

New Attack Strategy for the Shrinking Generator

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Abstract. This work shows that the cryptanalysis of the shrinking generator requires fewer intercepted bits than what indicated by the linear complexity. Indeed, whereas the linear complexity of shrunken sequences is between $A \cdot 2^{(S-2)}$ and $A \cdot 2^{(S-1)}$, we claim that the initial states of both component registers are easily computed with fewer than $A \cdot S$ shrunken bits. Such a result is proven thanks to the definition of shrunken sequences as interleaved sequences. Consequently, it is conjectured that this statement can be extended to all interleaved sequences. Furthermore, this paper confirms that certain bits of the interleaved sequences have a greater strategic importance than others, which must be considered as a proof of weakness of interleaved generators.

1 Introduction

Stream ciphers are considered nowadays the fastest encryption procedures. Consequently, they are implemented in many practical applications e.g. the algorithms A5 in GSM communications [10], the encryption system E0 in Bluetooth specifications [2] or the algorithm RC4 [15] used in Microsoft Word and Excel.

From a short secret key (known only by the two interested parties) and a public algorithm (the sequence generator), a stream cipher procedure is based on the generation of a long sequence of seemingly random bits. Such a sequence is called the keystream sequence.

For the encryption the sender realizes the bit-wise (Exclusive-OR) XOR operation among the bits of the original message or plaintext and the keystream sequence. The result is the ciphertext to be sent. For the decryption, the receiver generates the same keystream, realizes the same bit-wise XOR operation between the received ciphertext and the keystream sequence and obtains again the original message.

Most keystream generators are based on Linear Feedback Shift Registers (LFSRs) [8], which are linear structures characterized by their length (the number of memory cells), their characteristic polynomial (the feedback function) and their initial states (the seed or key of the cryptosystem). If the characteristic polynomial is a primitive polynomial [14], then the LFSRs generate Pseudo- Noise sequences (PN-sequences) with good characteristics of pseudorandomness.

For a survey on primitive LFSRs, PN-sequences, and shift equivalences the interested reader is referred to [8]. In stream cipher procedures, the PN- sequences are

combined by means of nonlinear functions in order to produce keystream sequences of cryptographic application. Combinational generators, nonlinear filters, clock-controlled generators, irregularly decimated generators ... are just some of the most popular nonlinear sequence generators. All of them produce keystreams with high linear complexity, long period and good statistical properties (see [6] and [3]).

Most cryptanalysis on stream ciphers are performed under a known plaintext hypothesis, that is to say, it is assumed that the attacker has direct access to a portion of the keystream sequence (the intercepted sequence). From the intercepted bits, the attacker has to deduce the cryptosystem key. Once the key is known, as the sequence generator is public, the whole keystream sequence can be reconstructed. The complexity of this attack is always compared with that of the key exhaustive search. If the former complexity is lesser, then the cryptosystem is said to be broken.

This work focuses on a particular kind of stream ciphers based on LFSRs: the class of shrinking generators. They are made out of two LFSRs and an irregular decimation. Shrinking generators have been thoroughly analyzed in several papers such as [17], [13] and [4]. Nevertheless, we present a new and efficient cryptanalytic attack requiring much lesser amount of intercepted bits than that of the previous attacks. The basic idea of this cryptanalysis consists in defining the output sequence of a shrinking generator as an interleaved sequence (see [9] and [12]). The characteristics of interleaved sequences reveals weaknesses that lead to practical attacks. In addition, we conjecture that these weaknesses can be extended to all interleaved sequence generators with application in cryptography.

The paper is organized as follows: in section 2, the description and characteristics of the shrinking generator is introduced. Interleaved configuration and related results are developed in section 3. A cryptanalytic attack against the shrinking generator that exploits the condition of interleaved sequence is presented in section 4, while the generalization of this technique to other cryptographic interleaved generators appears in section 5. Finally, conclusions in section 6 end the paper.

2 The Shrinking Generator

The so-called Shrinking Generator (SG) is a nonlinear keystream generator composed by two LFSRs [5] so that a control register SRS decimates the sequence produced by the other register SRA . S and A denote respectively their corresponding lengths and fulfil that $(S, A) = 1$ and $S < A$. $P_S(x)$ and $P_A(x) \in GF(2)[x]$ denote their corresponding primitive characteristic polynomials.

The sequence $\{s_i\}$ produced by SRS controls the bits of the sequence $\{a_i\}$ produced by SRA which are included in the output shrunken sequence $\{z_j\}$ according to the following rule: If $s_i = 1$ then $z_j = a_i$, and if $s_i = 0$, then a_i is discarded.

As different pairs of SRA/SRS initial states can generate the same shrunken sequence, in the sequel we assume that the first term of the sequence $\{s_i\}$ equals 1, that is $s_0 = 1$. According to [5], the period of the shrunken sequence is:

$$T = (2^A - 1)2^{(S-1)}, \quad (1)$$

its linear complexity, notated LC , satisfies the following inequality:

$$A \cdot 2^{(S-2)} < LC \leq A \cdot 2^{(S-1)}, \quad (2)$$

and its characteristic polynomial is of the form:

$$P_{ss}(x) = (P(x))^p \quad (3)$$

where $P(x)$ is an A -degree primitive polynomial in $GF(2)[x]$ and p is an integer in the interval $2^{(S-2)} < p \leq 2^{(S-1)}$. Moreover, it can be proven [16] that the shrunken sequence has also good distributional statistics. Therefore, this scheme has been traditionally used as keystream sequence generator with application in secret-key cryptography.

3 Interleaved Configuration

The $(2^A - 1) \cdot 2^{(S-1)}$ bits of a period of any shrunken sequence $\{z_j\}$ can be arranged into a $(2^A - 1) \times 2^{(S-1)}$ matrix that we will call *interleaved configuration* and will denote by IC . In fact,

$$IC = \begin{pmatrix} z_0 & z_1 & \cdots & z_{2^{S-1}-1} \\ z_{2^{S-1}} & z_{2^{S-1}+1} & \cdots & z_{2 \cdot 2^{S-1}-1} \\ z_{2 \cdot 2^{S-1}} & z_{2 \cdot 2^{S-1}+1} & \cdots & z_{3 \cdot 2^{S-1}-1} \\ z_{3 \cdot 2^{S-1}} & z_{3 \cdot 2^{S-1}+1} & \cdots & z_{4 \cdot 2^{S-1}-1} \\ \cdots & \cdots & \cdots & \cdots \\ z_{(2^A-2) \cdot 2^{S-1}} & z_{(2^A-2) \cdot 2^{S-1}+1} & \cdots & z_{(2^A-1) \cdot 2^{S-1}-1} \end{pmatrix}$$

Now the following result allows one to identify each element of the matrix IC with the corresponding term of the sequence $\{a_i\}$.

Theorem 3.1. The interleaved configuration matrix IC can be written in terms of the elements of the sequence $\{a_i\}$ such as follows:

$$IC = \begin{pmatrix} a_{o0} & a_{o1} & \cdots \\ a_{2^{S-1}+o0} & a_{2^{S-1}+o1} & \cdots \\ a_{2 \cdot 2^{S-1}+o0} & a_{2 \cdot 2^{S-1}+o1} & \cdots \\ a_{3 \cdot 2^{S-1}+o0} & a_{3 \cdot 2^{S-1}+o1} & \cdots \\ \cdots & \cdots & \cdots \\ a_{(2^A-2) \cdot 2^{S-1}+o0} & a_{(2^A-2) \cdot 2^{S-1}+o1} & \cdots \end{pmatrix}$$

where the additive sub-indices oj ($j = 0, 1, \dots, 2^{S-1} - 1$) depend on the bits of the sequence $\{s_i\}$ in the following way: if $s_i = 1$, then the corresponding sub-index oj equals the sub-index i , $oj = i$. All the sub-indices are taken module $2^A - 1$, that is to say, the period of the sequence $\{a_i\}$.

Proof. Since the period of the PN-sequence $\{s_i\}$ is $(2^S - 1)$, the number of bits with value 1 in a period is exactly 2^{S-1} , and all the elements of any column of IC come from the same term $s_i = 1$ of the PN-sequence, the above expression for the matrix IC in terms of the elements of $\{a_i\}$ is obtained. \square

According to the assumption $s_0 = 1$, the sub-index $o_0 = 0$. Next, the following result analyzes the characteristics of the columns of the matrix IC .

Theorem 3.2. The sequences $\{d_j\} = \{a_{k+oj} : k = 0, (2^S - 1), 2 \cdot (2^S - 1), \dots, (2^A - 2) \cdot (2^S - 1)\} (j = 0, 1, \dots, 2^{((S-1))} - 1)$ corresponding to the columns of the matrix IC are shifted versions of a unique PN-sequence whose characteristic polynomial is given by:

$$P_D(x) = (x + \alpha^N)(x + 2N)(x + \alpha^{2 \cdot N}) \dots (x + \alpha^{2^{(A-1)} \cdot N})$$

where N is an integer defined as $N = 2^0 + 2^1 + \dots + 2^{(S-1)}$ and $\alpha \in GF(2^A)$ a root of the primitive polynomial $P_A(x)$.

Proof. Every sequence $\{d_j\}$ corresponding to the j -th column of IC is a regular decimation of the PN-sequence $\{a_i\}$. More precisely, such a sequence is obtained by taking one out of $(2^S - 1)$ terms in $\{a_i\}$. The primality between A and S guarantees the primality between $(2^A - 1)$ and $(2^S - 1)$. Thus, the decimated sequence $\{d_j\}$ is also a PN-sequence. In addition, as every $\{d_j\}$ has been obtained from $\{a_i\}$ with a decimation ratio of value $(2^S - 1)$, then its characteristic polynomial $P_D(x)$ is the polynomial of the cyclotomic coset $(2^S - 1)$ in the Galois Field $GF(2^A)$ generated by the roots of the polynomial $P_A(x)$, see [4]. The starting point of each $\{d_j\}$ is given by the corresponding sub-index oj . \square

4 Cryptanalytic Attack

The cryptanalytic attack consists in the computation of the initial states of both registers SRA and SRS . In fact, from some known bits of the shrunken sequence we have to determine the first A bits $(a_0, a_1, \dots, a_{A-1})$ of the sequence $\{a_i\}$ as well as the first S bits $(s_0, s_1, \dots, s_{S-1})$ of the sequence $\{s_i\}$. This attack can be divided into two different steps. In the first one, the computation of the initial state of SRA is carried out. In the second step and based on the SRA initial state, we determine the corresponding initial state of the register SRS .

4.1 SRA Initial State

Previously to the computation of the initial state, the following result is introduced.

Lemma 4.1. Given A bits of the shrunken sequence corresponding to A successive elements of any column of IC , the remaining bits of such a column can be determined.

Proof. Theorem 3.2 determines $P_D(x)$ the characteristic polynomial of the PN-sequence corresponding to every column of IC . Thus, knowing A successive bits of any column and its characteristic polynomial, the linear recurrence relationship allows one to compute the remaining bits of such a column. \square

Now the computation of the SRA initial state is described in the next result.

Theorem 4.2. Given A bits of the shrunken sequence corresponding to A successive elements of the first column of IC , the bits of the initial state of the register SRA can be determined.

Proof. Lemma 4.1 shows that the knowledge of A successive elements of the first column of IC allows one to generate the remaining bits of such a column. On the other hand, from Theorem 3.1 we know that the $(n+1)$ -th element of the first column of IC corresponds to $a_{n \cdot (2^S - 1)}$, that is to say, the $(n \cdot (2^S - 1) + 1)$ -th term of the sequence generated by the register SRA . Consequently, we first solve the following system of modular equations in the unknowns n_i

$$n_i \cdot (2^S - 1) \equiv i \pmod{(2^A - 1)} (i = 0, 1, \dots, (A - 1)),$$

and then we compute successively the $(n_i + 1)$ -th ($i = 0, 1, \dots, (A - 1)$) elements of the first column of IC in order to obtain a_0, a_1, \dots, a_{A-1} , respectively. \square

4.2 SRS Initial State

The computation of the SRS initial state is described in the next result.

Theorem 4.3. Given $A \cdot S$ bits of the shrunken sequence corresponding to the top-left corner $(A \times S)$ sub-matrix of IC , the bits of the initial state of the register SRS can be determined.

Proof. Firstly, from the knowledge of the $(A \times S)$ sub-matrix of IC ,

$$SUB_{IC} = \begin{pmatrix} a_0 & a_{o1} & \cdots \\ a_{2^S-1} & a_{(2^S-1)+o1} & \cdots \\ a_{2 \cdot 2^S-1} & a_{2 \cdot (2^S-1)+o1} & \cdots \\ a_{3 \cdot 2^S-1} & a_{3 \cdot (2^S-1)+o1} & \cdots \\ \cdots & \cdots & \cdots \\ a_{(A-1) \cdot 2^S-1} & a_{(A-1) \cdot (2^S-1)+o1} & \cdots \end{pmatrix}$$

and according to Lemma 4.1, we can deduce the remaining bits of those S columns. Secondly, the relative shifts among columns may be computed from the comparison between consecutive columns. Since the sequence in every column of IC is exactly the same but starting at different points given by a_{oj} , as soon as a relative shift is found the sub-index oj may be easily computed. In addition, each sub-index oj indicates the position of the $(j+1)$ -th 1 in the initial state of SRS while the intermediate bits are 0's. Thus, the above procedure can be repeated for $j = 1, 2, \dots$ till we get $oj \geq (S - 1)$. In this way, the initial state of the register SRS is thoroughly determined. \square

4.3 Illustrative Example

Let us consider a shrinking generator characterized by:

(1) SRA with length $A = 5$, characteristic polynomial $P_A(x) = x^5 + x^4 + x^3 + x^2 + 1$ and output sequence $\{a_i\}$.

(2) SRS with length $S = 4$, characteristic polynomial $P_S(x) = x^4 + x^3 + 1$ and output sequence $\{s_i\}$.

(3) The characteristic polynomial of the shrunken sequence is $P_{ss}(x) = P_D(x)^p = (x^5 + x^3 + x^2 + x + 1)^8$.

Given 20 bits of the shrunken sequence corresponding to a (5×4) sub-matrix of IC

$$SUB_{IC} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

we can launch a cryptanalytic attack against the shrinking generator in order to obtain the initial states of both LFSRs. Table 1 shows the calculations carried out for cryptanalyzing the above described generator. The most left column represents the indices n_i numbered $(0, \dots, 2^A - 2 = 30)$. Next column shows from Theorem 3.1 the position of the terms (a_0, a_1, \dots, a_4) of the sequence $\{a_i\}$ in the first column $\{d_o\}$ of the matrix IC . The following columns of the Table 1 represent the matrix IC : in boldface the (5×4) sub-matrix with the known bits, the remaining bits of $\{d_o\}$ are the bits computed to determine the initial states of SRA and SRS , and the symbol - corresponds to unknown bits of the shrunken sequence.

Computation of the SRA Initial State: According to Theorem 4.2, we compute the positions of the $(n_i + 1)$ -th elements of the first column of IC by solving the equation system

$$n_i \cdot 15 \equiv \text{mod } 31 \quad (i = 0, 1, \dots, 4).$$

That is to say, $n_0 = 0, n_1 = 29, n_2 = 27, n_3 = 25, n_4 = 23$. Then, by means of the characteristic polynomial $P_D(x)$ we determine the values of the $(n_i + 1)$ -th $(i = 0, 1, \dots, 4)$ elements of the first column $\{d_o\}$ of IC . Consequently, $a_0 = 1, a_1 = 0, a_2 = 0, a_3 = 1, a_4 = 1$ (see Table 1). Therefore, the initial state of the register SRA $(1, 0, 0, 1, 1)$ has been determined.

Computation of the SRS initial state: According to Theorem 4.3, we compute the relative shifts between consecutive columns in the matrix IC :

- Computation of o_1 : We know a_1 at the $(29+1)$ -th position of the first column $\{d_o\}$ and compute its $S - 1 = 4$ successive bits. We compare these 5 bits $(0, 0, 1, 1, 0)$ with the first 5 bits $(0, 0, 1, 1, 0)$ of the second column $\{d_1\}$ (see Table 1). There is coincidence, thus $o_1 = 1$.

- Computation of o_2 : We know a_2 at the $(27+1)$ -th position of the $\{d_o\}$ and compute its 4 successive bits. We compare these 5 bits $(0, 1, 0, 0, 1)$ with the first 5 bits $(1, 0, 0, 1, 0)$ of the third column $\{d_2\}$. There is no coincidence, thus we analyze the following bit a_3 . We know a_3 at the $(25+1)$ -th position of $\{d_o\}$ and compute its 4 successive bits. We compare these 5 bits $(1, 0, 0, 1, 0)$ with the first 5 bits $(1, 0, 0, 1, 0)$ of $\{d_2\}$ (see Table 1). There is coincidence, thus $o_2 = 3$. Since $o_2 = 3 \geq S - 1$, we have determined the initial state of SRS . In fact, $s_0 = 1, o_1 = 1$ implies $s_1 = 1, o_2 = 3$ implies $s_2 = 0$ and $s_3 = 1$. Therefore, the SRS initial state is $(s_0, s_1, s_2, s_3) = (1, 1, 0, 1)$. Remark that only the knowledge of three columns of the sub-matrix has been necessary to identify the initial state of SRS . Indeed, this number equals the number of bits 1 in the initial state of the selector register. The maximum number of known bits corresponds to SRS initial state with all bits 1. In the remaining cases, less bits are sufficient.

Once the initial states of both register are determined, the whole shrunken sequence that is the keystream sequence can be computed.

Table 1. Matrix IC corresponding to the described SG.

n_i	$\{a_i\}$	d_0	d_1	d_2	d_3	d_4	\cdots	d_7
0	a_0	1	0	1	1	-	-	-
1		1	0	0	1	-	-	-
1		1	0	0	1	-	-	-
2		0	1	0	1	-	-	-
3		0	1	1	1	-	-	-
4		0	0	0	1	-	-	-
5		-	-	-	-	-	-	-
...		-	-	-	-	-	-	-
23	a_4	1	-	-	-	-	-	-
24		-	-	-	-	-	-	-
25	a_3	1	-	-	-	-	-	-
26		0	-	-	-	-	-	-
27	a_2	0	-	-	-	-	-	-
28		1	-	-	-	-	-	-
29	a_1	0	-	-	-	-	-	-
30		0	-	-	-	-	-	-

5 Generalization to Interleaved Sequences

First of all, we introduce the general definition of interleaved sequence [12].

Definition 5.1. Let $f(x)$ be a polynomial over $GF(q)$ of degree r and let m be a positive integer. For any sequence $\{u_k\}$ over $GF(q)$, we write $k = i \cdot m + j$ with $(i = 0, 1, \dots)$ and $(j = 0, \dots, m - 1)$. If every sub-sequence $\{u_j\}$ of $\{u_k\}$ defined as $\{u_{i \cdot m + j}\}$ is generated by $f(x)$, then the sequence $\{u_k\}$ is called an interleaved sequence over $GF(q)$ of size m associated with the polynomial $f(x)$.

Table 2. Interleaved sequence with 4 shifted versions of the same PN-sequence.

u_0	u_1	u_2	u_3
1	1	1	1
1	0	1	0
0	0	1	1
0	1	0	1
1	0	0	1
0	1	1	0
1	1	0	0

Table 2 shows the interleaved sequence $\{u_k\}$ over $GF(2)$ associated with the 3-degree characteristic polynomial $f(x) = x^3 + x + 1$ over $GF(2)$ and size $m = 4$. Reading by rows, the interleaved sequence is $\{u_k\} = \{1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 0\}$ while by columns the sequence is made out of $\{u_j\}$ ($j = 0, \dots, 3$) four shifted versions of the PN-sequence generated by $f(x)$.

Interleaved sequences are currently used as keystream sequences with application in cryptography. They can be generated in different ways:

(1) By a LFSR controlled by another LFSR (which may be the same one) e.g. multiplexed sequences [11], clock-controlled sequences [1], cascaded sequences [7], shrinking generator sequences [5] etc.

(2) By one or more than one LFSR and a feed-forward nonlinear function e.g. Gold-sequence family, Kasami (small and large set) sequence families, GMW sequences, Klapper sequences, No sequences etc. See [9] and the references cited therein.

In brief, a large number of well-known cryptographic sequences are included in the class of interleaved sequences. Next, the link between interleaved sequences and shrunken sequences is expressed in the following result.

Theorem 5.2. Shrunken sequences are interleaved sequences of size $2^{(S-1)}$.

Proof. Let $\{z_k\}$ be a shrunken sequence with characteristic polynomial $P(x)^p$ where $P(x)$ is an A -degree primitive polynomial and p is an integer in the interval $2^{(S-2)} < p \leq 2^{(S-1)}$. According to the interleaved configuration IC , we may express $\{z_k\}$ in terms of m sequences $\{z_j\}$ where $\{z_j\} = \{z_{i-m+j}\}$ with $i \geq 0, m = 2^{(S-1)}$ and $(j = 0, \dots, m-1)$. Since by Theorem 3.2 the sequences $\{z_j\}$ are generated by the same characteristic polynomial $P_D(x)$, we get that the shrunken sequence $\{z_k\}$ is an interleaved sequence of size $2^{(S-1)}$ associated with the polynomial $P_D(x)$. \square

The previous theorem proves that shrunken sequences are interleaved sequences. Moreover, section 4 shows that the knowledge of a number of bits of the shrunken sequence allows us to mount a cryptanalytic attack against the shrinking generator. As many cryptographic sequence generators produce interleaved sequences, then the previous considerations take us into the following conjecture:

Conjecture 5.3. Given a number of bits corresponding to an initial sub-matrix of the interleaved configuration IC of an interleaved sequence, it is possible to obtain the whole interleaved sequence. The confirmation of this conjecture would prove the weakness of interleaved generators for cryptographic purposes.

6 Conclusions

In this work a new cryptanalytic attack against the class of shrinking generators has been proposed. The amount of intercepted bits necessary to realize such an attack is much lesser than that of other standard cryptanalysis. The basic idea consists in defining the shrunken sequence as an interleaved sequence. Hence the weaknesses inherent to interleaved sequences can be advantageously used in the practical attack. A direct consequence of this technique is its generalization to other interleaved sequence generators of cryptographic purpose. In this way, the security of this kind of generators must be carefully checked.

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