organizations, he is faced with the need to choose between apparently different products. The financial advisers have to offer an agile and well-qualified service to be able to continue counting on the confidence of their customers and to increase their results consequently. In this paper, presents a further step in justifying such evalutation. Results from the proposed approach present a better undestanding of each system to decision makers for evaluating justification issues which sometimes cannot be defined.

## 1 INTRODUCTION

With increasing frequency it can be seen that new products appear on the market under many different forms that, either real or apparent, have different characteristics. It should not be forgotten that the strong competition characterising the financial world obliges those offering payment means to a great effort of diversification and differentiation of products that permits them, on the one hand, to cover the widest range of possible users and, on the other, provoke a flaw by means of the presentation of different products with the object of get around the laws of the perfect market.

Evidently that for each business, and even for each specific situation, there will be a different valuation of each one of the characteristics of the financial products (Zadeh, 1971).

In this context two fundamental elements appear that make up the problem:

1) Differentiation in the characteristics of each one of the financial products on offer.

2) Different estimate, by the acquirer, of each of the characteristics relative to the rest, which provides an order of preference.

Evidently, the degree of preference for each one of the characteristics relative to the others may sometimes be determined by means of measurements, that is, with an objective nature, but on other occasions it will be necessary to resort to subjective numerical situations, that is by means of valuations.

With all this an attempt is made to arrive at certain results that express the order of preference between different financial products to which a business may opt. The subjective nature of the estimated values should lead to certain conclusions that can be expressed by means of fuzzy sets (Bustince & Herrera, 2008).

## 2 PROBLEM FORMULATION

We start out from the existence of a finite and re-countable number of financial products $P_1, P_2, ..., P_n$, which each posses certain determined characteristics $C_1, C_2, ..., C_m$ in such a way that for each characteristic it is possible to establish a quantified (objective or subjective) relation of preferences.

Therefore for $C_j$ we have that: $P_1$ is preferred $\mu_1/\mu_2$ times over $P_2$, $\mu_1/\mu_3$ times over $P_3$, …, $\mu_1/\mu_n$ times over $P_n$, …, $P_n$ is preferred $\mu_n/\mu_1$ times over $P_1$, $\mu_n/\mu_2$ times over $P_2$, …, $\mu_n/\mu_{n-1}$ times over $P_{n-1}$.

With this previous relation of preferences we will be able to construct the following matrix, which will be reflexive and reciprocal by construction:



Vizuete Luciano E. and Mª Gil Lafuente A. (2008).
SUBJECTIVE PREFERENCES IN FINANCIAL PRODUCTS.
In *Proceedings of the Tenth International Conference on Enterprise Information Systems - AIDSS*, pages 407-410
DOI: 10.5220/0001676904070410

$$[C_{ij}] = \begin{bmatrix} 1 & \frac{\mu_1}{\mu_2} & \frac{\mu_1}{\mu_3} & \dots & \frac{\mu_1}{\mu_n} \\ \frac{\mu_2}{\mu_1} & 1 & \frac{\mu_2}{\mu_3} & \dots & \frac{\mu_2}{\mu_n} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{\mu_n}{\mu_1} & \frac{\mu_n}{\mu_2} & \frac{\mu_n}{\mu_3} & \dots & 1 \end{bmatrix} \qquad (1)$$

This matrix is also coherent or consistent, (Dubois & Prade, 1995) since the following is complied with:

$$\forall i, j, k \in \{1, 2, ..., n\},\ \frac{\mu_i}{\mu_j} \cdot \frac{\mu_j}{\mu_k} = \frac{\mu_i}{\mu_k} \qquad (2)$$

For this reason we are going to consider certain properties (Vasantha, 2007), those in which all the elements that are members of $R_0^+$ :

a) A positive square matrix posses a dominant value of its own 1 real positive which is unique for which what is complied is that $\lambda \geq n$, where n is the order of the square matrix.

b) The vector that corresponds to the dominant own value is found also formed by positive terms and when normalised, is unique.

When $\lambda$ is a number close to $n$ it is said that the matrix is nearly coherent; on the contrary it will be necessary to make an adjustment between the elements of the matrix (Gil Aluja, 1998 & 1999), if wanting to use this scheme correctly. It is considered that $\lambda - n$ or $\frac{\lambda - n}{n}$ is an index of coherence. As is very well known, when a reciprocal matrix is also coherent it complies with $[C_{ij}] \cdot [v_i]^T = n \cdot [v_i]^T$ where $[v_i]^T$ is the transpose of row $i$.

When the reciprocal matrix is not coherent, we write: $[C_{ij}] \cdot [v'_i]^T = \lambda \cdot [v'_i]^T$. We accept $[v'_i]$ as the result when the index of coherence $\frac{\lambda - n}{n}$ is sufficiently small.

For each characteristic $C_j$, $j = 1, 2, ..., m$ the corresponding reflexive and reciprocal matrix $[C_{ij}]$ is obtained. Once the m matrices are constructed the dominant own values $\lambda_j$ and their corresponding vectors $\lfloor X_{ij} \ \dots \ X_{nj} \rfloor$ must be found for each one, verifying if they posses sufficient consistency by means of the «index of coherence».

The elements of each corresponding own vector will give rise to a fuzzy sub-set:

$$\begin{array}{cc} & \begin{array}{ccccc} P_1 & P_2 & P_3 & \dots & P_n \end{array} \\ X_j = & \begin{array}{|c|c|c|c|c|} \hline x_{1j} & x_{2j} & x_{3j} & \dots & x_{4j} \\ \hline \end{array} \end{array}$$

which once normalised in sum equal to one will be:

$$\begin{array}{cc} & \begin{array}{ccccc} P_1 & P_2 & P_3 & \dots & P_n \end{array} \\ D_j = & \begin{array}{|c|c|c|c|c|} \hline p_{1j} & p_{2j} & p_{3j} & \dots & p_{4j} \\ \hline \end{array} \end{array}$$

The *m* own vectors are regrouped forming a Matrix 1, the form of which will be:

**Matrix 1**

[Pij]=

| | C1 | C2 | C3 | C4 | | Cm |
|---|---|---|---|---|---|---|
| P1 | P11 | P12 | P13 | P14 | … | P1m |
| P2 | P21 | P22 | P23 | P24 | … | P2m |
| | … | … | … | … | | … |
| Pn | Pn1 | Pn2 | Pn3 | Pn4 | … | Pnm |

Each column of this matrix brings to light the relative degree in which a characteristic is possessed by all the financial products. As we have already pointed out, this can be represented by a normalised fuzzy sub-set $D_j$. From this perspective there exist m fuzzy sub-sets (Kao & Liu, 2001). On the other hand each row expressed, for one product, the degree in which it possess each one of the characteristics, which is also represented by a fuzzy sub-set $Q_i$ such as:

$$\begin{array}{cc} & \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & \dots & C_m \end{array} \\ Q_i = & \begin{array}{|c|c|c|c|c|c|} \hline p_{i1} & p_{i2} & p_{i3} & p_{i4} & \dots & p_{im} \\ \hline \end{array} \end{array}$$

On the other hand, each business has a different appreciation of the importance that each characteristic has (Gil Lafuente, 2005). Evidently, this estimate can vary from one moment to another and its quantification has a basically subjective sense, therefore will be expressed by means of valuations.

The establishment of these valuations can be done by means of a comparison between the relative importance of a characteristic in relation to the rest. Therefore, for example, it can be said that a characteristic is two times as important as another, or has half the importance of a third.

In this way we can construct a Matrix 2, that obviously will be square, reflexive and anti-symmetrical. Since there are n products, its order will be $m \times m$:

**Matrix 2**

| | | C1 | C2 | C3 | C4 | | Cm |
|---|---|---|---|---|---|---|---|
| [Pij]= | C1 | 1 | a12 | a13 | a14 | … | a1m |
| | C2 | a21 | 1 | a23 | a24 | … | a2m |
| | | … | … | … | … | | … |
| | Cm | am1 | am2 | am3 | am4 | … | 1 |

Due to the condition of asymmetry the following will be complied with:

$$a_{ij} = \frac{1}{a_{ij}} \qquad (3)$$

Once the matrix 2 has been determined, we proceed to obtain the corresponding dominant value and vector. This vector will bring to light the preferences of the business relative to the characteristics, $y_j = \lfloor y_1 \; y_2 \; y_3 \; y_4 \; y_5 \rfloor$

In order for this vector to be susceptible to being used as a weighting element, we are going to convert it into another that possesses the property that the sum of its elements be equal to the unit. For this we do:

$$b_j = \frac{y_j}{\sum_{j=1}^{m} y_j}, \quad j = 1,\ 2,\ ...,\ m \qquad (4)$$

With which we arrive at $b_j = \lfloor b_1 \; b_2 \; b_3 \; b_4 \; b_5 \rfloor$

We are now in a position finally to arrive at the sought after result, by taking matrix $[p_{ij}]$ and multiplying it to the right by vector $[b_j]$. The result will be another vector, which will express the relative importance of each financial product for the business, taking into account its preferences for each one of the characteristics:

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1m} \\ p_{21} & p_{22} & p_{23} & \cdots & p_{2m} \\ p_{31} & p_{32} & p_{33} & \cdots & p_{3m} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{n1} & p_{n2} & p_{n3} & \cdots & p_{nm} \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \cdots \\ b_m \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \cdots \\ d_m \end{bmatrix} \qquad (5)$$

The result can also be expressed by means of a normal fuzzy sub-set, by doing:

| | | $P_1$ | $P_2$ | $P_3$ | $P_4$ | … | $P_m$ |
|---|---|---|---|---|---|---|---|
| H | = | $h_1$ | $h_2$ | $h_3$ | $h_4$ | … | $h_m$ |

This model on the contrary to all those that use as the only basis for selection, the price of the money, has as its greatest advantage the possibility of incorporating a wide range of elements that, in the reality of businesses, at times play a decisive role at the time of taking the decision to select a financial product from among those offered on the market. These elements normally do not have the same weight at the time of making a valuation.

# 3 APPLICATION OF THE PROPOSED MODEL

With the object of illustrating the model a case has been considered which we have linked to the one shown, in order to cover certain financial requirements, resorts to three credit institutions which propose as the most adequate, one financial product each (Vizuete & Gil Lafuente, 2007). Therefore there is a choice between three products $P_1,\ P_2,\ P_3$.

The characteristics of these products makes them different, but in certain aspects some are more attractive, but in others these are less favourable. Obviously, in the eyes of the businessman not all the characteristics have the same weight at the time of deciding to accept one or another (Kaufmann & Gil Aluja, 1987 & 1990). The five characteristics mentioned previously were considered as important: price of the money, payback period, possibilities for renewal, fractioning repayments, speed of granting.

**1.** With regard to the price of the money the following data is considered: for $P_1$ 20%, for $P_2$ 22% and for $P_3$ 18%. This then is objective data and it is logical to think that the preference would be for the lowest price in a proportional manner. In this way the following matrix can be constructed:

| | $P_1$ | $P_2$ | $P_3$ |
|---|---|---|---|
| $P_1$ | 1 | 11/10 | 9/10 |
| $P_2$ | 10/11 | 1 | 9/11 |
| $P_3$ | 10/9 | 11/9 | 1 |

Once this matrix has been constructed the corresponding dominant own value and vector must be obtained. Among the various procedures existing we are going to use the following:

$$\begin{bmatrix} 1 & 1{,}1 & 0{,}9 \\ 0{,}9090 & 1 & 0{,}8181 \\ 1{,}1111 & 1{,}2222 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2{,}7271 \\ 3{,}3333 \end{bmatrix} = 3{,}3333 \cdot \begin{bmatrix} 0{,}9 \\ 0{,}8181 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1{,}1 & 0{,}9 \\ 0{,}9090 & 1 & 0{,}8181 \\ 1{,}1111 & 1{,}2222 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0{,}9 \\ 0{,}8181 \\ 1 \end{bmatrix} = \begin{bmatrix} 2{,}6999 \\ 2{,}4543 \\ 2{,}9998 \end{bmatrix} = 2{,}9998 \cdot \begin{bmatrix} 0{,}9 \\ 0{,}8181 \\ 1 \end{bmatrix}$$

For normalisation of the sum equal to 1, in this way arriving at:

[Pi1]=

| P1 | P2 | P3 |
|---|---|---|
| 0,3311 | 0,3009 | 0,3679 |

The same process should be developed for Pi2, ..., Pi5. Once we have obtained these five vectors $[p_{ij}]$, $j = 1, 2, 3, 4, 5$, we group them and form the following matrix:

$[p_{ij}]=$

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| $P_1$ | 0,3311 | 0,3333 | 0,1681 | 0,4285 | 0,6483 |
| $P_2$ | 0,3009 | 0,4000 | 0,3572 | 0,1428 | 0,2296 |
| $P_3$ | 0,3679 | 0,2666 | 0,4746 | 0,4285 | 0,1219 |

With the following square, reflexive and reciprocal matrix can be arrived at matrix 3:

**Matrix 3**

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| $C_1$ | 1 | 2 | 6 | 8 | 4 |
| $C_2$ | 1/2 | 1 | 4 | 6 | 2 |
| $C_3$ | 1/6 | 1/4 | 1 | 3 | 1/2 |
| $C_4$ | 1/8 | 1/6 | 1/3 | 1 | 1/3 |
| $C_5$ | 1/4 | 1/2 | 2 | 3 | 1 |

In order to obtain the corresponding dominant own value and vector the same process can be used as followed before. In this way with the normalisation in sum equal to one:

$$\left[b_j\right] = \left[0,4704 \quad 0,2685 \quad 0,836 \quad 0,0430 \quad 0,1342\right]$$

Finally, if we take matrix $[p_{ij}]$ and multiply to the right by vector $[b_j]$, which in short constitutes a weighting, we arrive at:

$$[d_j] = \begin{bmatrix} 0,3311 & 0,3333 & 0,1681 & 0,4285 & 0,6483 \\ 0,3009 & 0,4000 & 0,3572 & 0,1428 & 0,2296 \\ 0,3679 & 0,2666 & 0,4746 & 0,4285 & 0,1219 \end{bmatrix} \times \begin{bmatrix} 0,4704 \\ 0,2685 \\ 0,0836 \\ 0,0430 \\ 0,1342 \end{bmatrix} = \begin{bmatrix} 0,3647 \\ 0,3157 \\ 0,3191 \end{bmatrix}$$

Taking into account that we have only considered four decimal points and the last one has not been rounded up, the sum of the elements of the last matrix does not give the unit as the result, which would have occurred if the rounding up were to have been done.

The result we have arrived at can also be expressed by means of a normal fuzzy sub-set, as follows:

P =

| $P_1$ | $P_2$ | $P_3$ |
|---|---|---|
| 1,0000 | 0,8656 | 0,8749 |

It will be seen in this fuzzy sub-set that financial product $P_1$ is preferable to products $P_2$ and $P_3$, although not too much. There is very little difference between $P_2$ and $P_3$.

# 4 CONCLUSIONS

In this paper, we have studied an example could be taken as typical since it shows what happens often in financial reality, when the decison maker is faced with the need to choose between apparently different products but which, when all is said and done, are very similar. This situation should not come as a surprise to us if it is thought that financial institutions attempt to compensate certain disadvantages of a product relative to other of the competition, by means of incentives to certain aspects that make it more attractive and allow in this way for its placing in the market under conditions of competitiveness.

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